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Consumption Externalities, Rental Markets and Purchase Clubs

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Topics in Mathematical Economics

The participants in a September 2002 Workshop on *Topics in Mathematical Economics* in honor of Birgit Grodal decided to have a series of papers appear on Birgit Grodal's 60'th birthday, June 24, 2003.

The Institute of Economics suggested that the papers became Discussion Papers from the Institute.

The editor of *Economic Theory* offered to consider the papers for a special Festschrift issue of the journal with Karl Vind as Guest Editor.

This paper is one of the many papers sent to the Discussion Paper series.

Most of these papers will later also be published in a special issue of *Economic Theory*.

Tillykke Birgit

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Consumption Externalities,
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Abstract A premise of general equilibrium theory is that private goods are rival. Nevertheless, many private goods are shared, e.g., through barter, through co-ownership, or simply because one person's consumption affects another person's wellbeing. We analyze consumption externalities from the perspective of club theory, and argue that, provided consumption externalities are limited in scope, they can be internalized through membership fees to groups. Our main applications are to rental markets and "purchase clubs" in which members share the goods that they have individually purchased.

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1 Introduction

One of the main distinctions in microeconomics is between private goods, which are rival, and public goods, which are nonrival. Rivalness means that only one agent benefits from or is harmed by consumption of the good. Nonrivalness means that a second agent can consume the good simultaneously, without impinging on the benefits received by the buyer.

Club theory mutes this sharp distinction. The essential idea in club theory is that, by sharing a club, members share its services and share externalities conferred by the attributes or activities of the club's members. When purchasing memberships, members anticipate the full suite of externalities, which are therefore internalized, since they have the option not to join. The club model has wide-ranging applicability, comprising educational opportunities, firms, schools, social activities, academic departments, and many other human activities that take place in groups. For a wide-ranging set of examples, see Ellickson, Grodal, Scotchmer and Zame (EGSZ 1999, 2001, 2003), and for how these idea relate to the more traditional literature on local public goods, see Scotchmer (2002).

An important type of nonrivalness is *consumption externalities*, or the notion that one agent's consumption of private goods can affect other agents' wellbeings. If consumption activities confer uncompensated externalities, an equilibrium will not generally be efficient; that is, the first welfare theorem of general equilibrium theory does not hold.

Nevertheless, consumption externalities are pervasive. When someone buys a dog that barks, all the neighbors suffer. When a college roommate cooks something smelly for dinner, the other roommates have to go out. When a roommate subscribes to your favorite magazine, you are pleased. When your best friend buys an Armani suit in your size, or better yet a country house, you contemplate the possibility of sharing it.

These consumption externalities can be internalized. As long as the consumption externalities are limited to a finite group of agents which can be conceived as a club, pricing of memberships creates an opportunity to internalize the consumption externalities.

The question is how to model the sharing of private goods, and provide for pricing. Different goods involve different protocols for sharing. Goods like power tools, ski equipment and sometimes cars are used only occasionally by each user. As long as the transactions costs are not exorbitant, it is more efficient to keep the good in use than to let it sit idle. Nevertheless, sharing may be inconvenient. If the good cannot be used simultaneously, then there must be a protocol for resolving conflicts or scheduling use. We would expect prices to reflect the priority that a member gets, or the overall inconvenience of the use, as measured, for example, by the ratio of total use to total goods.

For some goods there is rather little inconvenience due to sharing. Computer software and sometimes digital content (music, movies) can be used simultaneously when installed simultaneously on different users' computers. The only inconvenience is in keeping the sharing group small enough to avoid detection, since simultaneous use will typically violate the seller's intellectual property rights.

For shared consumption that generates pleasure for one person and discomfort for another, such as playing Beatles tunes at midnight or smoking cigarettes, the protocol of sharing might be to prohibit use at certain hours or in certain places.

The purpose of this paper is to show how shared consumption (or "externalities") can be accommodated within the club model, and also to show how the club model subsumes ordinary market solutions to sharing, such as rental markets. The technology of sharing will determine how the club model must be adapted or applied.

Section 2 reprises the group-formation model of EGSZ (2003). Section 3 gives informal examples, showing how group formation can internalize consumption externalities. We discuss four ways that the club model can be adapted to accommodate sharing, the appropriate one depending on the technology of sharing. Section 4 addresses purchase clubs, where the purchased (shared) goods are assumed to be proprietary. In this section, the club model is extended to allow proprietary pricing of private goods. Section 5 shows that club theory leads to a useful model of rental markets, with peak and off-peak pricing, and prices that reflect the inconvenience of competing with other users.

2 A Reprise of the Club Model

In order to understand the special cases and extensions developed in the next sections, it is necessary to refer to the full group-formation or club model, which is described here. Readers who are familiar with the EGSZ (2003) can skip to the next section.

Groups are described by an exogenous set of *grouptypes*.

To define grouptypes, let Ω be an abstract, finite set of *membership characteristics*, and let Γ be an abstract, finite set of *activities*.

A *grouptype* is a triple (π, γ, y) consisting of a *profile* $\pi : \Omega \to \mathbf{Z}_+ = \{0, 1, \ldots\}$, an activity $\gamma \in \Gamma$, and a vector of private goods $y \in \Re^N$. The negative elements of y represent net inputs, and the positive elements represent net outputs.

For $\omega \in \Omega$, $\pi(\omega)$ represents the number of members of the group having the membership characteristic ω. A membership characteristic specifies the role in the grouptype that the membership entails (such as teacher or student), as well as the personal qualities required for the membership, including attributes such as intelligence, cooperativeness, personal habits, computer skills, the ability to teach, and managerial skills. The personal attributes can either be inherent or acquired. In the applications of Sections 4 and 5, the membership characteristics are respectively contributions of a good that will be shared by members or the usage of a rental good. In each case it is natural to model the contributions as real numbers (which must be chosen, however, from a finite set), although in general no such structure is imposed on Ω.

We take as given a finite set of possible grouptypes $\mathcal{G} = \{(\pi, \gamma, y)\}$.

A *membership* is an opening in a particular grouptype for an agent of a particular membership characteristic; i.e., $(\omega, (\pi, \gamma, y))$ such that $(\pi, \gamma, y) \in \mathcal{G}$ and $\pi(\omega) \geq 1$. We write $\mathcal{M}$ for the (finite) set of memberships.

Each agent may choose many memberships in groups or none. A membership *list* is a function $\ell : \mathcal{M} \to \{0, 1, \dots\}$, where $\ell((\omega, (\pi, \gamma, y)))$ specifies the number of memberships of type $(\omega, (\pi, \gamma, y))$.

The set of agents is a nonatomic measure space $(A, \mathcal{F}, \lambda)$. That is, A is a set, $\mathcal{F}$ is a σ-algebra of subsets of A and λ is a non-atomic measure on $\mathcal{F}$ with $\lambda(A) < \infty$. A complete description of an agent $a \in A$ consists of a consumption set, an endowment of private goods and a utility function.

Agent a's *consumption set* X_a specifies the feasible bundles of private goods and feasible lists of memberships that the agent may choose. Private-goods consumption is non-negative. For many examples, the consumption set of an agent a can be written $X_a = \Re^N_+ \times \mathbf{Lists}(a)$ where $\mathbf{Lists}(a)$ is a finite set of lists, but in other cases, the consumption set is restricted by the list. As discussed in the next section, this may be natural with consumption externalities. We impose a bound M on how many memberships an agent can consume.

Feasibility will typically impose many restrictions, since memberships are not restricted in any other way. For example, the restrictions would prevent an agent from choosing memberships that are impossible, such as being simultaneously a sumo wrestler and a member of a ballet club.

Agent a's *endowment* is $(e_a, 0) \in X_a$. Agents are endowed with private goods but not with group memberships.

Agent a's *utility function* $u_a : X_a \longrightarrow \Re$ is defined over private goods consumptions and lists of group membership. For each $\ell \in \mathbf{Lists}(a)$, $u_a(\cdot, \ell)$ is continuous and strictly monotone ; i.e., utility is strictly increasing in private goods consumption.

