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Equilibria with Arbitrary Market Structure

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Topics in Mathematical Economics

The participants in a September 2002 Workshop on *Topics in Mathematical Economics* in honor of Birgit Grodal decided to have a series of papers appear on Birgit Grodal's 60'th birthday, June 24, 2003.

The Institute of Economics suggested that the papers became Discussion Papers from the Institute.

The editor of *Economic Theory* offered to consider the papers for a special Festschrift issue of the journal with Karl Vind as Guest Editor.

This paper is one of the many papers sent to the Discussion Paper series.

Most of these papers will later also be published in a special issue of *Economic Theory*.

Tillykke Birgit

Troels Østergaard Sørensen	Karl Vind
Head of Institute	Guest Editor

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Abstract

Fifty years ago Arrow introduced contingent commodities and Debreu observed that this reinterpretation of a commodity was enough to apply the existing general equilibrium theory to uncertainty and time. This interpretation of general equilibrium theory is the Arrow-Debreu model. The complete market predicted by this theory is clearly unrealistic, and Radner(1972) formulated and proved existence of equilibrium in a multiperiod model with incomplete markets. Hart(1975) showed that the lower bound on consumption sets assumed by Radner was an essential limitation, but needed for the result.

The problem raised by the Hart examples was eventually solved by Duffie and Shafer (1985,86) . In these papers are shown *generic* existence of equilibria in economies with incomplete markets.

In this paper the Radner result is extended in other directions. Radner assumed a specific structure of markets, independence of preferences, indifference of preferences, and total and transitive preferences. All of these assumptions are dropped here.

1 Introduction

Arrow(1953) (Arrow 1953) introduced contingent commodities and Debreu(1953) (Debreu 1953) observed that this reinterpretation of a commodity was enough to apply the existing general equilibrium theory to uncertainty and time. This interpretation of general equilibrium theory is the Arrow-Debreu model. The complete market predicted by this theory is clearly unrealistic, and Radner(1972) (Radner 1972) formulated and proved existence of equilibrium in a multiperiod model with incomplete markets. Hart(1975) (Hart 1975) showed that the lower bound on consumption sets assumed by Radner was an essential limitation, but needed for the result.

The problem raised by the Hart examples was eventually solved by Duffie and Shafer (1985,86) (Duffie and Shafer 1985,1986). In these papers are shown *generic* existence of equilibria in economies with incomplete markets.

In this paper the Radner result is extended in other directions. Radner assumed a specific structure of markets, independence of preferences, indifference of preferences, and total and transitive preferences. All of these assumptions are dropped here.

A *premarket* can be any linear subspace of the commodity space, and a *market* is then what can be bought at the premarket at a *price*. A *market structure* is any set of premarkets, and there are no assumptions needed for the market structure.

The preferences are defined on the state space, so they may depend on the trades on the markets - and not just on the sum of the trades over all markets -, on the trades of the other consumers, and on the prices. The preferences are not assumed to be total and transitive. The possibility that the preferences depend on how the consumer obtains his consumption, is needed if this type of theory is used to understand what the market structure will be, and it is useful if we want to understand the role of money.

The way the existence theorem is proved may be as important as the theorem. The economy with incomplete markets is translated into an economy with a complete market. The translation is that each **old** consumer is made into a set of **new** consumers, one for each market, and each commodity is made into a set of commodities, one for each market.

A **new** consumer will however not take as given the trades of the other copies of the same **old** consumer so a new equilibrium concept *Walras equilibrium with coordination* is needed. (Grodal and Vind 2001) gives an existence theorem for this equilibrium, and this theorem is applied here to give existence of first Walras equilibrium with coordination in the new economy and then by a simple translation an equilibrium with incomplete markets.

Letting the set of premarkets be very large (for example including subspaces spanned by coordinates and diagonals) we would with realistic assumptions on preferences get that only very few markets will be used in an equilibrium, and only under special assumptions would the market structure used in equilibrium be complete.

The way the theorem is proved shows that incomplete markets can be regarded both as a generalization of a complete market and - by a reinterpretation of commodities and consumers - as a special case of an economy with a complete market. This means that economies with incomplete markets can be regarded as special cases of economies with externalities, and more generally that the existing theory of Walras equilibria, optimality, fairness etc. can be used on the special case corresponding to an economy with incomplete markets.

