

DISCUSSION PAPERS
Institute of Economics
University of Copenhagen

03-33

**Late Informed Betting and
the Favorite-Longshot Bias**

**Marco Ottaviani and
Peter Norman Sørensen**

Stu­di­estræde 6, DK-1455 Copenhagen K., Denmark
Tel. +45 35 32 30 82 - Fax +45 35 32 30 00
<http://www.econ.ku.dk>

Late Informed Betting and the Favorite-Longshot Bias*

Marco Ottaviani[†] Peter Norman Sørensen[‡]

July 2003

Abstract

According to the favorite-longshot bias observed in parimutuel betting, the final distribution of bets overestimates the winning chance of longshots. This paper proposes an explanation of this bias based on late betting by small privately informed bettors. These bettors have an incentive to protect their private information and bet at the last minute, without knowing the bets simultaneously placed by the others. Once the distribution of bets is revealed, if bets are more informative than noisy, all bettors can recognize that the longshot is less likely to win than indicated by the distribution of bets.

*We thank David Goldreich, Denis Gromb, Johnnie Johnson, Tim van Zandt, Bill Zame, Jeff Zwiebel, and seminar participants at the 2003 North American Summer Meetings of the Econometric Society, London Business School, London School of Economics, Tilburg, and Southampton for helpful comments.

[†]Economics Subject Area, London Business School, Sussex Place, Regent's Park, London NW1 4SA, England. Phone: +44-20-7262-5050 ext. 3778. Fax: +44-20-7402-0718. E-mail: mottaviani@london.edu. Web: <http://faculty.london.edu/mottaviani>.

[‡]Institute of Economics, University of Copenhagen, Studiestræde 6, DK-1455 Copenhagen K, Denmark. Phone: +45-3532-3056. Fax: +45-3532-3000. E-mail: peter.sorensen@econ.ku.dk. Web: <http://www.econ.ku.dk/sorensen>.

1. Introduction

In parimutuel betting, a bet placed on an outcome entitles the bettors to an equal share of the total money bet (“pool”) when that outcome is realized, after deductions for tax and expenses (“track take”).¹ Because the holders of winning tickets divide the pool, the payoff odds are determined by the proportion of bets on the different outcomes. The distribution of bets should then reflect the probability of each outcome as assessed by the market.

Starting with Griffith (1949), horse race betting data have been used to test this proposition. Comparing the empirical winning chance with the proportion of money bet on the horses, the market odds have been shown to be highly correlated with the empirical odds. But according to the widely observed favorite-longshot bias, horses with short odds (i.e., favorites) win more even frequently than the odds indicate, while horses with long odds (i.e., longshots) win less frequently. In some cases, even an uninformed bettor who only observes the final bet distribution may be able to profit from placing an extra bet on the favorite. The favorite-longshot bias is often seen as a challenge to traditional economic theory, according to which the market mechanism should produce prices that efficiently incorporate all information (Thaler and Ziemba (1988)).

This paper proposes a new explanation of the puzzle based on simultaneous last minute betting by privately informed bettors. When many (few) players have bet on the same outcome, now the favorite (longshot), each player learns that many (few) other players possessed private information in favor of this outcome, and so they realize that they should have bet even more (less) on this outcome. This is exactly the effect captured by the favorite-longshot bias. If the market closes immediately after the informed bets are placed, the market’s tâtonnement process cannot incorporate this private information and reach a rational expectations equilibrium.

Our explanation is compatible with the observation that the late bets contain a large amount of information about the horses’ finishing order, as documented by Asch, Malkiel and Quandt (1982). We show that the timing incentives depend on the presence of market power and concerns about information revelation.² On the one hand, bettors have an incentive to bet early in order to prevent competitors from unfavorably changing the

¹The main betting methods used in horse-race tracks are fixed odds betting and parimutuel (or pool) betting (cf. Dowie (1976)). In fixed odds betting, bookmakers accept bets at specific, but changing, odds throughout the betting period. This implies that the return to any individual bet is not affected by bets placed subsequently. In parimutuel betting, the return to a bet depends instead on the final total bets placed on the same horse, so that all bettors (but possibly the last one) do not know with certainty the odds. Since its invention in France by Pierre Oller in the second half of the nineteenth century, parimutuel betting has become the most common wagering procedure at major horse-racing tracks throughout the world (but not in the UK, where fixed odds betting attracts the lion’s share of the bets). It is also typically adopted in greyhound tracks, jai alai games, soccer, basketball, and many other games.

²Alternatively, the bets placed at the end could be more informative because more information becomes publicly available on the likely performance of the horses.

odds. On the other hand, waiting is attractive because it allows bettors hide their own private information and possibly glean information from others. When informed bettors are “small”, the second effect overrides the first and bets are simultaneously placed at the closing time.

The insights gained in our analysis of parimutuel markets with private information can be applied to new markets for financial hedging. As explained by Economides and Lange (2001), the parimutuel mechanism is particularly apt for trading contingent claims and has been recently employed in the Iowa Electronic Markets³ and Parimutuel Derivative Call Auction markets.⁴ An advantage of these markets is that the intermediary managing the parimutuel market is not exposed to any risk. On the flip side, market participants are subject to risk on the terms of trade and might have incentive to delay their orders.

Potters and Wit (1996) propose an explanation closely related to ours. Their privately informed bettors are allowed the chance to adjust the bets at the final market odds, but ignore the information contained in the bets.⁵ In their setting, the favorite-longshot bias arises as a deviation from the rational expectations equilibrium. Our bettors instead fully understand the informational connection, but are not allowed to adjust their bets after they observe the final market odds. Feeney and King (2001) and Koessler and Ziegelmeyer (2002) have also recently proposed game theoretic models of parimutuel betting with asymmetric information, focusing mostly on the case of sequential betting with exogenous order. We instead focus on simultaneous betting and offer insights on the forces driving the timing of bets.

A number of alternative theories have been formulated to explain the favorite-longshot bias. First, Griffith (1949) suggested that individuals subjectively ascribe too large probabilities to rare events. Second, Weitzman (1965) and Ali (1977) hypothesized that individual bettors are risk loving, and so are willing to give up a larger expected payoff when assuming a greater risk (longer odds). Third, Isaacs (1953) noted that an informed monopolist bettor would not bet until the marginal bet has zero value. Fourth, Hurley and McDonough (1995) noted a sizeable track take limits the amount of arbitrage by bettors with superior information and so tends to result in relatively too little bets placed on the favorites.⁶ Fifth, Shin (1991) and (1992) explained the favorite-longshot bias in fixed odds betting as a response of uninformed bookmakers to private information possessed by some bettors

³The Iowa Electricity Markets are real-money futures markets in which contract payoffs depend on economic and political events such as elections. See <http://www.biz.uiowa.edu/iem>.

⁴Starting in October 2002, Deutsche Bank and Goldman Sachs have been hosting Parimutuel Derivative Call Auctions of options on economic statistics. See Baron and Lange (2003) for a report on the performance of these markets.

⁵Ali's (1977) Theorem 2 also features privately information bettors who ignore the information of others.

⁶For a more extensive review of these explanations see the survey by Sauer (1998).

(“insiders”).⁷ See Section 6 for a more detailed discussion of the merits and shortcomings of the different theories.

Our findings are illustrated in the simplest setting with two horses. After formulating the model in Section 2, in Section 3 we focus on the simple case of simultaneous betting with a finite number of partially informed players deciding on which horse to place their unit bets when the pool has no pre-existing bets and zero track take. By considering the case with a finite number of players forced to bet simultaneously, we obtain a crisp illustration of how the sign and magnitude of the favorite-longshot bias depends on the informativeness of the signal and the number of players.

We then endogenize the timing, by allowing the players to decide when to publicly place their bets. In general, bets not only affect odds but also possibly reveal information to the other bettors. We analyze these two effects in isolation by considering in turn two versions of the model. Section 4 shows that early betting results when players affect the market odds but are not concerned about revealing information.

In order to isolate the information revelation effect and abstract from the individual bettors’ effect on odds, in Section 5 we then consider a continuum of small informed bettors. The analysis of the dynamic betting game relies on the characterization of the equilibrium in the static simultaneous betting game with positive track take and pre-existing bets. The equilibrium of the dynamic game features an extreme form of delay, with small partially informed bettors placing late bets. When analyzing the last minute betting game with a continuum of players, we also obtain some testable comparative statics predictions of the theory for changes in the amount of pre-existing bets, level of the track take, and mass of informed bettors. We conclude in Section 6 by discussing the predictions of our theory and some avenues for future research. The Appendix collects the more technical proofs.

2. Model

We consider a horse race with $K = 2$ horses. The outcome that horse x wins the race is identified with the *state*, $x \in \{-1, 1\}$.