A *state* of an economy is a measurable mapping

$$(x, \mu) : A \longrightarrow \Re^N \times \Re^{\mathcal{M}}$$

A state specifies choices of private goods and lists of group memberships for each agent. Feasibility of a state of the economy entails consistent matching of agents and feasible consumptions.

Consistent matching of agents will be expressed in terms of an *aggregate membership vector* $\bar{\mu} \in \Re^{\mathcal{M}}$, representing the total number of memberships of each type chosen by the agents collectively. We say that an aggregate membership vector $\bar{\mu} \in \Re^{\mathcal{M}}$ is *consistent* if for every qrouptype $(\pi, \gamma, y) \in \mathcal{G}$, there is a real number $\alpha(\pi, \gamma, y)$, representing the "number" (measure) of groups of type (π, γ, y), such that

$$\bar{\mu}(\omega, (\pi, \gamma, y)) = \alpha(\pi, \gamma, y)\pi(\omega)$$

for each $\omega \in \Omega$. Consistency means that there are no partially filled groups (except possibly for choices by a set of agents with measure zero).

The state (x, μ) is *feasible* if it satisfies the following requirements:

(i) **Individual feasibility** $(x_a, \mu_a) \in X_a$ for each $a \in A$

(ii) **Material balance**

$$\int_A x_a \, d\lambda(a) \leq \int_A e_a \, d\lambda(a) + \int_A \sum_{(\omega,(\pi,\gamma,y)) \in \mathcal{M}} \mu_a(\omega, (\pi, \gamma, y)) \frac{y}{|\pi|} \, d\lambda(a)$$

(iii) **Consistency** The aggregate vector of memberships $\int_A \mu_a \, d\lambda(a)$ is consistent.

Condition (ii) means that private consumption plus private expenditures on the costs of acquiring characteristics do not exceed endowments plus net production.

Associated with a feasible state is a collection $\{\alpha(\pi,\gamma,y)|(\pi,\gamma,y)\in\mathcal{G}\}$ which describes the measures of the groups of various types. Because the set of agents is a continuum, there will either be "no" groups of a given type (π,γ,y) in a feasible state of the economy, $\alpha(\pi,\gamma,y)=0$, or there will be "many" (indeed, infinitely many) groups of this type, $\alpha(\pi,\gamma,y)>0$. Because members of a group care only about the membership characteristics of other members, and not about their identities, it is not necessary to identify the agents belonging to each individual club.

Both private goods and group memberships are priced, so prices (p,q) lie in $\Re^N_+\times\Re^{\mathcal{M}}$; p is the vector of prices for private goods and q is the vector of prices for group memberships. Prices of group memberships may be positive, negative or zero. Membership prices have different interpretations in different examples. They may be required to pay for the infrastructure of the group or its activities, to remunerate a member for his opportunity cost of membership, in particular, wages, or may, when negative, be required to compensate a member for membership when his presence confers positive externalities on other members. A negative price means that he is paid to be a member. In Section 4 below, a negative price might mean that the member of a purchase club is partially reimbursed by other members for the purchases he contributes.

A *group equilibrium* consists of a feasible state (x,μ) and prices $(p,q)\in\Re^N_+\times\Re^{\mathcal{M}}, p\neq 0$ such that

(1) **Budget feasibility for agents** For almost all $a\in A$,

$$(p,q)\cdot(x_a,\mu_a)\leq p\cdot e_a$$

(2) **Optimization by agents** For almost all $a\in A$:

$$(x'_a,\mu'_a)\in X_a \text{ and } u_a(x'_a,\mu'_a)>u_a(x_a,\mu_a)\Rightarrow(p,q)\cdot(x'_a,\mu'_a)>p\cdot e_a$$

(3) **Budget balance for grouptypes** For each $(\pi,\gamma,y)\in\mathcal{G}$:

$$\sum_{\omega\in\Omega}\pi(\omega)\ q(\omega,(\pi,\gamma,y))+p\cdot y=0$$

Thus, at an equilibrium individuals optimize subject to their budget constraints and the sum of membership prices in a given grouptype is exactly equal to the net cost or surplus generated by the use or production of private goods, $p \cdot y$.

EGSZ (2003) prove the first welfare theorem for this model as well as core/competitive equivalence. (Or, more accurately, they point out that the theorems of EGSZ (1999) can be adapted for the (2003) elaboration of the (1999) model.)

In the examples of the EGSZ papers, the characteristics Ω generally represent qualifications needed to perform certain functions within groups, such as teaching or dancing, or simply conferring externalities. The externalities within groups generally arise from these characteristics. However, as discussed in the introduction, another type of externality that can arise within groups comes from the consumption bundles of private goods that are chosen by the group members. We now discuss how the model can be extended to internalize consumption externalities.

3 Consumption Externalities

The group formation model can accommodate consumption externalities (or shared consumption) by using the following adaptations:

- Using consumption sets, consumption of private goods can be restricted in a way that depends on group memberships.
- The activity vector γ can specify how private goods, modeled in y as inputs, are shared.
- The membership characteristic can obligate the member for certain purchases that must be shared with other members, with the terms of sharing specified by γ.

- The membership characteristic can entitle the member to certain specified usage of the shared good.

Some examples follow.

Example 1. Suppose that a group of friends share a house. Their joint consumption of the house is a type of club, (π, γ, y), and their enjoyment of it depends on the consumption and activities of the members, such as whether they throw late-night parties, play Beatles tunes, or smoke cigarettes. A shared house which seeks to avoid these consumption activities, and hence to avoid the consumption externalities, can be described by the following clubtype: The input/output vector y consists of the house. The activity γ consists of a commitment that no member shall listen to Beatles tunes at midnight. Alternatively, the membership characteristic ω may commit the member not to play Beatles tunes. The consumption set may prohibit a member of such a household from purchasing cigarettes.

If households (families) could form endogenously rather than being fixed in advance, then this example is closely related to the household consumption model of Gersbach and Haller (2001). They study exogenously formed households whose members care about the consumption vectors of all other members. The individual consumption vectors are a joint decision of the household, using a combined budget constraint. In the club model, consumption of private goods is the private decision of the member, but consumption can be constrained in the consumption set in a way that depends on memberships. In the club model, the transfers are explicit through positive and negative membership fees. In the Gersbach and Haller model, the transfers among household members are implicit in the combined budget constraint.

Example 2. Suppose that a group of students share a house, and divide tasks in advance, so that someone must be the cook, someone must take out the garbage, someone must bring sports equipment, and someone who is smart and versatile will do their collective homework. These commitments could be built into the membership characteristics ω. Some characteristics, like being the cook, could be acquired skills, and others, like doing all their homeworks, require innate abilities and also learned skills. Not everyone could feasibly choose such a membership. If the cook requires cookbooks, those could be part of the input vector y, or they could be considered an investment required to have the "cook" membership. That is, the cook's consumption set must specify that he consumes cookbooks. Bringing the cookbooks will presumably cause him to have a lower membership price in equilibrium than if the cookbooks were provided as an input in y. Similarly, the person who contributes the sports equipment must presumably invest in it, and his membership price should reflect this investment. If different members bring different sports equipment, their personal characteristics will reflect their contributions. The activity γ must specify the organizational arrangements under which they decide how to ration the sports equipment.

Example 3. The friends may band together for the dedicated purpose of sharing music or software CD's, in order to avoid purchasing duplicate copies. This is a purchase club, described in Section 4. We model purchase clubs as shared usage of goods that are not subject to congestion, but for which the sharing group is limited in size. The motivation is shared purchases of digital products that can be installed separately on several computers and used simultaneously. In fact, if there are no congestion costs, the digital product could be shared by an unboundedly large group of people. We assume that sharing on this scale cannot happen, because it would be detected and stopped by the copyright holder. Because sharing violates license terms, it usually takes the form that friends buy software or content in the realization that they will barter the use of it for a similar digital product that someone else has bought.

Since CD's are sold at proprietary prices, which should be taken as a datum of the economy, the shared CD's cannot be modeled in the input vector y. Instead we model the contributions of proprietary goods in the membership characteristic ω. If there is a sharing protocol to be worked out, it could be modeled in γ. If the sharing protocol involves priorities, the priorities could be specified in the membership characteristic, along with the contributions.