2 The economy

$$E = \left(L, C, I, (M_i)_{i \in I}, (X_c, \mathcal{P}_c, I_c)_{c \in C}\right)$$

is the economy studied

Notation	**Interpretation**
L, finite set, $h \in L$	commodities
C, finite set, $c \in C$	consumers
I, finite set, $i \in I$	market institutions
$M_i \subset \mathbb{R}^L$, linear subspace, $i \in I$	premarket i,
$X_i = \left\{x \in \mathbb{R}^{LIC} \mid \sum_c x(i,c) = 0\right\}$	feasible set for market i
$X_c \subset \mathbb{R}^{LIC}$	feasible set for consumer c
$\mathcal{P}_c : X_c \to 2^{X_c}$	preferences of consumer c
$I_c \subset I$	premarkets available to consumer c
$T = \left(\mathbb{R}^L \setminus \{0\}\right)^I$	prices

3 Definitions

Definition 1 (Natural premarket) *A premarket is* natural *if it is spanned by points from* $\mathbb{R}^L_+$

Definition 2 (Premarket structure) *A* premarket structure $(M_i)_{i\in I}$ *is any finite family of natural premarkets.*

Definition 3 (Market i) *Given a* $p = (p_i)_{i\in I} \in T = \left(\mathbb{R}^L \setminus \{0\}\right)^I$ *we define* market i *as*

$$M_i(p) = \{z \in M_i \mid p_i z = 0\}$$

Definition 4 (Market structure) *The* market structure *is then*

$$(M_i(p))_{i\in I}$$

Definition 5 (Complete) *A premarket is* complete *if*

$$M = \mathbb{R}^L,$$

a market structure with a price p is complete *if*

$$\dim \sum_{i\in I} M_i(p) \geq L - 1$$

and a premarket structure is complete with respect to $T_0 \subset T$ *if the market structure is compete with respect to all $p \in T_0$ i.e.*

$$\dim \sum_{i\in I} M_i(p) \geq L - 1 \textit{ for all } p \in T_0 \textit{ with } p_i M_i \neq \{0\}$$

4 The state space

Definition 6 (State space) *The state space for E is now $\mathbb{R}^{LIC}$*

Notation 7 *The notation*

$$x = (x(i))_{i \in I} = (x(c))_{c \in C} = (x(h))_{h \in H} =$$

$$(x(i,c))_{(i,c) \in IC} = (x(h,i,c))_{(h,i,c) \in LIC} \in \mathbb{R}^{LIC}$$

will be used for elements in the state space.

Definition 8 (Market feasible) *A state x is **market feasible for c given the price** p if*

$$x \in V_c(p) = \left\{ x \in \mathbb{R}^{LIC} \left| \begin{array}{ll} x(i,c) \in M_i(p), & \forall i \in I_c, \\ x(i,c) = 0, & \forall i \notin I_c \end{array} \right. \right\}$$

Definition 9 (Feasible state-price) *A **state and a price** (x,p) **is feasible** if*

$$x \in F(E) = \bigcap_{c \in C} X_c \cap \bigcap_{i \in I} X_i \cap \bigcap_{c \in C} V_c(p) = X_C \cap X_I \cap V_C(p)$$

5 Equilibrium

Definition 10 (Equilibrium) *A **state and a price** (x,p) **is equilibrium in** E if it is feasible and there does not exist a $y \in \mathbb{R}^{LIC}$ and a $c \in C$ such that*

$$y \in X_c \cap V_c(p) \cap \mathcal{P}_c(x) \;\; \textit{where}$$

$$y(i,d) = x(i,d) \;\; \textit{for } d \neq c.$$

Definition 11 ($Eq(E)$) *The **equilibrium set** will be denoted $Eq(E) \subset \mathbb{R}^{LIC} \times T$*

Definition 12 (Walras equilibrium) *A **state and a price** (x,p) **is Walras equilibrium in** E if it is equilibrium in E and*

$$M_i(p) = \{z \in M_i \,|\, qz = 0\}$$

for some $q \in \mathbb{R}^{L} \setminus \{0\}$ for all $i \in I$

Definition 13 ($W(E)$) *The set of **Walras equilibria** will be denoted*

$$W(E) \subset \mathbb{R}^{LIC} \times T$$

Definition 14 (Complete Walras equilibrium) *A* ***state and a price*** (x, p) ***is a complete Walras equilibrium in*** E *if it is Walras equilibrium in* E *and the market structure is complete (with respect to* p*)*

Definition 15 **(**$CW(E)$**)** *The set of* ***complete Walras equilibria*** *will be denoted*

$$CW(E) \subset \mathbb{R}^{LIC} \times T$$

The relations

$$CW(E) \subset W(E) \subset Eq(E) \subset F(E) \times T \subset \mathbb{R}^{LIC} \times \mathbb{T}$$

are obvious

6 Examples

Example 16 (CompleteWalras equilibrium with one market) *Let* $I = \{1\}$ *and* $M_1 = \mathbb{R}^L$*, and let* $I_c = I$ *for all* $c \in C$*, then* equilibrium in E *is obviously* a complete Walras equilibrium (with one market)

Example 17 (The prototype two period financial market)