The players are informed bettors (or *insiders*) of size N , either a finite set $\{1, \dots, N\}$ (in Sections 3 and 4) or a continuous interval $[0, N]$ (in Section 5). All players have a common *prior belief* $q = \Pr(x = 1)$, possibly formed after the observation of a common signal. In addition, each player i is privately endowed with *signal* s_i .⁸ These signals are assumed to be identically and independently distributed conditionally on state x .

⁷Relative to favorites, longshots attract a relatively higher proportion of insiders and pay out more conditional on winning. To counteract this more severe adverse selection problem, posted odds on longshots are relatively shorter. In a regular financial market there would be a higher bid-ask spread (Glosten and Milgrom (1985)).

⁸Private (or inside) information is believed to be pervasive in horse betting. See e.g., Crafts (1985).

Upon observation of signal s , the prior belief q is updated according to Bayes' rule into the *posterior belief* $p = \Pr(x = 1|s)$. The posterior belief p is assumed to be distributed according to the continuous c.d.f. G with p.d.f. g on $[0, 1]$.⁹ By the law of iterated expectations, the prior must satisfy $q = E[p] = \int_0^1 pg(p) dp$. The conditional p.d.f. can be derived from $g(p|x = 1) = pg(p)/q$ and $g(p|x = -1) = (1 - p)g(p)/(1 - q)$ — these relations are necessary, since Bayes' rule yields $p = qg(p|x = 1)/g(p)$ and $1 - p = (1 - q)g(p|x = -1)/g(p)$. Note that $g(p|x = 1)/g(p|x = -1) = (p/(1 - p))((1 - q)/q)$ reflects the property that high beliefs in outcome 1 are more frequent when outcome 1 is true. More strongly, strict monotonicity of the likelihood ratio in p implies that $G(p|x = 1)$ is strictly higher than $G(p|x = -1)$ on the support, in the sense of first order stochastic dominance: the difference $G(p|x = 1) - G(p|x = -1) < 0$ for all p such that $0 < G(p) < 1$.

Some of the results are derived under additional assumptions. The posterior distribution is said to be *symmetric* if $G(p|x = 1) = 1 - G(1 - p|x = -1)$, i.e. the chance of posterior p conditional on state $x = 1$ is equal to the chance of posterior $1 - p$ conditional on state $x = -1$. The signal distribution is said to be *unbounded* if $0 < G(p) < 1$ for all $p \in (0, 1)$.

Our results are well illustrated by the *linear signal* example with conditional p.d.f. $f(s|x = 1) = 2s$ and $f(s|x = -1) = 2(1 - s)$ for $s \in [0, 1]$, with corresponding c.d.f. $F(s|x = 1) = s^2$ and $F(s|x = -1) = 2s - s^2$. This signal structure can be derived from a binary signal with precision distributed uniformly. In this example, the posterior odds ratio is

$$\frac{p}{1 - p} = \frac{q}{1 - q} \frac{f(s|x = 1)}{f(s|x = -1)} = \frac{q}{1 - q} \frac{s}{1 - s}. \quad (2.1)$$

After receiving the signal, the players have the opportunity to bet on $x = -1$ or $x = 1$. The players are assumed to be risk neutral and to maximize the expected value of their winning. We denote by a_x the (possibly zero) amount of exogenously given *pre-existing bets* on state x that have already been placed in advance by unmodeled *outsiders* (or “noise bettors”). The total amount bet by insiders and outsiders is placed in a pool, from which a proportional *track take* τ is taken, before distributing the pool evenly to the winning bets. If no bets were placed on the winning outcome, no payment is made. Let b_{xi} denote the amount bet by player i on outcome x , and let b_x denote the total amount bet by insiders on outcome x . If x is the winner, every unit bet on outcome x receives the payout

$$(1 - \tau) \frac{a_x + b_x + a_{-x} + b_{-x}}{a_x + b_x}. \quad (2.2)$$

⁹In the presence of discontinuities in the posterior belief distribution, the only symmetric equilibria might involve mixed strategies. Our results can be extended to allow for these discontinuities.

3. Favorite-Longshot Bias

The goal of this section is to provide the simplest setting to illustrate how the favorite-longshot bias may arise from informed betting and how its sign depends on the interplay of noise and information. In this model there is a finite number N of informed players forced to simultaneously submit exactly one bet each, there is no prior betting ($a_x = 0$), and there is no track take ($\tau = 0$).¹⁰ These additional assumptions allow us to characterize the equilibrium and study how it is affected by changes in the informativeness of the signal and the number of players (Section 3.1). We then show how the sign and magnitude of the favorite longshot bias depend on the interplay of noise and information (Section 3.2).

3.1. Equilibrium Characterization

Consider a rational bettor with posterior p . The expected payoff of a bet on outcome $y \in \{-1, 1\}$ is $U(y|p) = pW(y|x=1) + (1-p)W(y|x=-1) - 1$ where $W(y|x)$ is the expected payoff of a bet on outcome y conditional on state x being realized. Note that $U(1|p) = pW(1|x=1) - 1$ since $W(y|x=-y) = 0$, as a bet on outcome y pays out nothing in state $-y$. The conditional winning payoff is random, because of the randomness of the others' signals and corresponding bets.

The best reply of each individual bettor has the following cutoff characterization. There exists a threshold posterior belief $p^* \in [0, 1]$ such that for $p < p^*$ it is optimal to bet on $x = -1$, and for $p > p^*$ it is optimal to bet on $x = 1$. Clearly $\partial U(1|p) / \partial p = W(1|x=1) > 0$. As the best response must be a cutoff strategy, we restrict attention without loss of generality to cutoff strategies and characterize the symmetric Bayesian Nash equilibrium:

Proposition 1 *With $N \geq 1$ insiders, there exists a unique symmetric equilibrium, in which all bettors with $p > \hat{p}_N$ bet for horse 1, and bettors with $p < \hat{p}_N$ bet for horse -1 , where \hat{p}_N is the solution to*

$$\frac{p}{1-p} = \frac{1-G(p|x=1)}{G(p|x=-1)} \frac{1-(1-G(p|x=-1))^N}{1-(G(p|x=1))^N}. \quad (3.1)$$

As N tends to infinity, \hat{p}_N tends to the unique solution to

$$\frac{p}{1-p} = \frac{1-G(p|x=1)}{G(p|x=-1)}. \quad (3.2)$$

Proof. See the Appendix. □

¹⁰If players were not forced to bet, the logic of no trade theorem implies that there would be no betting in equilibrium. In Section 5's model the presence of positive outside bets allows the insiders to make positive expected return when betting.

With a large number of players, there is no uncertainty on the conditional distribution of the opponents. In order to directly derive the symmetric equilibrium in the limit, consider the expected payoff achieved by a bettor with posterior p who bets on outcome 1. If winning, the bettor share the pool with all those who also picked 1. Since all bettors use the same cutoff strategy \hat{p} , the fraction of bettors who picked 1 in state $x = 1$ is $1 - G(\hat{p}|x = 1)$. The expected payoff from a bet on outcome 1 is then $p / (1 - G(\hat{p}|x = 1))$. Similarly, the expected payoff from -1 is $(1 - p) / G(\hat{p}|x = -1)$. The payoff of an indifferent bettor satisfies equation (3.2).

3.2. Bias

The symmetric equilibrium strategy with N insiders has cutoff posterior belief \hat{p}_N . The resulting binomial distribution of bets is easily derived. For any $k = 0, \dots, N$,

$$\Pr(k \text{ bet } 1 \text{ and } N - k \text{ bet } -1|x) = \binom{N}{k} (1 - G(\hat{p}_N|x))^k G(\hat{p}_N|x)^{N-k}.$$

Since $G(\hat{p}_N|x = -1) > G(\hat{p}_N|x = 1)$, the distribution of bets is higher when $x = 1$ than when $x = -1$ in the first order stochastic dominance order. This is a simple implication of the fact that higher private beliefs are more frequent when the true outcome is higher, resulting in more bets by the insiders on the higher outcome.

The *market odds ratio* for outcome 1, when k have bet on outcome 1, is $\rho = (N - k) / k$. A bet on outcome 1 pays out $1 + \rho$ when winning — this parimutuel rule automatically balances the book since the total payback equates the total amount bet, $k(1 + \rho) = N$. The *implied market probability* for outcome 1 is k/N , equal to the fraction of money bet on the corresponding horse. A Bayes-rational observer of the final bets distribution would instead update to the *Bayesian odds ratio*,

$$\frac{1 - q \Pr(\text{exactly } k \text{ bet } 1|x = -1)}{q \Pr(\text{exactly } k \text{ bet } 1|x = 1)} = \frac{1 - q}{q} \left(\frac{1 - G(\hat{p}_N|x = -1)}{1 - G(\hat{p}_N|x = 1)} \right)^k \left(\frac{G(\hat{p}_N|x = -1)}{G(\hat{p}_N|x = 1)} \right)^{N-k}.$$

In general, this Bayesian odds ratio is different from the market odds ratio.