Example 4. Suppose that the shared good is partially rival, in the sense that intense use causes conflict or congestion that must be resolved. In that case, we would expect membership prices to depend on both the member's intensity of use, the overall usage, and perhaps other aspects that reflect when demand is likely to be high. Here we are thinking of, for example, a sailboat. The demand to use sailboats is mostly on weekends and summer days. The price during the afternoon on a balmy Saturday in August might have to be very high in order to avoid other types of rationing, while the price for an early morning sail on a Tuesday in January might have to be very low in order to keep the sailboat in use. Some group types commit to overall low usage, so that a sailboat is more likely to be free when the user wants it.

Here the sharing protocol can be specified by the organizational structure or activity, γ, for example, "first come first served," or "call ahead for a reservation," and γ could also specify an overall level of usage. The membership characteristic ω could specify a particular member's intensity of usage, perhaps depending on whether the usage is peak or off-peak. The vector y represents the input vector of shared goods themselves.

This example is developed in Section 5, where we link it to the more standard model of rental markets.

4 Purchase Clubs

The most straightforward way to model shared purchases is to model them as inputs in the input/output vector y. However that is not consistent with the model unless the shared goods are competitively supplied. The application below is to shared goods that are proprietary, with prices given as a datum of the economy. We therefore give a slight modification to the EGSZ model, in which contributions of goods purchased at proprietary prices are modeled as membership characteristics.

Copyright owners have argued for many years that their profits are undermined when users share. Their calculation of the loss usually involves the assumption that every unauthorized user would otherwise purchase a legitimate copy at the prevailing price. Both common sense and the economics literature challenge this view. What is argued in the literature (Besen and Kirby (1989), Varian (2000), Bakos, Brynjolfsson and Lichtman (1999)) is that proprietors will anticipate the sharing behavior, and set different prices if the good is sold to individual users than if sold to users who are expected to share it. These papers argue, somewhat provocatively, that sharing may actually *increase* the proprietor's profit. We revisit this question, using a variant of the club model that allows for proprietary pricing.

Before presenting the model, we begin with an extended example. The example and theorem that follow are provided for a substantive purpose as well as an illustrative one. The example shows that whether sharing enhances profit depends on the groups that form. The theorem that follows relies on the main characterization of a group equilibrium, which is that groups will form in a way that is collectively efficient – efficient for the buyers, that is. Group formation that is efficient for the buyers is probably not efficient for the sellers. Indeed, this is more or less what the theorem shows. In the example, the sellers' profits may be enhanced if group formation is, for example, random, but profit will not be enhanced if group formation is systematic in some way that serves the interests of the buyers. The theorem shows that the profit available to the sellers is exactly the same with sharing of purchases as without, provided the purchase groups form efficiently in equilibrium, and the selling price can depend on the size of the group.

This result would not survive in the form given if the shared goods involved marginal costs of supply, as sharing would then reduce industry costs, and the proprietor would presumably share in the benefits. This is the focus of the related work by Besen and Kirby (1989) and Varian (2000).

Instead of assuming that consumers form different sharing groups for different digital content, as in BBL, we give a model in which a group may share several products instead of one. This shift in focus gives more flexibility in how we can think of members of a group making side payments. If several products are involved, one can think of the members as contributing purchases, and trading the use of their individually owned products. If only one product is involved, explicit side payments are required.

4.1 Purchase Clubs: An Example in Three Parts

We will consider purchase clubs that share CD's of two kinds, classical and jazz. In the three subparts to this example, the willingness to pay (WTP) for these two types of CD are positively correlated, negatively correlated and uncorrelated "within" individuals. Positive (negative) correlation means that individuals with high WTP for classical CDs would have high (low) WTP for jazz. No correlation means that the WTP for one type of music is uninformative as to the agent's WTP for the other.

Assume that for each CD, half the population has WTP a and the other half has WTP x, $x < a$. We will assume that $\frac{3}{8}(a+x) > \frac{1}{2}a$ and $\frac{3}{8}(a+x) > x$. How the willingnesses to pay are distributed among the agents will depend on whether their willingnesses to pay for jazz and classical are negatively, positively or not correlated.

Our benchmark will be the profitability of selling separately to single buyers. At price $p = a$, half the agents buy, so per-person profit is $\frac{a}{2}$ for each CD. At $p = x$ everyone buys, so the expected profit per person is x. Profit cannot be improved over $max\{\frac{a}{2}, x\}$ by selling at any price between x and a. We compare this benchmark with a situation where sharing groups of size 2 can form. In each subcase, we work out the profit opportunities with randomly matched groups and homogeneous groups, and compare with the profit available by selling to single buyers.

The "taste spaces" in the first and second special cases (positive and negative correlation) contain two types of WTP $\{v_1, v_2\}$, and the "taste space" in the third special case (independence) contains four types of WTP, $\{v_1, v_2, v_3, v_4\}$. In each case, the taste vectors are equally represented in the population. Each taste vector specifies a WTP for each type of CD. We first describe the aggregate willingnesses to pay of the groups that form, and, for the case that groups form randomly, the probabilities. For each subcase, we work out the profit opportunities at each price, and conclude at the end that random matching may improve profit opportunities, as compared to selling to single buyers, but homogeneous groups will not.

POSITIVE WITHIN-PERSON CORRELATION

Tastes (WTP's):

	v_1	v_2
WTP classical	a	x
WTP jazz	a	x

If the groups of size 2 form randomly, there will be three types of groups, $\{(2,0),(0,2),(1,1)\}$ with the following willingnesses to pay:

Group WTP's with random matching

probability	1/4	1/2	1/4
group type	$(2,0)$	$(1,1)$	$(0,2)$
WTP classical	$a+a$	$a+x$	$x+x$
WTP jazz	$a+a$	$a+x$	$x+x$

Profits with randomly matched groups

price	profit per person per CD
$p = 2a$	$\frac{1}{2}\frac{1}{4}(2a) = \ a/4$
$p = a+x$	$\frac{1}{2}\frac{3}{4}(a+x) = \ 3(a+x)/8$
$p = 2x$	x

Under the conditions on the parameters a and x that we specified, the most profitable price is $p = a + x$.

If homogeneous groups form instead of random matching, there will be equal numbers of groups $\{(0,2),(2,0)\}$. These groups have WTP $(2a, 2a)$ and $(2x, 2x)$ respectively.

Profits with homogeneous groups

price	profit per person per CD	which groups buy
$p = 2a$	$\frac{1}{2}\frac{1}{2}2a = a/2$	$(2, 0)$
$p = a + x$	$\frac{1}{2}\frac{1}{2}(a + x) = (a + x)/4$	$(2, 0)$
$p = 2x$	x	$(2, 0), (0, 2)$

NEGATIVE WITHIN-PERSON CORRELATION

Tastes(WTP):

	v_1	v_2
WTP classical	a	x
WTP jazz	x	a

Group WTP's with random matching

probability	1/4	1/2	1/4
group type	$(2, 0)$	$(1, 1)$	$(0, 2)$
WTP classical	$a + a$	$a + x$	$x + x$
WTP jazz	$x + x$	$a + x$	$a + a$

Profits with random matching

price to group	profit per person per CD
$p = 2a$	$\frac{1}{2}\frac{1}{4}(2a) = \ a/4$
$p = a + x$	$\frac{1}{2}\frac{3}{4}(a + x) = \ 3(a + x)/8$
$p = 2x$	x

Profits with homogeneous groups

price to group	profit per person per CD	which groups buy
$p = 2a$	$\frac{1}{2}\frac{1}{2}2a = a/2$	$(2, 0)$
$p = a + x$	$\frac{1}{2}\frac{1}{2}(a + x) = (a + x)/4$	$(2, 0)$
$p = 2x$	x	$(2, 0), (0, 2)$

NO WITHIN-PERSON CORRELATION

Tastes (WTP)

	v_1	v_2	v_3	v_4
WTP classical	a	a	x	x
WTP jazz	a	x	a	x

Group WTP's with random matching

Grouptype	Prob	WTP classical WTP jazz	Grouptype	Prob	WTP classical WTP jazz
(2,0,0,0)	1/16	$a+a$ $a+a$	(1,0,1,0)	1/8	$a+x$ $a+a$
(0,2,0,0)	1/16	$a+a$ $x+x$	(1,0,0,1)	1/8	$a+x$ $a+x$
(0,0,2,0)	1/16	$x+x$ $a+a$	(0,1,1,0)	1/8	$a+x$ $a+x$
(0,0,0,2)	1/16	$x+x$ $x+x$	(0,1,0,1)	1/8	$a+x$ $x+x$
(1,1,0,0)	1/8	$a+a$ $a+x$	(0,0,1,1)	1/8	$x+x$ $a+x$

Profits with random matching

price to group	profit per person per CD
$p = 2a$	$a/4$
$p = a + x$	$3(a + x)/8$
$p = 2x$	x

Group WTP's in homogeneous groups:

probability	1/4	1/4	1/4	1/4
group types	$(2, 0, 0, 0)$	$(0, 2, 0, 0)$	$(0, 0, 2, 0)$	$(0, 0, 0, 2)$
WTP classical	$a + a$	$a + a$	$x + x$	$x + x$
WTP jazz	$a + a$	$x + x$	$a + a$	$x + x$

Profits with homogeneous groups

price to group	profit per person per CD
$p = 2a$	$a/2$
$p = a + x$	$(a + x)/4$
$p = 2x$	x

Perhaps remarkably, the following result holds in all three cases:

Remark 1 *Regardless of how willingness to pay for CDs of different types is correlated within agents, it is more profitable to sell to randomly assembled groups of size 2 than to single agents. However it is not more profitable to sell to groups of size 2 if there is no taste variation within groups.*

Hence, whether maximum profit increases or decreases when goods are shared depends on how the agents assemble themselves into groups in equilibrium. This is addressed in the next subsection. Since every pair of agents in the model below can have different tastes, there is no concept of forming groups with homogeneous tastes. It is not the homogeneity of tastes that erases any profit advantage to selling to groups, but rather the fact that groups form endogenously in a way that is collectively efficient for the members, conditional on the proprietary prices.

4.2 Purchase Clubs: A Theorem

In developing the example, it was convenient to describe the members of groups by their tastes. However membership prices cannot depend on tastes, as tastes are unobservable. The membership characteristics will be the contributions of proprietary goods.

Suppose there are C goods that can be purchased and shared. Proprietors market these goods at prices $r = (r_1, ..., r_C) > 0$. We will compare two situations:

that the proprietors sell to individual agents, and that the proprietors sell to groups of maximum size k.

The set Ω will serve various purposes in the model that follows. Most importantly, the elements $\omega \in \Omega$ will represent the contributions that a member might make to a group. Let

$$\Omega = \left\{ z \in Z_{+}^{C} \mid z \leq (Mk, Mk....Mk) \right\}$$

for a given $k > 1$ where M is the maximum number of memberships in an agent's consumption set.

In the example, $C = 2$ (jazz and classical), and we assumed that group types would consume shared goods $\{(0,0),(0,1),(1,0),(1,1)\}$. That is, no group purchased more than one unit of each shared good. That is the outcome we expect if access to a single unit of any shared good is sufficient as a matter of preferences. However it is technically convenient to define Ω so that multiple contributions of each shared good are allowed, and it is technically convenient to allow groups to collectively purchase more than one unit of each shared good.

A *purchase club type* (π, γ, y) is a club type such that the membership characteristics $\omega \in \Omega$ are interpreted as contributions, and $\sum_{\omega \in \Omega} \pi(\omega)\omega$ is the vector of goods shared by members of such a group. Recall that, in general, the expression $\sum_{\omega \in \Omega} \pi(\omega)\omega$ has no meaning, as the characteristic ω need not be a number. If a member chooses a membership for which $\omega > 0$, then he contributes at least one shared good, and may be paid in equilibrium by members who choose $\omega = 0$.

The *contributions* of an agent who consumes a list ℓ of memberships are

$$\sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi)\omega.$$

Given the prices r, the *cost* of the contributions in list ℓ is

$$\sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) r \cdot \omega$$

The conditions that define a group equilibrium (Section 2) must be slightly rewritten to account for the contributions of purchased goods in budget feasibility.

A *purchase-club equilibrium* at prices r consists of a feasible state (x, μ) and prices $(p, q) \in \Re^N_+ \times \Re^{\mathcal{M}}, p \neq 0$ such that

(1) **Budget feasibility for agents** For almost all $a \in A$,

$$(p, q) \cdot (x_a, \mu_a) + \sum_{(\omega,(\pi,\gamma,y)) \in \mathcal{M}} \mu_a(\omega, (\pi, \gamma, y)) \; r \cdot \omega \qquad \leq p \cdot e_a$$

(2) **Optimization by agents** For almost all $a \in A$:

$$(x'_a, \mu'_a) \in X_a \text{ and } u_a(x'_a, \mu'_a) \quad > u_a(x_a, \mu_a)$$

$$\Rightarrow (p, q) \cdot (x'_a, \mu'_a) + \sum_{(\omega,(\pi,\gamma,y)) \in \mathcal{M}} \mu'_a(\omega, (\pi, \gamma, y)) \; r \cdot \omega \quad > p \cdot e_a$$

(3) **Budget balance for grouptypes** For each $(\pi, \gamma, y) \in \mathcal{G}$:

$$\sum_{\omega \in \Omega} \pi(\omega) \; q(\omega, (\pi, \gamma, y)) + p \cdot y = 0$$

We will say that, for an arbitrary list ℓ, a consumption bundle (x, ℓ) *is budget-feasible* for a particular agent $a \in A$ if condition (1) holds for (x, ℓ) and $(x, \ell) \in X_a$.

For simplicity, we shall assume there is a single private good, the numeraire, and shall refer to equilibrium as $(x, \mu), q$.

Although it is not necessary in general, we will specialize the model to isolate the points of interest. Let the set Γ be a singleton, specifying that members will share their purchases, so that we can suppress the activity $\gamma \in \Gamma$ in the description of the group type. Let $y = 0$, since the shared goods are described in the membership characteristics. Since there is no input/output vector, a group type is only described by the profile π which describes how many members contribute each vector ω of shared goods. We will also assume there is an exogenous bound k on the size of sharing groups, and this is the k in the definition of Ω. (In the example, $k = 2$). Since γ is a singleton and $y = 0$, the sets of possible group types and memberships are

$$\begin{aligned} \mathcal{G} &= \{\pi : \Omega \to Z_+ \mid |\pi| \leq k\} \\ \mathcal{M} &= \{(\omega, \pi) \mid \omega \in \Omega, \ \pi \in \mathcal{G}\} \end{aligned}$$

We will use the notation ω^ℓ to refer to the *consumption* of an agent (distinct from the *contributions* of the agent) if he consumes a list ℓ:

$$\omega^\ell = \sum_{(\omega,\pi)\in\mathcal{M}} \ell(\omega, \pi) \left(\sum_{\omega\in\Omega} \pi(\omega)\omega \right) \tag{1}$$

Utility functions $u_a : X_a \to \Re$, are defined by

$$u_a(x, \ell) = U_a(x, \omega^\ell) \tag{2}$$

where $U_a : \tilde{X}_a \to \Re$ represents utility as a function of the goods themselves, and

$$\begin{aligned} X_a &= \Re_+ \times \left\{\ell \in Z_+^{\mathcal{M}} : |\ell| \leq M\right\} \\ \tilde{X}_a &= \left\{(x, z) \in \Re_+ \times Z_+^C \mid z = \omega^\ell, \ (x, \ell) \in X_a\right\} \end{aligned}$$

When referring to an equilibrium $(x, \mu), q$, we will say that a goods bundle (x, ω) is *budget-feasible* for $a \in A$ if $\omega = \omega^\ell$ for some budget-feasible $(x, \ell) \in X_a$.

A1: Preferences can be defined as in (2), where for all $a \in A$, (i) $U_a(\cdot, z)$ is increasing in the first argument at each $z \in \Omega$, and (ii) for each $x \geq 0$, if $z \notin \Omega$, then there exists $z' \in \Omega$, $z' < z$,[1] such that $U_a(x, \ z') \geq U_a(x, \ z)$.