As the final distribution of market bets is not perfectly known when the bets are placed, the market odds are not good estimators of the *empirical odds*, estimated from the race outcomes. The Bayesian odds instead incorporate the information revealed in the betting distribution and adjust for noise, and so are better estimators of the empirical odds compared to the market odds. If our model is correctly specified, the Bayesian odds should be equal on average to the empirical odds. The favorite-longshot bias identified in the data suggests that the difference between the market odds ratio and our Bayesian odds ratio is systematic: when the market odds ratio ρ is large (“long”), it is smaller than the corresponding Bayesian odds ratio. Thus, a longshot is less likely to win than suggested by the market odds.

Our structural model allows us to uncover a systematic relation between Bayesian and market odds depending on the interplay between the amount of noise and information contained in the bettors' signal. To appreciate the role played by *noise*, note that market odds can range from zero to infinity, depending on the realization of the signals. For example, if most bettors happen to draw a low signal, the market odds of outcome 1 will be very long. If the signals contain little information, the Bayesian odds are close to the prior odds even if the market odds are extreme. In this case, the deviation of the market odds from the prior odds are largely due to the randomness contained in the signal, so that the reverse of the favorite longshot bias is present (i.e., the market odds are more extreme than the posterior odds).

As the number of bettors increases, the realized market odds contain more and more *information*, so that the posterior odds are more and more extreme for any market odds different from 1. We can therefore establish:

Proposition 2 *Let $\rho^* > 0$ be defined by*

$$\rho^* = -\frac{\log\left(\frac{1-G(\hat{p}|x=-1)}{1-G(\hat{p}|x=1)}\right)}{\log\left(\frac{G(\hat{p}|x=-1)}{G(\hat{p}|x=1)}\right)} \quad (3.3)$$

where \hat{p} is the unique solution to the limit equilibrium condition (3.2). Take as given any market odds ratio $\rho \in (0, \infty)$. As the number of insiders tends to infinity, ρ is strictly smaller (resp. greater) than the associated Bayesian odds ratio if and only if $\rho > \rho^*$ (resp. $\rho < \rho^*$).

Proof. Let $\rho > \rho^*$ be given. The desired inequality is

$$\rho = \frac{N-k}{k} < \frac{1-q}{q} \left(\frac{1-G(\hat{p}_N|x=-1)}{1-G(\hat{p}_N|x=1)} \right)^k \left(\frac{G(\hat{p}_N|x=-1)}{G(\hat{p}_N|x=1)} \right)^{N-k}.$$

Taking the natural logarithm and re-arranging, we arrive at the inequality

$$\frac{1}{N} \log(\rho) - \frac{1}{N} \log\left(\frac{1-q}{q}\right) < \frac{1}{1+\rho} \log\left(\frac{1-G(\hat{p}_N|x=-1)}{1-G(\hat{p}_N|x=1)}\right) + \frac{\rho}{1+\rho} \log\left(\frac{G(\hat{p}_N|x=-1)}{G(\hat{p}_N|x=1)}\right).$$

The left hand side tends to zero as N tends to infinity. The right hand side tends to a positive limit, precisely since $\rho > \rho^*$. \square

For long market odds ρ , the market odds are shorter than the Bayesian odds, and vice versa, in accordance with the favorite longshot bias. The turning point ρ^* is a function of how much more informative is the observation that the private belief exceeds \hat{p} than the observation that it falls short of \hat{p} . The observation of ρ^* insiders with beliefs below

\hat{p} exactly offset the observation of one insider with beliefs above \hat{p} , as can be seen from expression (3.3).

In Proposition 2 the realized market odds ratio $\rho \in (0, \infty)$ is held constant as the number of players N tends to infinity. Since the probability distribution of ρ is affected by changes in N , it is natural to wonder whether this probability distribution of realized market odds can become so extreme that it is irrelevant to look at fixed non-extreme realizations. This is not the case, because in the limit as N goes to infinity the fraction of bets on outcome 0 is positive and equal to $G(\hat{p}|x)$, while the remaining fraction $1 - G(\hat{p}|x)$ is betting on outcome 1. By the strong law of large numbers, the noise vanishes and the market odds ratio ρ tends almost surely to the limit ratio $G(\hat{p}|x) / (1 - G(\hat{p}|x))$ in state x as the number of informed bettors increase. The observation of the bet distribution eventually reveals the true outcome, so that the Bayesian odds ratio becomes more extreme (either diverging to infinity or converging to zero) as N tends to infinity. It then becomes more likely that the realized market odds are less extreme than the posterior odds. This fact supports the favorite longshot bias as the theoretical prediction of our simple model.

In the special case with symmetric prior $q = 1/2$ and symmetric signal distribution (implying that $G(1/2|x = 1) = 1 - G(1/2|x = -1)$), the symmetric equilibrium has $\hat{p}_N = \hat{p} = 1/2$ for all N . The turning point is then $\rho^* = 1$. In this simplified case, we can further illuminate the fact that the favorite longshot bias arises when the realized bets contain more information than noise.

Proposition 3 *Assume the prior belief is $q = 1/2$ and that the signal distribution is symmetric. Take as given any market odds ratio $\rho \in (0, \infty)$. If the signal informativeness $G(1/2|x = -1) / G(1/2|x = 1)$ is sufficiently large or the number of bettors N is large enough, ρ is strictly smaller (resp. greater) than the associated Bayesian odds ratio if and only if $\rho > 1$ (resp. $\rho < 1$).*

Proof. Let $\rho > 1$ be given. The desired inequality is

$$\rho = \frac{N - k}{k} < \left(\frac{1 - G(1/2|x = -1)}{1 - G(1/2|x = 1)} \right)^k \left(\frac{G(1/2|x = -1)}{G(1/2|x = 1)} \right)^{N-k} = \left(\frac{G(1/2|x = -1)}{G(1/2|x = 1)} \right)^{N-2k}.$$

Taking the natural logarithm and re-arranging, we arrive at the inequality

$$\frac{\rho + 1}{\rho - 1} \log(\rho) < N \log \left(\frac{G(1/2|x = -1)}{G(1/2|x = 1)} \right). \quad (3.4)$$

Since $\rho > 1$ and $G(1/2|x = -1) > G(1/2|x = 1)$, all terms are positive. The right hand side of (3.4) tends to infinity when the informativeness ratio $G(1/2|x = -1) / G(1/2|x = 1)$ or the number of bettors N tend to infinity. \square

As illustrated by the key inequality (3.4), the favorite longshot bias arises when bettors are many (large N) or well informed (large $G(1/2|x = -1)/G(1/2|x = 1)$). Since the left-hand side of (3.4) is a strictly increasing function of ρ , it is harder to satisfy the inequality for longer market odds. This is natural, since the insiders must reveal more information through their bets in order for the Bayesian odds to become very long.

It is worth remarking that our result also holds when the insiders' information contains a common error.¹¹ To illustrate this point, modify the model so that the true outcome is z , while our previously used x is a binary signal of z . The private signal is only informative about x , but conditionally on x , its distribution is independent of z . We show that in the limit with infinite N , the symmetric equilibrium features the favorite-longshot bias:

Proposition 4 *Assume that G is symmetric and that the state x is a symmetric binary signal of the outcome z of the race, with $\Pr(x = 1|z = 1) = \Pr(x = -1|z = -1) \equiv \pi > 1/2$. The Bayesian odds ratio is more extreme than the market odds ratio associated to the symmetric equilibrium of the limit game with an infinite number of players.*

Proof. See the Appendix. □

Illustration. We now illustrate our findings in the linear signal example. Through the monotonic translation of signals into posterior beliefs in (2.1), the cutoff posterior belief defining the equilibrium corresponds to a cutoff private signal \hat{s}_N . The equilibrium condition (3.1) in terms of the cutoff private signal is

$$\frac{q}{1-q} = P(\hat{s}_N, N) \equiv \frac{1 - \hat{s}_N}{\hat{s}_N} \frac{1 - F(\hat{s}_N|x = 1)}{F(\hat{s}_N|x = -1)} \frac{1 - (1 - F(\hat{s}_N|x = -1))^N}{1 - (F(\hat{s}_N|x = 1))^N}.$$

Figure 3.1 plots the equilibrium signal cutoff $\hat{s}_N(q)$ for different values of N . A single bettor ($N = 1$) optimally bets on the horse that is more likely to win according to the posterior belief. Note that for $q < 1/2$, equilibrium betting is biased in favor of the “ex-ante longshot” $x = 1$. This is due to the fact that a player wins when $y = x$. But according to the logic of the winner’s curse, conditionally on x the opponents receive information in favor of x , and so they are more likely to bet on x . There is thus a positive correlation of the true state and the number of bettors on it. This creates an incentive to bet on the longshot, which tends to receive fewer bets. Observe that full rationality thus works to reduce the favorite-longshot bias which would arise from the non-strategic betting rule $\hat{s} = 1 - q$. This adds to the strength of Proposition 2 which derived the favorite longshot bias under the assumption of full rationality.