Part (ii) of this assumption is satisfied if consumers are just as well off consuming Mk units of any shared good (or any other number less than Mk) as any larger number of units. If members of each group can use the shared good simultaneously,

[1] The notation $z' < z$ means that $z' \leq z$ and $z'_i < z_i$ for at least one element i.

as is the case when they install computer software or digital music separately on all their computers, we would expect that one unit of each shared good is sufficient.

The next three claims characterize a purchase-club equilibrium. Claim 2 describes prices such that, in equilibrium, agents are indifferent as to which membership they have in a group type that is used in equilibrium. Different memberships in a given group type require different contributions. Members who contribute shared goods pay low prices (perhaps negative prices), and members who contribute no shared goods pay high prices, to just an extent that they are indifferent.

Claim 2 *Suppose that A1 holds. Let $(x, \mu), q$ be a purchase-club equilibrium at prices $r > 0$. Then*
(i) For each group type π, q satisfies (3) for at least one membership (ω, π).

$$q(\omega, \pi) \leq \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega - r \cdot \omega \tag{3}$$

(ii) If group type π is used in equilibrium ($\alpha(\pi) > 0$), q satisfies (3) with equality for all memberships in that group type.
(iii) If group type π is used in equilibrium and $\pi(\omega_1), \pi(\omega_2) > 0$, then for all $a \in A$

$$q(\omega_1, \pi) + r \cdot \omega_1 = q(\omega_2, \pi) + r \cdot \omega_2 = \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega$$

Proof: (i) If (3) holds with equality, budget balance is satisfied. (Multiply both sides of (3) by $\pi(\omega)$ and sum on ω.) If (3) does not hold with equality, then by budget balance, (3) holds as an inequality for at least one membership in a given grouptype π.

(ii) Suppose to the contrary that for a given π such that $\alpha(\pi) > 0$,

$$\begin{aligned} q(\omega_1, \pi) &> \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega - r \cdot \omega_1 \\ q(\omega_2, \pi) &< \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega - r \cdot \omega_2 \end{aligned}$$

where $\pi(\omega_1), \pi(\omega_2) > 0$.

An agent's total payments when he chooses a membership are the cost of the contributions plus the membership fee, $q(\omega, \pi) + r \cdot \omega$. Since all memberships in a given group type π give access to the same shared goods $\sum_{\omega \in \Omega} \pi(\omega)\omega$, every agent is better off buying a membership in such a group type that reduces the total payments, namely (ω_2, π) instead of (ω_1, π). This is a contradiction, since $\pi(\omega_1) > 0$.

(iii) follows from (ii). $\square$

Claim 3 *Suppose that A1 holds. Let $(x, \mu), q$ be a purchase-club equilibrium at prices $r > 0$. Let $\{\omega^{\mu_a} | a \in A\}$ be the consumptions of shared goods defined by (1). Then*
(i) $\omega^{\mu_a} \in \Omega$ for almost every $a \in A$.
(ii) If the grouptype π is used in equilibrium ($\alpha(\pi)>0$), then $|\pi| = k$.
(iii) For almost every $a \in A$, the consumption of private goods satisfies

$$x_a = e_a - \frac{r}{k} \cdot \omega^{\mu_a} \tag{4}$$

Proof: For a given $\omega \in \Omega$, there is a list $\ell \in$ **Lists** such that $\omega^\ell = \omega$ and the goods bundle $(x, \omega) \in \tilde{X}_a$ is budget-feasible if

$$e_a - \frac{r}{k} \cdot \omega^\ell \geq x \geq 0 \tag{5}$$

To construct ℓ, let π be a group type such that $|\pi| = k$ and $\omega^\ell = \omega = \sum_{\omega' \in \Omega} \pi(\omega')\omega'$. Construct ℓ with a single membership in this grouptype, namely a membership for which (3) holds. Then (5) implies budget feasibility:

$$\begin{aligned}
x &\leq e_a - \frac{r}{k} \cdot \omega^\ell = e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \frac{r}{|k|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega \\
&= e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega \\
&\leq e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \left[\, r \cdot \omega + \; q(\omega, \pi)\right]
\end{aligned}$$

Using Claim 2(ii), equilibrium consumption can be characterized as follows for almost every $a \in A$:

$$\begin{aligned} x_a &= e_a - \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\,[\, r \cdot \omega + \ q(\omega,\pi)] \\ &= e_a - \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\frac{r}{|\pi|} \cdot \sum_{\omega\in\Omega} \pi(\omega)\omega \\ &\leq e_a - \frac{r}{k} \cdot \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi) \sum_{\omega\in\Omega} \pi(\omega)\omega = e_a - \frac{r}{k} \cdot \omega^{\mu_a}. \end{aligned} \tag{6}$$

(i) Suppose $\omega^{\mu_a} \notin \Omega$ for a set of agents of positive measure, say $\bar{A} \subseteq A$. For each such $a \in \bar{A}$, by A1 there exists $\omega' \in \Omega$, $\omega' < \omega^{\mu_a}$, such that $U_a(x_a, \omega^{\mu_a}) \leq U_a(x_a, \omega')$. Using (6), $0 \leq x_a \leq e_a - \frac{r}{k} \cdot \omega^{\mu_a} < e_a - \frac{r}{k} \cdot \omega'$. Using (5), there is a list ℓ with associated consumption $\omega^\ell = \omega'$ such that $(x_a, \ell) \in X_a$ is budget feasible. Further, there is a budget-feasible $(x, \ell) \in X_a$ such that $e_a - \frac{r}{k} \cdot \omega' \geq x > x_a$. But then $U_a(x_a, \omega^{\mu_a}) \leq U_a(x_a, \omega') < U_a(x, \omega')$, which contradicts the efficiency of equilibrium.

(ii) Suppose that there is a group type π such that $|\pi| < k$ and $\alpha(\pi) > 0$. Let $\bar{A} \subseteq A$ be the set of agents with memberships in this grouptype π. Using (i), we can assume without loss of generality that for each $a \in \bar{A}$, $\omega^{\mu_a} \in \Omega$. Since (6) holds as a strict inequality, $e_a - \frac{r}{k} \cdot \omega^{\mu_a} > x_a \geq 0$. Using (5), there is a list ℓ with a single membership in a club of type π, $|\pi| = k$, such that $\omega^\ell = \omega^{\mu_a}$. (x_a, ω^ℓ) is budget feasible because $e_a - \frac{r}{k} \cdot \omega^\ell = \ e_a - \frac{r}{k} \cdot \omega^{\mu_a} > x_a \geq 0$. Further, for every $a \in \bar{A}$, there is a budget-feasible $\left(x_a^\ell, \ell\right)$, $e_a - \frac{r}{k} \cdot \omega^\ell \geq x_a^\ell > x_a$, such that $U_a(x_a^\ell, \omega^\ell) = U_a(x_a^\ell, \omega^{\mu_a}) > U_a(x_a, \omega^{\mu_a})$, which contradicts the efficiency of equilibrium.

(iii) But if $|\pi| = k$, then, using Claim 2(ii), $x_a = e_a - \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\,[\, r \cdot \omega + \ q(\omega,\pi)]$ $= e_a - \frac{r}{|k|} \cdot \omega^{\mu_a}$ so (4) holds. $\square$

Our objective is to compare the group equilibrium at prices r to a market in which proprietors sell to individual agents at prices r/k. To study the market with individual buyers, we define the agents' demand sets as follows. For each $a \in A$ and

$r > 0$

$$D^a(r) = \{f \in \Omega \ \mid \ e_a - r \cdot f \geq 0 \text{ and for all } \omega \in \Omega, \text{ either} \tag{7}$$

$$U_a \left(e_a - r \cdot f, \ f\right) \geq U_a \left(e_a - r \cdot \omega, \ \omega\right) \text{ or } e_a - r \cdot \omega < 0\}$$

By A1, there is no loss of generality in restricting to demand vectors in Ω.

Aggregate demand is the integral of a selection from individual demand sets. A *demand selection at prices* r is an integrable function $f : A \to \Omega$ such that $f(a) \in D^a(r)$ for each $a \in A$. The *aggregate demand correspondence* is

$$D(r) = \left\{ \int_A f(a) d\lambda(a) \mid f \text{ is a demand selection at prices } r \right\}$$

The following Claim and Proposition hold under the assumpton that at each $r > 0$, there is an integrable demand selection. (If there is no notion of aggregate demand, the proposition has no meaning.)