¹¹In most situations there is often a common element of uncertainty. For instance, in horse races, the insider information might show that one horse is in far better condition than publicly assessed, yet the actual race contains an unforeseeable element of randomness, implying that this particular horse does not necessarily win.

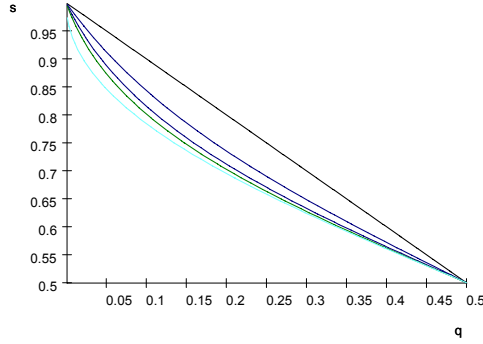


Figure 3.1: The equilibrium cutoff signal \hat{s}_N in the linear signal example is plotted against the prior belief $q \in [0, 1/2]$ for $N = 1, 2, 3, 4, 100000$, in progressively thinner shade. The downward sloping diagonal ($\hat{s}_1 = 1 - q$) corresponds to the optimal rule ($\hat{p}_1 = 1/2$) for a single bettor. As the number of players increases, the cutoff signal decreases and converges to the limit $\hat{s}(q)$.

To further illustrate the favorite longshot bias, Figure 3.2 plots how the expected payoff of a bet on outcome 1 varies with the market odds in the symmetric case with $q = 1/2$. As $F(1/2|x = 1) = 1/4$ and $F(1/2|x = -1) = 3/4$, the Bayesian odds ratio reduces to 3^{N-2k} . The implied Bayesian probability is $3^{2k}/(3^N + 3^{2k})$. Since k have bet 1, the expected return from an extra bet on outcome 1 is $((N + 1)/(k + 1)) 3^{2k}/(3^N + 3^{2k})$. Notice the similarity of the curve generated in this stylized example with Thaler and Ziemba's (1988) Figure 1, plotting the empirical expected return for horses with different market odds. When the market assigns long odds (a small probability) to an outcome (say, $x = 1$), that outcome wins less frequently than indicated by the market odds.

4. Early Betting with Market Power

So far we have analyzed a game of simultaneous betting. We now turn to a dynamic setting and investigate the factors conducive to simultaneous equilibrium play. We continue with the model set up in Section 2 allowing for non-negative track take τ and prior bets a_x . Assume that time is discrete and that betting is open in a commonly known finite window of time, with periods denoted by $t = 1, \dots, T$.¹² Assume also that the total amount of bets placed by the outsiders, a_{-1} and a_1 , are deterministic, unaffected by the amounts bet by the insiders, and commonly known. Following Hamilton and Slutsky's (1990) "extended

¹²The assumption of discrete time is made for technical convenience, but seems inessential. Typically, betting is open for a period before the beginning of the race and the bet distribution (and corresponding provisional odds) are displayed at regular intervals. For example, the UK's Tote updates the display every thirty seconds.

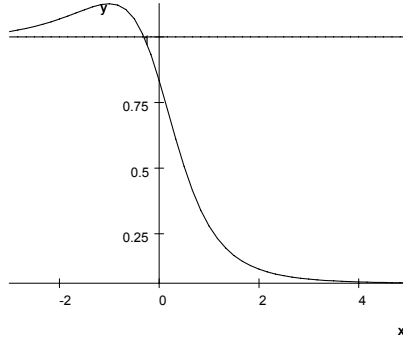


Figure 3.2: The expected return of a bet on outcome 1 is plotted against the natural logarithm of the market odds ratio in the the linear signal example with prior $q = 1/2$ and $N = 4$ players.

game with action commitment”, players decide when and how to bet, with the assumption that players who bet late can observe the firm bets placed in earlier periods.

The timing of bets is affected by the interplay of two opposing forces. First, players want to bet early, in order to capture a good market share of profitable bets, as in a Cournot oligopoly game. Second, players want to bet late, in order not to reveal their private information to the other bettors and maybe observe others.¹³ In the present Section 4 we focus on the first incentive by considering a model with a finite number of large bettors who share the same information and are able to influence odds. In this first case, we show that in equilibrium all (but at most one) bettors place their bets early. In the next Section 5 we isolate the second incentive by considering a continuum of small bettors. In that second case we show that in equilibrium informed bets are placed late.

To study the effect of market power, assume that insiders can place bets of arbitrary non-negative size. A player who can make a sizable bet faces an adverse movement in the odds, and should consider this effect when deciding how much to bet. This market power channel introduces an incentive to bet early, before other players place their bet to one’s detriment.

Following Hurley and McDonough (1995), assume that there is no private information, in that the N rational bettors share the same information about the state. In our setup of Section 2, this is the degenerate case with no (or completely uninformative) private signal, so that $p = q = \Pr(x = 1)$ for sure. The amounts bet by other bettors cannot then reveal any information. If the common prior belief, the track take and prior bets are such that $q(1 - \tau) > a_1 / (a_1 + a_{-1})$, the prior bets are so favorable that it is profitable to place bets

¹³This effect is also present in an open auction with fixed deadline. See also the discussion in Roth and Ockenfels (2001) and Medrano and Vives (2001).

on outcome 1, but it is unprofitable to bet on outcome -1 . If bettors $i = 1, \dots, N$ place the amounts b_1, \dots, b_N on outcome 1, player i 's payoff is

$$U_i(b_i) = q(1 - \tau) \frac{a_1 + a_{-1} + \sum_{i=1}^N b_i}{a_1 + \sum_{i=1}^N b_i} b_i - b_i.$$

The model allows a direct mapping into Cournot model of imperfect competition. To see this, interpret the amount b_i as the quantity produced at constant marginal unit cost by firm i . The market for the output has the inverse demand curve

$$p(b) = q(1 - \tau) \frac{a_1 + a_{-1} + b}{a_1 + b}, \quad (4.1)$$

where b is the aggregate quantity produced. Bettors suffer inframarginal losses from increasing their own bets and so do not bet until the marginal price of an extra unit equates the marginal cost. In simultaneous equilibrium, the market's subjective probability $(a_1 + b) / (a_1 + a_{-1} + b)$ for outcome 1 is lower than the profit-eliminating $q(1 - \tau)$, for the usual reason that demand is above marginal cost ($p(b) > 1$).

To derive the equilibrium timing, we can appeal to a result by Matsumura (1999):

Proposition 5 *With N informed bettors, there are two subgame perfect equilibria. In the first, all bettors place early bets. In the second equilibrium, all but one bettor place early bets.*

Proof. The following three conditions about the two-stage betting game with exogenous timing and arbitrary pre-existing bets can be verified to be satisfied (details available on request):

1. In any two-stage game with exogenous sequencing, there exists a pure strategy equilibrium and the equilibrium is unique.
2. If the number of followers is one, this follower strictly prefers the Cournot outcome to the follower's outcome.
3. If the number of leaders is one, this leader strictly prefers the leader's outcome to the Cournot outcome.

This game then verifies the three assumptions of Matsumura's (1999) Proposition 3, proving the result. \square

It follows that almost all bettors move simultaneously at the earliest possible instance, e.g. after they have received their public information.¹⁴ Market power gives an incentive

¹⁴Following Matsumura, players do not observe their simultaneous opponents before placing bets. If we relax this assumption, then the lone follower would deviate to produce simultaneously with all the others.

to move early, to capture a good share of the money on the table. This prediction is at odds with Asch, Malkiel and Quandt's (1982) observation of late informed betting. As shown in the next section, individual bettors have an incentive to bet late if they instead have private information about the money on the table.

5. Late Betting with Private Information

In this section we isolate the incentive to bet late for informational reasons by deliberately removing market power. We show that there is no advantage to betting early for individuals who can only bet a small fixed amount of money and so have no inframarginal bets. As bettors are only marginal, the final payout will not depend on a given player's bet. Thus the bet will not directly affect the attractiveness of the bets to other players. However, an early bet has the potential to send a signal to other players about one's private information. But it is unattractive to send such a signal, since other players will tend to follow the signal and erode the value of one's private information. Moreover, delaying a marginal bet gives an informational advantage if one observes the others' bets placed in the meantime.

In order to completely remove the players' market power, from now on we assume that the insider population is a continuum of size N and that each player can place a size-one bet at most once.¹⁵ Equivalently, the insiders can only place a relatively small bet due to liquidity constraints. With a continuum of players, the actual distribution of beliefs in the population equals the probability distribution G . Compared to Section 3, we now allow the insiders the option to abstain from betting. While players have discretion over whether to bet or not, the smallness of each individual allows us to consider the effect of private information in isolation, without consideration for market power.