Claim 4 *Suppose that A1 holds. Let* $(x, \mu), q$ *be a purchase-club equilibrium at prices* $r > 0$*, and let* $\{\omega^{\mu_a}\}_{a \in A}$ *be the associated consumptions of shared goods. Then (8) holds for almost every agent* $a \in A$*, where* x_a *satisfies (4).*

$$U_a \left(x_a, \omega^{\mu_a}\right) \geq U_a \left(e_a - \frac{r}{k} \cdot \omega, \ \omega\right) \text{ for all } \omega \in \Omega \tag{8}$$

Proof: Suppose the inequality (8) does not hold for a set of agents of positive measure, $\bar{A} \subseteq A$. We will find a feasible state of the economy $(\tilde{x}, \tilde{\mu})$ for which

$$U_a \left(\tilde{x}_a, \omega^{\tilde{\mu}_a}\right) \geq U_a \left(x_a, \omega^{\mu_a}\right) \text{ for all } a \in A \tag{9}$$

with strict inequality for $a \in \bar{A}$. This contradicts that the equilibrium (x, μ) is efficient.

Let f be a demand selection at prices $\frac{r}{k}$. Using the definition of a demand selection and Claim 3(i), the following holds for all $a \in A$ and holds with strict inequality for the agents $a \in \bar{A}$.

$$U_a\left(e_a - \frac{r}{k} \cdot f(a),\ f(a)\right) \geq U_a\left(e_a - \frac{r}{k} \cdot \omega^{\mu_a},\ \omega^{\mu_a}\right) = U_a\left(x_a, \omega^{\mu_a}\right) \tag{10}$$

Therefore we can complete the proof by constructing a feasible state $(\tilde{x}, \tilde{\mu})$ such that for each $a \in A$, $\omega^{\tilde{\mu}_a} = f(a)$, $\tilde{x}_a = e_a - \frac{r}{k} \cdot f(a)$.

For each $\bar{\omega} \in \Omega$, let $A^{\bar{\omega}} \equiv \{a \in A \mid f(a) = \bar{\omega}\}$. Suppose that $A^{\bar{\omega}}$ has positive measure. (If $A^{\bar{\omega}}$ has measure zero it is irrelevant.) Assign each $a \in A^{\bar{\omega}}$ to a single membership, in a group type $\pi^{\bar{\omega}} \in \mathcal{G}$ defined as follows. Let $\pi^{\bar{\omega}}(\bar{\omega}) = 1, \pi\left(\omega^0\right) = k-1$, where $\omega^0 = (0, 0, .., 0)$, and let $\pi^{\bar{\omega}}(\omega) = 0$ for $\omega \notin \{\bar{\omega}, \omega^0\}$. Then $|\pi^{\bar{\omega}}| = k$. For each $a \in \bar{A}^{\bar{\omega}}$, $\bar{\omega} = \sum_{\omega \in \Omega} \pi^{\bar{\omega}}(\omega)\omega = f(a)$. For the feasible state $(\tilde{x}, \tilde{\mu})$, (9) holds for all $a \in A$, and holds strictly for $a \in \bar{A}$. This contradicts that (x, μ) is efficient, and therefore contradicts that $(x, \mu), q$ is an equilibrium. $\square$

The inequality (8) characterizes agents' consumption of shared goods in a group equilibrium. Combined with (4), it becomes

$$U_a\left(e_a - \frac{r}{k} \cdot \omega^{\mu_a},\ \omega^{\mu_a}\right) \geq U_a\left(e_a - \frac{r}{k} \cdot \omega,\ \omega\right) \text{ for all } \omega \in \Omega \tag{11}$$

which looks very much like the definition of the demand correspondence (7) for individual purchases at prices $\frac{r}{k}$. This is the basis of the argument that follows, which says that proprietors have the same profit opportunities in both market circumstances.

A complication, however, is that neither the individual demand correspondence nor group equilibrium is necessarily unique. Consumers may be indifferent between these equilibria, but the proprietors will not be. Assuming that the proprietors price above marginal cost, they prefer more sales to fewer. Similarly, the several group equilibria at prices r will generate the same total utility for agents, but will generate different total profit for the proprietors.

Our objective below is to prove an "equivalence" from the proprietor's point of view between selling to individuals and selling to groups. We must define a notion of equivalence that accounts for the problem of multiple equilibria.

We show that, despite the multiple equilibria, the profit possibilities are the same whether the proprietors sell to individuals or to groups. Aggregate sales in the group equilibrium can be defined as[2]

$$\omega(x,\mu) = \int_A \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\ \omega\ \ d\lambda(a)$$

If z represents an aggregate demand vector at prices r/k and $\omega(x,\mu)$ represents aggregate sales to members of groups in a group equilibrium $(x,\mu),q$ at prices r, then the profits in the two situations are the same if (12) holds. The following proposition says that there is always an equivalence of that type.

$$z = k\omega\,(x,\mu) \tag{12}$$

Proposition 5 [Profit Equivalence] *Suppose that A1 holds.*
(i) Let $(x,\mu),q$ be a purchase-club equilibrium at prices $r>0$. Then $k\omega(x,\mu) \in D(r/k)$.
(ii) Let $z\in D(r/k)$ be an aggregate demand vector at prices $r/k>0$. Then there exists a purchase-club equilibrium $(x,\mu),q$ at prices r, with aggregate purchases $\omega(x,\mu)=z/k$.

Proof: (i) If $\{\omega^{\mu_a}\}_{a\in A}$ are the consumptions associated with the group equilibrium, they satisfy

$$\frac{1}{k}\int_A \omega^{\mu_a}\ d\lambda(a) = \int_A \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\ \omega\ \ d\lambda(a) = \omega(x,\mu) \tag{13}$$

[2]Since (x,μ) is an equilibrium, μ is integrable. Hence the sets $\{a\in A\ \mid\ \mu_a(\omega,\pi)=t\}$ for $t=0,1,...,M$, are measurable. But then the functions defined by $\mu_a(\omega,\pi)\omega$ and $\mu_a(\omega,\pi)\left[\sum_{\omega\in\Omega}\pi(\omega)\omega\right]$ are also integrable, hence the function defined by ω^{μ_a} is integrable.

This is because there are k agents consuming every purchased good. Since $\{\omega^{\mu_a}\}_{a\in A}$ satisfy (11), they are also a demand selection at prices $\frac{r}{k}$. Hence $k\omega(x,\mu) \in D(r/k)$.

(ii) The aggregate demand can be written $z = \int_A f(a)\, d\lambda(a)$ for a demand selection f at prices $\frac{r}{k}$. For the selection f, we will construct an equilibrium $(x,\mu), q$ as in the proof of Claim 4, using prices q described by (3) with equality. To show that (x_a,μ_a) is optimal for $a \in A$, we must show that $U_a(x_a,\omega^{\mu_a}) \geq U_a(x_a^\ell,\omega^\ell)$ for any budget-feasible (x_a^ℓ,ℓ). Due to the choice of q and $|\pi| \leq k$, any budget-feasible $(x_a^\ell,\ell) \in X_a$ satisfies $x_a^\ell \leq e_a - \frac{r}{k}\cdot\omega^\ell$.