A behavior strategy specifies after each publicly observed history and privately observed signal whether to bet now on horse 1, bet now on horse -1 , or abstain in this period. A perfect Bayesian equilibrium specifies a behavior strategy for each player, such that every player's strategy is optimal given the other players' strategies. Perfection requires that the continuation strategy should be optimal after any publicly observed history, given rational beliefs. We analyze this timing game using backwards induction, and therefore begin by considering the last period.

5.1. Informed Betting in the Last Stage

This section focuses on last-minute simultaneous betting by a continuum of small informed bettors. By allowing for a positive track take, the option that rational informed bettors

¹⁵Issues of market manipulation cannot arise in this setting with unit irrevocable bets. See also Camerer (1998) on the limited effectiveness of market manipulation in parimutuel markets.

withhold from betting, and the presence of pre-existing bets, the rational bettors make non-negative expected returns when betting. We can then develop testable comparative statics predictions of the theory. For the purpose of our dynamic analysis, we are particularly interested in the observation that the higher is the prior belief in outcome x , the shorter are the equilibrium market odds for outcome x .

The distribution G of private beliefs now refers to the distribution in the sub-population that has not bet before the last period. The information contained in the publicly observed history of the game is applied to the prior belief of the previous period, resulting in a publicly updated new prior belief q common to all players. The distribution F of signals in the remaining population is simply the re-normalized version of the original distribution truncated with the early bettors. The belief distribution G is then re-derived as in Section 2. We will eventually show that this distribution F is the same as the original distribution. A possible change in history which affects the prior q therefore has a predictable effect on G . If q is increased, all signals are updated to higher beliefs than before, and so G is more favorable.

In simultaneous Bayesian Nash equilibrium, every bettor correctly predicts the fraction of the informed who bet on each outcome in each state. Denote by b_y^x the amount bet by the insiders on outcome y when state x is true. If state x is true, the payout to bets on outcome x is $W(x|x) = (1 - \tau)(a_1 + a_{-1} + b_1^x + b_{-1}^x) / (a_1 + b_1^x) > 0$. Observe directly that $1/W(1|1) + 1/W(-1|-1) = 1/(1 - \tau)$ — the fractions of the betted money paid to winners sum to the total fraction of money paid out.

Consider now the decision problem of a player with belief p . The expected payoff from betting on outcome 1 is $U(1|p) = pW(1|1) - 1$. On outcome -1 the expected payoff is $U(-1|p) = (1 - p)W(-1|-1) - 1$. The payoff from not betting is $U(0|p) = 0$. Immediately, we observe that $U(1|p) - U(0|p)$ and $U(0|p) - U(-1|p)$ are increasing in p . The best response is therefore a cutoff policy. There exists some $\hat{p}_{-1}, \hat{p}_1 \in [0, 1]$ such that for $p < \hat{p}_{-1}$ it is optimal to bet on $x = -1$, and for $p > \hat{p}_1$ it is optimal to bet on $x = 1$.

If interior to the belief distribution, the two cutoff values can be determined from the indifference conditions $0 = \hat{p}_1 W(1|1) - 1$ and $0 = (1 - \hat{p}_{-1}) W(-1|-1) - 1$. Since $W(x|x) > 0$ we immediately find that $\hat{p}_1 > 0$ and $\hat{p}_{-1} < 1$. When $\tau > 0$, the identity $1/W(1|1) + 1/W(-1|-1) = 1/(1 - \tau)$ implies $\hat{p}_{-1} < \hat{p}_1$ — a positive track take thus implies that some players refrain from betting.

Under some circumstances, there will be no betting on a given outcome in equilibrium. For instance, when the prior bets heavily favor outcome 1 or the track take is very large (i.e., $(1 - \tau)(a_1 + a_{-1} + N) \leq a_1$), we have $W(1|1) < 1$, so that in equilibrium no bets are placed on outcome 1. Likewise, when $(1 - \tau)(a_1 + a_{-1} + N) \leq a_{-1}$ there is no betting on outcome -1 . We now provide sufficient conditions for the existence of an equilibrium with positive betting on both outcomes:

Proposition 6 *Assume that the belief distribution is unbounded and that*

$$0 < \tau < \min \left\{ \frac{a_1}{a_1 + a_{-1}}, \frac{a_{-1}}{a_1 + a_{-1}} \right\}.$$

There exists a unique symmetric Nash equilibrium in which all players use interior thresholds $0 < \hat{p}_{-1} < \hat{p}_1 < 1$.

Proof. See the Appendix. □

With interior thresholds, each individual bets more frequently on outcome y when y is true: $b_1^1 > b_1^{-1}$ and $b_{-1}^{-1} > b_{-1}^1$. Given the presence of a continuum of insiders, there is no noise in the betting, resulting in an extreme version of the favorite longshot bias. Thus, the amounts bet by the insiders fully reveal the true outcome. The favorite longshot bias derived in Section 3 then carries over in the presence of prior bets and a positive track take.

If $\tau > a_1 / (a_1 + a_{-1})$, the prior bets have given market odds so unbalanced, that even a bettor who knows that outcome -1 will happen for sure is unwilling to bet on outcome -1 . In this sense, prior bets are such that there is no arbitrage opportunity. It is possible, however, that rational bets placed on outcome 1 would be sufficient to make betting on outcome -1 attractive — the proposition did not address the case $a_1 / (a_1 + a_{-1}) < \tau < (a_1 + N) / (a_1 + a_{-1} + N)$. Even when $a_1 / (a_1 + a_{-1}) < \tau$, if no bettor received private beliefs near $p = 0$, there might not be any one willing to bet on outcome -1 . To obtain an interior equilibrium with positive betting on both outcomes, the proposition assumes that the market odds are balanced relative to the track take, and that some bettors receive arbitrarily strong signals in favor of either outcome.

Our first comparative statics result is an intuitive property that is very helpful for the dynamic analysis. If a player makes an early bet on outcome 1 in equilibrium, this will signal to the market that he has private information in favor of this outcome. This favorable signal implies that the later players have a higher prior belief q than if the signal had not been sent. Intuitively, they are then more inclined to bet on outcome 1, driving down the payout $W(1|1)$. The signal therefore tends to reduce the profit to the signaling player. For now, we characterize the effect on the outcome of the last stage of the game:

Proposition 7 *Assume that the belief distribution is unbounded and that*

$$0 < \tau < \min \left\{ \frac{a_1}{a_1 + a_{-1}}, \frac{a_{-1}}{a_1 + a_{-1}} \right\}.$$

A marginal increase in q implies that \hat{p}_1 and \hat{p}_{-1} both weakly increase, that $W(1|1)$ weakly decreases, and that $W(-1|-1)$ weakly increases.

Proof. See the Appendix. □

In a symmetric setting, we can derive more detailed comparative statics results:

Proposition 8 *Assume that the distribution of private beliefs is unbounded and symmetric. Assume symmetry of the prior bets, $a_1 = a_{-1} \equiv a > 0$, and that $0 < \tau < 1/2$. There exists a unique symmetric-policy Nash equilibrium, where bettors with beliefs exceeding the threshold $\hat{p}_1 \in (1/2, 1)$ bet on outcome 1, bettors with beliefs below $1 - \hat{p}_1$ bet on outcome -1 , and all other bettors place no bet. The threshold \hat{p}_1 is a function of τ and a/N , increasing in τ and decreasing in a/N . A decrease in a/N , or a decrease in τ , both imply more extreme market odds and result in a reduced favorite-longshot bias.*

Proof. See the Appendix. □

Intuitively, a greater track take makes rational betting less attractive, so that only bettors with higher beliefs in an outcome find it attractive. With a smaller amount of uninformed betting, or a greater total population of rational players, the bets placed by the extreme-signal informed players have a greater impact on the market odds, thereby making informed betting less attractive for individuals with a given signal. Although the increase in N or decrease in a reduce the fraction $2(1 - \hat{p}_1)$ of active bettors, the informed population gains size relative to the prior bets, and so their bets have a larger influence on the market odds. For very large values of N , the market probability for outcome 1 tends only to $1/(1 - \tau)$ — the positive track take prevents the informed population from fully correcting the odds.

The symmetric setting has the appealing property that the initial market belief in outcome 1, $a_1/(a_1 + a_{-1})$ equals the prior belief $q = 1/2$. A priori, then, the market odds are correct, and there is no scope for betting on the basis of public information alone. Nevertheless, privately informed individuals can profit from betting. In the symmetric model we have $b_{-1}^1 = b_1^{-1} < b_1^1 = b_{-1}^{-1}$, so the market probability satisfies $(a + b_1^1)/(a + a + b_1^1 + b_{-1}^1) > 1/2 > (a + b_1^{-1})/(a + a + b_1^{-1} + b_{-1}^{-1})$. The favorite-longshot bias is clear: when the market's implied probability of an outcome exceeds $1/2$, but remains well below 1, the Bayesian (and empirical) probability of the outcome is 1.

Illustration. In the linear signal example with fair prior ($q = 1/2$), balanced pre-existing bets ($a_1 = a_{-1} = a$), and track take $\tau \leq 1/2$, the unique symmetric-policy Nash equilibrium has a simple explicit expression, with cutoff signal

$$\hat{s}_1 = \frac{(1 - \tau)(1 + a/N) - \sqrt{(1 + a/N)(\tau^2 + (1 - \tau)^2 a/N)}}{(1 - 2\tau)} \in [1/2, 1).$$

5.2. Timing

Having analyzed the last period play, we can finally argue in favor of late informed betting.

Proposition 9 *Assume that the belief distribution is unbounded and that*

$$0 < \tau < \min \left\{ \frac{a_1}{a_1 + a_{-1} + N}, \frac{a_{-1}}{a_1 + a_{-1} + N} \right\}.$$

Postponing all betting to the last period is a perfect Bayesian equilibrium.

Proof. The following strategy profile constitutes such a perfect Bayesian equilibrium. After any history, all remaining players postpone their betting to the last period and play then a simultaneous Bayesian Nash equilibrium with the updated belief distribution G . If some players have already moved, they are removed from the distribution of the private signal F , and if the early-movers signalled some information then the public prior is updated and G changed accordingly.

To prove that this is an equilibrium, consider any public history at time $t < T$. In one case, the resulting belief distribution is no longer thought unbounded. In this case, the true state must have been revealed when the extreme-belief players were moving. Then, all remaining players will in the final period bet on the winning outcome, either until there are no more players, or until its return is driven to zero. Every player is therefore indifferent to betting now or later, and might as well postpone.

In the other case, the belief distribution is still unbounded. The bets c_x of players who already bet will in the final period be treated as pre-existing, but notice that the condition $\tau < a_x / (a_x + a_{-x} + N) \leq (a_x + c_x) / (a_x + a_{-x} + c_x + c_{-x})$ is still satisfied in the last period. Suppose that some player considers a deviation to bet now, without loss of generality on outcome 1. Since outcome 1 is more attractive the higher is the private belief, the off-path belief of all other players is to update q to a higher value. Since the player is marginal, the same continuum of players as otherwise is still present, and according to their equilibrium strategy they proceed to the simultaneous game at time T . By Proposition 7, the higher continuation belief implies a weak decrease in the payout $W(1|1)$ for bets on outcome 1. But this player is then at least as well off delaying the bet on outcome 1, thereby earning the non-decreased payout in state 1. It is then optimal for this player to postpone. \square

6. Conclusion

We have proposed a novel explanation of both the favorite longshot bias and the timing of informative bets, based on a simple model with initial bets from “noise” bettors and

late bets from small privately informed profit maximizing bettors. As a by-product of our analysis, we have developed some tractable models of simultaneous betting in parimutuel markets with private information.

The first insight gained from our analysis is that the market odds are typically different from the empirical odds if bettors place bets without knowing the final distribution of market bets. The sign and extent of the favorite longshot bias depends on the interaction of noise and information. In the presence of little private information, posterior odds are close to prior odds, even with extreme market odds, so that deviations of market odds from prior odds are mostly due to the noise contained in the signal. In this case, the market odds tend to be more extreme than the posterior odds, resulting in a reversed favorite-longshot bias. As the number of bettors increases, the realized market odds contain more information and less noise. For any fixed market odds, the posterior odds are then more extreme, resulting in increased favorite-longshot bias. The favorite longshot bias always arises with a large number of bettors, provided that they have some private information.

Using the market odds to evaluate the rationality of the bettors is equivalent to assuming too much information on their side. It is tantamount to requiring that bettors know the final distribution of bets, which they do not with simultaneous betting. If betting were allowed to unexpectedly reopen, additional bets would be placed to rebalance the market odds toward the posterior odds, eliminating the puzzle.

In order to test this first prediction of our theory, one could exploit the existing variation across betting environments. The presence and degree of private information tend to vary consistently depending on the nature of the underlying sport or prominence of the event. For example, there are probably more punters with inside information about the outcome of horse races rather than football matches. The amount of noise present depends on the number of punters, as well as on the observability of the bets previously posted. Our model predicts a reverse favorite-longshot bias if bettors have little or no private information and cannot observe the bets placed by others (e.g., in lotto games).¹⁶

Similarly to Isaacs' (1953) market power explanation and Hurley and McDonough's (1995) limited arbitrage explanation, our informational resolution of the favorite longshot bias is specific to the parimutuel market structure. These three theories do not apply to fixed odd betting, in which the bias is also often observed, and so complement other explanations proposed for the bias in fixed odds betting markets. Sharing with Shin (1992) the assumption of privately informed bettors, we provide a parimutuel counterpart of his adverse selection explanation. The behavioral and risk loving explanations instead predict the presence of the bias regardless of the market structure, but cannot account for the varying extent of the bias in different countries and are mute on the timing issues.

¹⁶The preponderance of noise might account for some of Metrick's (1996) findings in NCAA betting.

Secondly, this paper has contributed to the analysis of endogenous timing. The incentive of informed traders to postpone their trades to the last minute is driven by the fact that in parimutuel betting all trades are executed at the same final price. In fixed odds betting and normal financial markets (e.g., as modeled in Kyle's (1985) continuous auction model), competition among informed traders drive them to trade as early as possible, thereby revealing their information early.¹⁷ If at all needed, subsequent arbitrage trading then eliminates the favorite-longshot bias.

We have identified (in Section 5) a scenario with many small bettors in which *all* informed bets are placed at the end of the betting period, but for our insight to apply it is enough that *some* informed bets are placed simultaneously at the end. We have also pointed out (in Section 4) that large bettors have a tempering incentive to place early bet, especially if they are not concerned about the information revealed to others. The analysis of the interplay of these two incentives is an interesting topic for future research.

Our theoretical findings seem compatible with experimental results recently obtained by Plott, Wit and Yang (2003) in laboratory parimutuel markets. Their experimental subjects were endowed with limited monetary budget and given private signals informative about the likelihood of the different outcomes. Subjects could place bets up to their budget before the random termination of the markets. Compared to our model, the presence of a random termination time gives bettors an additional incentive to move early in order to reduce the termination risk. Although the experimental subjects were explained Bayes' rule, not all profitable bets were made and some favorite-longshot bias was observed. According to the logic of our theory, the market odds were not equalized to the posterior odds because some of the informed bettors were possibly postponing the placement of their limited budget, gambling on the termination to happen later.

Persistent cross-country differences in the observed biases could be attributed to varying degrees of market participation and informational asymmetry, patterns in the coexistence of parallel (fixed odd and parimutuel) betting schemes, and degrees of randomness in the closing time in parimutuel markets. As also suggested by Gabriel and Marsden (1990) and Bruce and Johnson (2000), bettors might have different incentives to place their bets on the parimutuel system rather than with the bookmakers depending on the quality of their information. The consistently different extent of favorite longshot bias depending on the market rules observed in the UK points to their relevance in determining the behavior of market participants on the supply and demand side.

The incentives to reveal information depend on the market structure and might explain the long-term performance of different trading institutions. The investigation of the implications of these results for the design of parimutuel markets is left to future research.

¹⁷See Holden and Subrahmanyam (1992).

7. References

- Ali, Mukhtar M., “Probability and Utility Estimates for Racetrack Bettors,” *Journal of Political Economy*, 1977, 85(4), 803–815.
- Asch, Peter, Burton G. Malkiel and Richard E. Quandt, “Racetrack Betting and Informed Behavior,” *Journal of Financial Economics*, 1982, 10(2), 187–194.
- Bruce, Alistair C. and Johnnie Johnson, “Investigating the Roots of the Favourite-Longshot Bias: An Analysis of Decision Making by Supply- and Demand-Side Agents in Parallel Betting Markets,” *Journal of Behavioral Decision Making*, 2000, 13, 413–430.
- Baron, Ken and Jeffrey Lange, “From Horses to Hedging,” *Risk* (*www.risk.com*), February 2003, 73–77.
- Camerer, Colin F., “Can Asset Markets Be Manipulated? A Field Experiment with Race-track Betting,” *Journal of Political Economy*, 1998, 106(3), 457–482.
- Crafts, Nicholas, “Some Evidence of Insider Knowledge in Horse Race Betting in Britain,” *Economica*, 1985, 52(207), 295–304.
- Economides, Nicholas and Jeffrey Lange, “A Parimutuel Market Microstructure for Contingent Claims Trading,” 2001, Discussion Paper no. EC-01-13, Stern School of Business, New York University.
- Feeney, Rob and Stephen P. King, “Sequential Parimutuel Games,” *Economics Letters*, 2001, 72, 165–173.
- Gabriel, Paul E. and James R. Marsden “An Examination of Market Efficiency in a Betting Market,” *Journal of Political Economy*, 1990, 98(4), 874–885.
- Glosten, Lawrence and Paul Milgrom, “Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders,” *Journal of Financial Economics*, 1985, 14(1), 71–100.
- Griffith, R. M., “Odds Adjustment by American Horse-Race Bettors,” *American Journal of Psychology*, 1949, 62(2), 290–294.
- Hamilton, Jonathan H. and Steven M. Slutsky, “Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria,” *Games and Economic Behavior*, 1990, 2, 29–270.
- Holden, Craig and Avindhar Subrahmanyam, “Long-Lived Private Information and Imperfect Competition,” *Journal of Finance*, 1992, 47(1), 247–270.
- Hurley, William and Lawrence McDonough, “Note on the Hayek Hypothesis and the Favorite-Longshot Bias in Parimutuel Betting,” *American Economic Review*, 1995, 85(4), 949–955.