Using the constructed $(x,\mu), q$ and the definition of a demand selection, for each $a \in A$, $U_a(x_a,\omega^{\mu_a}) = U_a(e_a - \frac{r}{k}\cdot f(a), f(a)) \geq U_a(e_a - \frac{r}{k}\cdot\omega,\omega)$ for all $\omega \in \Omega$ such that $e_a - \frac{r}{k}\cdot\omega \geq 0$. If $\omega^\ell \in \Omega$, a budget-feasible (x_a^ℓ,ℓ) satisfies $U_a(x_a,\omega^{\mu_a}) \geq U_a(e_a - \frac{r}{k}\cdot\omega^\ell,\omega^\ell) \geq U_a(x_a^\ell,\omega^\ell)$. If $\omega^\ell \notin \Omega$, then by A1 there exists $\omega \in \Omega$ such that $U_a(e_a - \frac{r}{k}\cdot\omega^\ell,\omega) \geq U_a(e_a - \frac{r}{k}\cdot\omega^\ell,\omega^\ell)$. But since $e_a - \frac{r}{k}\cdot\omega > e_a - \frac{r}{k}\cdot\omega^\ell$, $U_a(e_a - \frac{r}{k}\cdot\omega,\omega) > U_a(e_a - \frac{r}{k}\cdot\omega^\ell,\omega) \geq U_a(e_a - \frac{r}{k}\cdot\omega^\ell,\omega^\ell) \geq U_a(x_a^\ell,\omega^\ell)$. But then $U_a(x_a,\omega^{\mu_a}) \geq U_a(e_a - \frac{r}{k}\cdot\omega,\omega) > U_a(x_a^\ell,\omega^\ell)$ for some $\omega \in \Omega$. Hence $(x,\mu), q$ is a purchase-club equilibrium at prices r. To complete the proof, notice that $z = \int_A f(a)\, d\lambda(a) = k\omega(x,\mu) \in D(r/k)$. $\square$

5 Rental Markets

Example 4 in Section 3 suggests that the club model can be interpreted as a rental market. Our objective here is to elaborate that example, and show circumstances in which sharing groups are equivalent to how we would conceive of a rental market in ordinary general equilibrium theory.

The easiest way to think of rental markets is that there is an amortized cost of keeping the rental good continuously in use. The competitive price of using it will reflect this amortized cost. If this is all there is to it, then general equilibrium theory as conceived by Arrow and Debreu can easily account for rental markets, even if demand is not time-invariant. If, for example, there are peak and off-peak demand periods (in the case of sailboats, balmy summer days and dark winter days), then we might think of rentals in the two periods as jointly produced, but different, goods. Price cannot equal "marginal cost" in both periods, since the price in the two periods will be different.

We now show how the club model accommodates rental markets, allowing the quality of the rentals (in the sense of inconvenience due to congestion) to be endogenous, and differentiating prices according to peak and off-peak periods.

Pricing in the club model is more flexible than in a rental market. Prices in an ordinary rental market are linear on units of usage, although possibly different in peak and off-peak periods. We show conditions under which rental prices in a group equilibrium can also be interpreted as linear prices on usage.

Let elements of Ω represent *usage*. In particular, for fixed k, represent usage by

$$\Omega = \{(\omega_p, \omega_o) | \omega_p \in \{0, 1, 2, ...k\}, \omega_o \in \{0, 1, 2, ...k\}\},$$

where the membership characteristic $(\omega_p, \omega_o) \in \Omega$ represents the number of units of rental of each type, peak and off-peak. As in the model of the previous section, this model specializes Ω to be a space of numbers rather than an abstract space.

A *rental group type* is (π, γ, y), where y represents the rental goods bought in a competitive market, and $\gamma \in \Gamma$ specifies the total usage offered by the rental group at both peak and offpeak times. In particular, $\Gamma = \left\{\{1, 2, ...\bar{\gamma}_p\} \times \{1, 2, ...\bar{\gamma}_o\}\right\}$, and γ can be written (γ_p, γ_o). A *feasible* rental group type (π, γ, y) satisfies $\pi \in \Pi(\gamma)$, where

$$\Pi(\gamma) = \left\{ \pi : \Omega \to Z_+ \quad | \sum_{(\omega_p, \omega_o) \in \Omega} \pi(\omega_p, \omega_o)\omega_p = \gamma_p, \sum_{(\omega_p, \omega_o) \in \Omega} \pi(\omega_p, \omega_o)\omega_o = \gamma_o \right\}$$

What we have in mind are rental groups that offer the same rental goods, such as a single sail boat, else total usage $\gamma \in \Gamma$ would not be related to congestion in any obvious way. We shall therefore assume that all grouptypes have the same input vector y. We shall thus leave y out of the description of a group type, although it remains in the budget balance condition for each rental grouptype.[3] The feasible set of grouptypes and memberships are

$$\begin{aligned} \mathcal{G} &= \{(\pi,\gamma) \mid \gamma = (\gamma_p,\gamma_o) \in \Gamma, \pi \in \Pi(\gamma)\} \\ \mathcal{M} &= \{((\omega_p,\omega_o),(\pi,\gamma)) \mid (\omega_p,\omega_o) \in \Omega, \quad (\pi,\gamma) \in \mathcal{G}\} \end{aligned}$$

Let $\omega^\ell = (\omega_p^\ell, \omega_o^\ell) : \Gamma \to Z_+ \times Z_+$ represent *usage* associated with the list ℓ. For each $\hat{\gamma} \in \Gamma$,

$$\begin{aligned} \omega_p^\ell(\hat{\gamma}) &= \sum_{\{(\pi,\gamma)\in\mathcal{G}\mid\gamma=\hat{\gamma}\}} \ell((\omega_p,\omega_o),(\pi,\hat{\gamma})) \;\; \omega_p \qquad (14) \\ \omega_o^\ell(\hat{\gamma}) &= \sum_{\{(\pi,\gamma)\in\mathcal{G}\mid\gamma=\hat{\gamma}\}} \ell((\omega_p,\omega_o),(\pi,\hat{\gamma})) \;\; \omega_o \end{aligned}$$

Consumption sets are constrained in that individual usage has an upper bound:

$$X_a = \Re_+ \times \textbf{Lists} \text{ where } \textbf{Lists} = \left\{\ell \in Z_+^{\mathcal{M}} : \omega_p^\ell(\gamma), \omega_o^\ell(\gamma) \le K, \text{ each } \gamma \in \Gamma\right\}$$

for some positive number K.

The condition under which we can prove that the club equilibrium is equivalent to a rental market is if utility functions $u_a : X_a \to \Re$, can be expressed as

$$u_a(x,\ell) = U_a(x,\omega^\ell) \qquad (15)$$

where $U_a : \tilde{X}_a \to \Re$ represents utility as a function of the other private goods and rental usage, and

$$\tilde{X}_a = \Re_+^N \times \left\{(\omega_p^\ell, \omega_o^\ell) : \ell \in \textbf{Lists}\right\}.$$

We say that a group equilibrium $(x,\mu),(p,q)$ is *equivalent to equilibrium in a rental market* if there exist rental prices $(\theta_p,\theta_o) : \Gamma \to R \times R$ such that, if $\ell \in$ **Lists**, the

[3] If the inputs y could vary for each level of service γ, all group types used in equilibrium would be those with the minimum $p \cdot y$. An alternative formulation would allow preferences to depend on usage in each type of club characterized by (γ, y) rather than γ, so that the combination (γ, y) determines the quality of the rental rather than just γ.

price of the list ℓ satisfies

$$(\theta_p, \theta_o) \cdot \left(\omega_p^\ell, \omega_o^\ell\right) = \ell \cdot q \tag{16}$$

The important feature of rental markets is that they impose a restriction on prices. The membership price $q((\omega_p, \omega_o\ ), (\pi, \gamma))$ will reflect the member's peak usage and offpeak usage, as well as the congestion. In the club model, there is no *a priori* restriction that the price $q((\omega_p, \omega_o\ ), (\pi, \gamma))$ can be conceived as a linear price on usage, and that the linear price is the same as that of other users, scaled by usage. In a rental market, that is the natural restriction. We now show that all agents pay a price that is the same linear function of usage, regardless of usage, and regardless of how they divide usage among different rental units.

Proposition 6 *Suppose that preferences can be expressed as (15). Then for every group equilibrium* $(x, \mu), (p, q')$ *there is another group equilibrium* $(x, \mu), (p, q)$ *that is equivalent to an equilibrium in a rental market.*

Proof of Proposition 6: To define the prices (θ_p, θ_o) in the rental market, we first define some distinguished group types that offer rentals in individual units, rather than selling usage in bulk. For each $\gamma \in \Gamma$, let (π^γ, γ) be a group type such that $\pi^\gamma(0,1) = \gamma_o$, $\pi^\gamma(1,0) = \gamma_p$, and $\pi(\omega_p, \omega_o) = 0$ for $(\omega_p, \omega_o) \notin \{(0,1),(1,0)\}$. For example, a membership $((0,1), (\pi^\gamma, \gamma))$ is a single off-peak use.