- Isaacs, Rufus, “Optimal Horse Race Bets,” *American Mathematical Monthly*, 1953, 60(5), 310–315.
- Koessler, Frédéric and Anthony Ziegelmeyer, “Parimutuel Betting under Asymmetric Information,” 2002, THEMA mimeo, U. Cergy-Pontoise.
- Kyle, Albert, “Continuous Auctions and Insider Trading,” *Econometrica*, 1985, 53(6), 1315–1335.
- Matsumura, Toshihiro, “Quantity-Setting Oligopoly with Endogenous Sequencing,” *International Journal of Industrial Organization*, 1999, 17, 289–296.
- Medrano, Luis A. and Xavier Vives, “Strategic Behavior and Price Discovery,” *Rand Journal of Economics*, 2001, 32(2), 221–248.
- Metrick, Andrew, “March Madness? Strategic Behavior in NCAA Basketball Tournament Betting Pools,” *Journal of Economic Behavior and Organization*, August 1996, 30, 159–172.
- Ockenfels, Axel and Al Roth “Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet,” *American Economic Review*, 2002, 92(4), 1093–1103.
- Plott, Charles, Jorgen Wit and Winston C. Yang, “Parimutuel Betting Markets as Information Aggregation Devices: Experimental Results,” *Economic Theory*, 2003, 22(2), 311–351.
- Potters, Jan and Jörgen Wit, “Bets and Bids: Favorite-Longshot Bias and the Winner’s Curse,” 1996, CentER Discussion Paper 9604.
- Sauer, Raymond D., “The Economics of Wagering Markets,” *Journal of Economic Literature*, 1998, 36(4), 2021–2064.
- Shin, Hyun Song, “Optimal Betting Odds Against Insider Traders,” *Economic Journal*, 1991, 101(408), 1179–1185.
- Shin, Hyun Song, “Prices of State Contingent Claims with Insider Traders and the Favourite-Longshot Bias,” *Economic Journal*, 1992, 102(411), 426–435.
- Thaler Richard H. and William T. Ziemba, “Anomalies: Parimutuel Betting Markets: Racetracks and Lotteries,” *Journal of Economic Perspectives*, 1988, 2(2), 161–174.
- Weitzman, Martin , “Utility Analysis and Group Behavior: An Empirical Analysis,” *Journal of Political Economy*, 1965, 73(1), 18–26.

Appendix A: Proofs

Proof of Proposition 1. We look for a symmetric equilibrium, in which each player adopts the same cutoff \hat{p} . Consider the best reply \check{p} of a player against all other players using \hat{p} . Given that the other players use the cutoff \hat{p} , the best response cutoff \check{p} is such that the bettor with belief \check{p} is indifferent between betting on either of the two horses, $U(1|\check{p}) = U(-1|\check{p})$, i.e.

$$\frac{\check{p}}{1 - \check{p}} = \frac{W(-1|x = -1)}{W(1|x = 1)}. \quad (\text{A.1})$$

Due to the assumption that players are forced to bet one unit each, the total pool of money to be shared among the winners is always equal to N . The conditional expected payoff when winning is obtained by using the cutoff strategies adopted by all opponents and the posterior distribution:

$$\begin{aligned} W(x|x) &= N \sum_{k=0}^{N-1} \frac{1}{k+1} \Pr(k \text{ others bet } x \text{ and } N-1-k \text{ others bet } -x|x) \\ &= N \sum_{k=0}^{N-1} \frac{1}{k+1} \binom{N-1}{k} \pi^k (1-\pi)^{N-1-k} \end{aligned}$$

where π denotes the probability that a single opponent bets on outcome x in state x . Note that

$$\begin{aligned} &\sum_{k=0}^{N-1} \frac{1}{k+1} \binom{N-1}{k} \pi^k (1-\pi)^{N-1-k} = \frac{1}{N} \sum_{k=0}^{N-1} \binom{N}{k+1} \pi^k (1-\pi)^{N-1-k} \\ &= \frac{1}{N\pi} \sum_{k=1}^N \binom{N}{k} \pi^k (1-\pi)^{N-k} = \frac{1}{N} \frac{1 - (1-\pi)^N}{\pi}, \end{aligned}$$

by using $\sum_{k=0}^N \binom{N}{k} \pi^k (1-\pi)^{N-k} = 1$. Thus the conditional expected payoffs when winning are

$$W(1|x = 1) = \frac{1 - (G(\hat{p}|x = 1))^N}{1 - G(\hat{p}|x = 1)}, \quad (\text{A.2})$$

and

$$W(-1|x = -1) = \frac{1 - (1 - G(\hat{p}|x = -1))^N}{G(\hat{p}|x = -1)}. \quad (\text{A.3})$$

At a symmetric equilibrium we have $\check{p} = \hat{p}$, yielding equation (3.1) after substitution of (A.2) and (A.3) into the equilibrium condition (A.1). Uniqueness of the solution follows from the fact that the left hand side of (3.1) is strictly increasing, ranging from zero to infinity as p ranges over $(0, 1)$, while the right hand side of (3.1) is weakly decreasing in p since $W(1|x = 1) = \sum_{k=0}^{N-1} (G(\hat{p}|x = 1))^k / N$ is increasing in \hat{p} and similarly $W(-1|x = -1)$ is decreasing in \hat{p} .

Finally, let p^* be the unique solution to the limit equation (3.2) and let an arbitrary $\varepsilon > 0$ be given. By monotonicity, at $p^* + \varepsilon$ the left hand side of (3.2) exceeds the right hand side. By pointwise convergence of the right hand side of (3.1) to the right hand side of (3.2), for sufficiently large N , the left hand side of (3.1) exceeds the right hand side at $p^* + \varepsilon$. A symmetric argument shows that for sufficiently large N , the right hand side of (3.1) exceeds the left hand side at $p^* - \varepsilon$. It follows that $\hat{p}_N \in (p^* - \varepsilon, p^* + \varepsilon)$ when N is sufficiently large. \square

Proof of Proposition 4. With a signal realization s that induces private belief p we now have $\Pr(x = z = 1|s) = \Pr(z = 1|x = 1)\Pr(x = 1|s) = \pi p$. If all players use symmetric strategies, we have $W(y = 1|x = z = 1) = W(y = -1|x = z = -1) \equiv \alpha$ and $W(y = 1|x = 1, z = -1) = W(y = -1|x = 1, z = -1) \equiv \beta$. The expected payoff from a bet on outcome 1 is $U(y = 1|p) = \pi p\alpha + (1 - \pi)(1 - p)\beta$ while $U(y = -1|p) = \pi(1 - p)\alpha + (1 - \pi)p\beta$. Now $U(y = 1|p) - U(y = -1|p)$ is weakly increasing in p if and only if $\pi\alpha \geq (1 - \pi)\beta$.

We now show that $\pi\alpha \geq (1 - \pi)\beta$. If instead $\pi\alpha < (1 - \pi)\beta$, a symmetric equilibrium should necessarily have a cut-off at $1/2$, but since those with $p > 1/2$ should then bet on outcome -1 , there would be more bets on this outcome when $x = 1$, and so $\alpha > \beta$. Since also $\pi > 1 - \pi$, this is incompatible with $\pi\alpha < (1 - \pi)\beta$, a contradiction.

If $\pi\alpha \geq (1 - \pi)\beta$ holds with equality, then $\pi > 1 - \pi$ implies $\alpha < \beta$. If the inequality is strict, then the unique equilibrium has a cut-off at $1/2$, and since more people bet on outcome 1 when $x = 1$, we get again $\alpha < \beta$. In the limit with infinite N , there is no uncertainty about how much is bet on outcome y given x . Since the remaining amount is bet on $y = -1$, we obtain the relation $1 = 1/W(y = z|x = z) + 1/W(y = z|x \neq z) = 1/\alpha + 1/\beta$. Since $W(y = 1|x = z = 1) = \alpha < \beta$, outcome 1 is the favorite when $x = 1$. Having observed that outcome 1 has odds $W(y = 1|x = z = 1)$, one can infer that $x = 1$, and so the Bayesian probability for outcome $z = 1$ is $\pi = \Pr(z = 1|x = 1)$. The expected return on the favorite, $\pi\alpha$, is then immediately weakly greater than the expected return on the longshot, $(1 - \pi)\beta$, by the inequality $\pi\alpha \geq (1 - \pi)\beta$. More strongly, the favorite-longshot bias carries over as $\pi \geq 1/\alpha$, so that the favorite has greater Bayesian odds than market odds. This inequality is true, since $1 = 1/\alpha + 1/\beta$ can be solved for $\beta = \alpha/(\alpha - 1)$ and $\pi\alpha \geq (1 - \pi)\beta$ then boils down to $\pi \geq 1/\alpha$. \square

Proof of Proposition 6. In a symmetric equilibrium, all individuals use the same cutoff policy. The amounts bet are then $b_1^1 = N(1 - G(\hat{p}_1|x = 1))$, $b_1^{-1} = N(1 - G(\hat{p}_1|x = -1))$, $b_{-1}^1 = NG(\hat{p}_{-1}|x = 1)$, and $b_{-1}^{-1} = NG(\hat{p}_{-1}|x = -1)$. The indifference conditions charac-

terizing \hat{p}_1 and \hat{p}_{-1} give the two equilibrium conditions:

$$\hat{p}_1 = \frac{1}{1 - \tau} \frac{a_1 + N(1 - G(\hat{p}_1|x = 1))}{a_1 + a_{-1} + N(1 - G(\hat{p}_1|x = 1)) + NG(\hat{p}_{-1}|x = 1)} \quad (\text{A.4})$$

and

$$\hat{p}_{-1} = 1 - \frac{1}{1 - \tau} \frac{a_{-1} + NG(\hat{p}_{-1}|x = -1)}{a_1 + a_{-1} + N(1 - G(\hat{p}_1|x = -1)) + NG(\hat{p}_{-1}|x = -1)}. \quad (\text{A.5})$$

First, we show existence. The product set of \hat{p}_{-1} in $[1 - (a_{-1} + N)/(1 - \tau)(a_1 + a_{-1} + N), 1]$ with \hat{p}_1 in $[0, (a_1 + N)/(1 - \tau)(a_1 + a_{-1} + N)]$ is non-empty, convex, and compact. With the convention that $G(p) = 0$ when $p < 0$ and $G(p) = 1$ when $p > 1$, the right hand sides of (A.4) and (A.5) continuously map this set into itself. We already noted that $\hat{p}_1 > 0$ and $\hat{p}_{-1} < 1$. It is immediate to see that $\hat{p}_1 \leq (a_1 + N)/(1 - \tau)(a_1 + a_{-1} + N)$, and $\hat{p}_{-1} \geq 1 - (a_{-1} + N)/(1 - \tau)(a_1 + a_{-1} + N)$. Brouwer's fixed point theorem then implies the existence of a solution to (A.4) and (A.5) within this product set. But such a solution must further satisfy $\hat{p}_1 < 1$. To see this, assume on the contrary that $\hat{p}_1 \geq 1$. Then the right hand side of (A.4) is $a_1/(1 - \tau)(a_1 + a_{-1} + NG(\hat{p}_{-1}|y = 1)) \leq a_1/(1 - \tau)(a_1 + a_{-1}) < 1$, by the assumption that $\tau < a_{-1}/(a_1 + a_{-1})$. But then the right hand side of (A.4) is strictly less than one, contradicting that $\hat{p}_1 \geq 1$ solves (A.4). A similar argument proves that $\hat{p}_{-1} > 0$.

Second, we show uniqueness. Assume that there are two different solutions $(\hat{p}_{-1}, \hat{p}_1)$ and $(\check{p}_{-1}, \check{p}_1)$ to equations (A.4) and (A.5), and assume without loss of generality that $\hat{p}_1 > \check{p}_1$. From the indifference conditions $\hat{p}_1 \hat{W}(1|1) - 1 = \check{p}_1 \check{W}(1|1) - 1 = 0$ it follows that $\hat{W}(1|1) < \check{W}(1|1)$. From the identity $1/W(1|1) + 1/W(-1|-1) = 1/(1 - \tau)$, then $\hat{W}(-1|-1) > \check{W}(-1|-1)$. From the indifference conditions $(1 - \hat{p}_{-1})\hat{W}(-1|-1) - 1 = (1 - \check{p}_{-1})\check{W}(-1|-1) - 1 = 0$ we conclude $\hat{p}_{-1} > \check{p}_{-1}$. Then $\hat{b}_{-1}^1/N = G(\hat{p}_{-1}|x = 1) \geq G(\check{p}_{-1}|x = 1) = \check{b}_{-1}^1/N$. From $\hat{b}_{-1}^1 \geq \check{b}_{-1}^1$ and $(1 - \tau)(a_1 + a_{-1} + \hat{b}_1^1 + \hat{b}_{-1}^1)/(a_1 + \hat{b}_1^1) = \hat{W}(1|1) < \check{W}(1|1) = (1 - \tau)(a_1 + a_{-1} + \check{b}_1^1 + \check{b}_{-1}^1)/(a_1 + \check{b}_1^1)$ it follows that $\hat{b}_1^1 > \check{b}_1^1$. Thus $1 - G(\hat{p}_1|x = 1) > 1 - G(\check{p}_1|x = 1)$, which is possible only if $\hat{p}_1 < \check{p}_1$. This contradiction establishes the result. \square

Proof of Proposition 7. Observe that $0 = \hat{p}_1 W(1|1) - 1$, so \hat{p}_1 increases if and only if $W(1|1)$ decreases. Assume on the contrary, that $W(1|1)$ increases. By the identity $1/W(1|1) + 1/W(-1|-1) = 1/(1 - \tau)$, then $W(-1|-1)$ decreases. By $0 = (1 - \hat{p}_{-1})W(-1|-1) - 1$, then \hat{p}_{-1} decreases. Since q increases, $b_{-1}^1/N = G(\hat{p}_{-1}|x = 1)$ weakly decreases. Since $W(1|1) = (1 - \tau)(a_1 + a_{-1} + b_1^1 + b_{-1}^1)/(a_1 + b_1^1)$ increased, it must be that $b_1^1/N = 1 - G(\hat{p}_1|x = 1)$ decreases. Since q increases, this is possible only if \hat{p}_1 increases. But this implies the contradiction that $W(1|1)$ decreases. We have

thus proved that \hat{p}_1 weakly increases and $W(1|1)$ weakly decreases. Again, by the identity $1/W(1|1) + 1/W(-1|-1) = 1/(1-\tau)$, then $W(-1|-1)$ weakly increases, and by $0 = (1 - \hat{p}_{-1})W(-1|-1) - 1$, then \hat{p}_{-1} weakly increases. \square

Proof of Proposition 8. Using the assumptions and $\hat{p}_1 = 1 - \hat{p}_{-1}$, condition (A.5) directly reduces to condition (A.4). Either condition can be rewritten as

$$(1 - \tau)\hat{p}_1 = \frac{a/N + 1 - G(\hat{p}_1|y = 1)}{2a/N + 1 - G(\hat{p}_1|y = 1) + 1 - G(\hat{p}_1|y = -1)}. \quad (\text{A.6})$$

The right hand side of (A.6) is continuous in \hat{p}_1 . At $1/2$ it strictly exceeds the left hand side, while the opposite is true at 1 . Thus there exists a solution to this equation in $(1/2, 1)$. At any solution, the right hand side is a strictly decreasing function of \hat{p}_1 . To see this, take the logarithm of the right hand side, differentiate and use symmetry of G to arrive at the desired inequality

$$\frac{a/N + 1 - G(\hat{p}_1|y = 1)}{a/N + 1 - G(\hat{p}_1|y = -1)} < \frac{g(\hat{p}_1|y = 1)}{g(\hat{p}_1|y = -1)} = \frac{\hat{p}_1}{1 - \hat{p}_1},$$

or equivalently

$$\hat{p}_1 > \frac{a/N + 1 - G(\hat{p}_1|y = 1)}{2a/N + 1 - G(\hat{p}_1|y = 1) + 1 - G(\hat{p}_1|y = -1)},$$

which is implied by (A.6) and $\tau > 0$. This proves the uniqueness of the equilibrium.

We now turn to the comparative statics results, still analyzing from equation (A.6). The direct effect on the left hand side of an increase in τ is negative, so an increase in τ implies an increase in \hat{p}_1 . In turn, the market odds on the right hand side was decreased. An increase in a/N will decrease the right hand side, so \hat{p}_1 falls. Since the left hand side falls, the market odds ratio on the right hand side also falls. \square