Let $(x, \mu), (p, q')$ be a group equilibrium. To define the prices in the rental market, for each $\gamma \in \Gamma$ let

$$\begin{aligned} \theta_p(\gamma) &= q'((1,0), (\pi^\gamma, \gamma)) \\ \theta_o(\gamma) &= q'((0,1), (\pi^\gamma, \gamma)) \end{aligned} \tag{17}$$

For the prices (17), we will show that (16) holds for each list that is chosen in equilibrium by a set of agents of positive measure. However (16) does not necessarily hold for lists that are not chosen. To guarantee that (16) holds for all lists, we will construct another equilibrium $(x,\mu),(p,q)$. The prices q' and q will differ only for memberships in group types that are not used in equilibrium; that is, group types for which $\alpha(\pi,\gamma)=0$. Let

$$\begin{aligned} q(\omega,(\pi,\gamma)) = \quad & q'(\omega,(\pi,\gamma)) \\ & \text{for all } \omega\in\Omega \text{ if } \alpha(\pi,\gamma)>0. \\ \\ q(\omega,(\pi,\gamma)) = \quad & \omega_p \;\; q'((1,0),(\pi^\gamma,\gamma)) + \omega_o \;\; q'((0,1),(\pi^\gamma,\gamma)) \\ & \text{for all } \omega\in\Omega \text{ if } \alpha(\pi,\gamma)=0 \end{aligned}$$

It follows that for each $\gamma\in\Gamma$

$$\begin{aligned} \theta_p(\gamma) &= q((1,0),(\pi^\gamma,\gamma)) \\ \theta_o(\gamma) &= q((0,1),(\pi^\gamma,\gamma)) \end{aligned}$$

Claim 7 *$(x,\mu),(p,q)$ is a group equilibrium. For each list $\ell\in$* **Lists** *which is not chosen in equilibrium by a set of agents of positive measure, (16) holds for q and θ.*

Proof of Claim 7: We show that almost every agent's optimizing choice (x_a,μ_a) is the same under price systems (p,q) and (p,q'), and that (16) holds for q and θ.

Let ℓ be a list that is not chosen in the equilibrium $(x,\mu),(p,q')$ by any group of agents with positive measure. For this list and almost all agents $a\in A$, there is no (x,ℓ) that is budget feasible at prices q' and strictly preferred to (x_a,μ_a). We must show that there is also no (x,ℓ) that is budget-feasible at prices q and strictly preferred to (x_a,μ_a). Suppose to the contrary that there is. But then we can construct a list ℓ' and a budget-feasible (x,ℓ') that is strictly preferred to (x_a,μ_a) at prices q', which is a contradiction.

To construct the list ℓ', assign memberships as follows: for each $\gamma\in\Gamma$, $\ell'((0,1),(\pi^\gamma,\gamma)) = \omega_o^\ell(\gamma)$, $\ell'(1,0),(\pi^\gamma,\gamma)) = \omega_p^\ell(\gamma)$. Then the lists ℓ and ℓ' provide the same usage $\omega^\ell=\omega^{\ell'}$, and

$$\ell\cdot q = \ell'\cdot q = \ell'\cdot q' = (\theta_p,\theta_o)\cdot\left(\omega_p^\ell,\omega_o^\ell\right).$$

(The cost $\ell\cdot q'$ can be greater or smaller than this.)

It holds that $\mu_a \cdot q = \mu_a \cdot q'$ for almost all $a \in A$. If (x, ℓ) is budget-feasible at prices q and strictly preferred to (x_a, μ_a), then (x, ℓ') is budget feasible at prices q and strictly preferred to (x_a, μ_a), so (x, ℓ') is budget-feasible at prices q' and strictly preferred to (x_a, μ_a), which contradicts that (x, μ),(p, q') is an equilibrium. $\square$

We have thus shown that (16) holds for lists ℓ that are not chosen in equilibrium by a set of agents of positive measure. We must argue that (16) also holds for lists that are chosen in the equilibrium $(x, \mu), (p, q)$, namely $\{\mu_a | a \in A\}$.

Construct a consistent list assignment $\{\tilde{\mu}_a | a \in A\}$ with the same individual usage as in the equilibrium lists $\{\mu_a | a \in A\}$, but in individual units rather than as aggregated memberships. For each $a \in A$, $\gamma \in \Gamma$, let

$$\begin{aligned}
\tilde{\mu}_a((1,0), (\pi^\gamma, \gamma)) &= \omega_p^{\mu_a}(\gamma) \\
\tilde{\mu}_a((0,1), (\pi^\gamma, \gamma)) &= \omega_o^{\mu_a}(\gamma) \\
\tilde{\mu}_a((\omega_p, \omega_o), (\pi, \gamma)) &= 0 \qquad \text{for all other memberships}
\end{aligned}$$

Then the following holds by construction for almost all $a \in A$.

$$\tilde{\mu}_a \cdot q = (\theta_p, \theta_o) \cdot \left(\omega_p^{\mu_a}, \omega_o^{\mu_a}\right) \tag{18}$$

Since $\omega^{\mu_a} = \omega^{\tilde{\mu}_a}$ for all $a \in A$, the following holds for almost every $a \in A$. Otherwise there would be a budget-feasible $(\tilde{x}_a, \tilde{\mu}_a)$ that would be preferred to (x_a, μ_a).

$$\mu_a \cdot q \leq \tilde{\mu}_a \cdot q \tag{19}$$

The assignment $\tilde{\mu}$ is consistent because μ is consistent. Since $\omega^{\mu_a}(\gamma) = \omega^{\tilde{\mu}_a}(\gamma)$ for each $\gamma \in \Gamma$ and every $a \in A$, the number of groups associated with $\tilde{\mu}$ is the same as the number associated μ, and each has the same cost $p \cdot y$. Since q balances the budget for each group type, μ and $\tilde{\mu}$ must generate the same revenue:

$$\int_A \mu_a \cdot q \; d\lambda(a) = \int_A \tilde{\mu}_a \cdot q \; d\lambda(a) \tag{20}$$

But this proves that (19) cannot hold with strict inequality for a set of agents with positive measure. Using Claim 7, (18) and (19), which holds with equality for almost all $a \in A$, we can conclude that (16) holds for all lists $\ell \in \mathbf{Lists}$. $\square$

5.1 Conclusion

The club model can account for consumption externalities in various ways. By consumption externalities, we mean that each member of a club cares about the private-goods consumption of other members. The models above elaborate that idea by introducing different technologies of sharing, and showing how the technologies of sharing can be reflected in group types and membership characteristics.

The term "consumption externality" suggests that each agent makes a consumption decision without considering its impact on others. The club model forces him to consider the impact. Groups that want to avoid negative externalties that arise from private consumption decisions will have memberships that involve a commitment to avoid consumption of certain private goods. Groups that want to generate positive externalities due to private consumption decisions will have membership characteristics that require certain kinds of consumption. These commitments can be built into feasible consumption sets, which can constrain the consumption of private goods in a way that is linked to memberships in groups.

The technology of sharing private goods was more precise in what we called purchase clubs and rental clubs. In the case of automobiles, sailboats and other durable goods which cannot be used simultaneously by all members of a group, the terms of sharing must be specified in the group type and the membership characteristics. Nothing requires that congestible durable goods be shared in rental groups rather than purchase groups; in fact, there is no clear distinction between those two concepts. We only chose those terms to suggest familiar market institutions, and to give different names to models based on different sharing technologies. The key point is that, if users care about total congestion as well as their own usage, then membership prices must reflect both. And membership prices may also reflect the externality-producing private goods that a member brings as part of his membership.

In the purchase-club and rental-club models, we respectively treated proprietary pricing and congestion costs. Of course proprietary pricing and congestion can be combined in the same model: Goods that are purchased at proprietary prices can nevertheless be subject to congestion. A group equilibrium will be efficient for the users conditional on the proprietary prices, but this is a conditional notion of efficiency. Each copy of a proprietary good that is subject to congestion may be used "too much" in equilibrium, to conserve on paying the proprietary price.

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