See (Duffie and Shafer 1985,1986). Let $t = 0, 1$ be two points in time. At time $t = 1$ there are $\{1, 2, \cdots, S\} = \mathbf{S}$ states of nature. At time 0 and at each of the states there are ℓ commodities, so the commodity space is $\mathbb{R}^L$ where $L = \ell(1 + S)$. The market structure consists of spot markets for each $s \in \{1, 2, \cdots, S\}$ and for $t = 0$ a spot market for the commodities and for k real assets $a^1, a^2, \cdots, a^k \in \mathbb{R}^L$. Denote by A the $L \times k$ matrix with a^j as the j'column.
In our terminology we have in the economy a premarket structure $(M_s)_{s=0}^S$, which is available to all consumers. For $s \in \mathbf{S}$ the premarket is defined by

$$M_s = \left\{ z \in \mathbb{R}^L \,|\, z(h, s') = 0, \text{ for } \forall h \text{ and } s \neq s' \right\}$$

for $t = 0$ the premarket M_0 is defined by

$$M_0 = \left\{ z \in \mathbb{R}^L \,\middle|\, y \in R^{\ell+k} \text{ with } \widetilde{A}y = z \right\}$$

where

$$\widetilde{A} = \underset{L \times (\ell + k)}{\begin{pmatrix} \begin{matrix} E \\ 0 \end{matrix} & A \end{pmatrix}}$$

where $\underset{\ell \times \ell}{E}$ and $\underset{s\ell \times \ell}{0}$ are unit and zero matrices

7 The existence theorem

Theorem 18 *Let E be the economy as described above.*
Assume:

For each $c \in C$ that $(X_c, \mathcal{P}_c)$ satisfies

X_c is convex, $0 \in intX_c$ (interior relative to spanX_c), and $\left\{x(c) \in \mathbb{R}^{LI} \,|x \in X_c\right\}$ is bounded below.

$\mathcal{P}_c$ is irreflexive and has open graph and convex values

For all $x \in X_c$

$$\mathcal{P}_c(x) \supset \{x\} + \overline{\mathbb{R}}_{+}^{LI} \backslash \{0\}$$

There exist $(X^c, \mathcal{P}^c)$ such that $intX^c \supset X_c$, $\mathcal{P}_c(x) = \mathcal{P}^c(x) \cap X_c$ for all $x \in X_0$, and $(X^c, \mathcal{P}^c)$ satisfies (i) and (ii)

Then $Eq(E) \neq \emptyset$

Proof. Define the economy

$$\mathsf{E} = \left(\mathsf{L}, \mathsf{C}, (\mathsf{X_c}, \mathsf{P_c}, e_{\mathsf{c}})_{\mathsf{c} \in \mathsf{C}}, g\right)$$

from the economy

$$E = \left(L, C, I, (M_i)_{i \in I}, (X_c, \mathcal{P}_c, I_c)_{c \in C}\right)$$

by

$L \times I = \mathsf{L}$	a finite set of $\nu-$commodities
$C \times I = \mathsf{C}$	a finite set of $\nu-$consumers
$\nu : \mathbb{R}^{LI} \to \mathbb{R}^{\mathsf{LC}}$	$\nu(x)(\mathsf{h}, \mathsf{c}) = \nu(x)(h, i, c, j) = x(h, i, c), i = j$
$\nu\left(X_c \cap \prod_{i \in I_c} M_i\right) = \mathsf{X_c} \subset \mathbb{R}^{\mathsf{LC}}$	the feasible set for consumer c
$\mathsf{P_c} : \mathsf{X_c} \to 2^{\mathsf{X_c}}$,	the preferences of consumer c
$e_{\mathsf{c}} : \mathsf{Y} \times \mathsf{Y} \to \mathsf{Y}$	coordination function
$g : \mathsf{Y} \times \mathsf{Y} \to 2^{\mathsf{C}}$	approval function
$T = \left(\mathbb{R}^L \backslash \{0\}\right)^I \subset \mathbb{R}^{IL} \backslash \{0\}$	prices

Most of the notation explains itself.

ν maps $\left(\mathbb{R}^{LC}\right)^I$ into the diagonal of $\left(\mathbb{R}^{LC}\right)^{2I}$.

Definition 19 (y) $\in \mathsf{P_c}(\nu(x))$ *if and only if* $y \in \mathcal{P}_c(x), \mathsf{c} = (c, i)$ *(so all* c *coming from the same c have the same preferences).*

Definition 20 (e_{c}) $e_{\mathsf{c}}(x, y) = \left(\left(x(d)_{d \neq c}\right), y(c)\right)$ *for* $\mathsf{c} = (c, i)$. *All new consumers coming from the same old consumer coordinates their trades, and no one else coordinate.*

Definition 21 (g) $g(x, y) = \{\mathsf{c} \in \mathsf{C} \,| x(c) \neq y(c), \mathsf{c} = (c, i)\}$

E is now an economy with coordination with a feasible set

$$F(\mathsf{E}) = \bigcap_{\mathsf{c}\in\mathsf{C}} \mathsf{X_c} \cap \left\{ \mathsf{x} \in \mathbb{R}^{\mathsf{LC}} \middle| \sum_{\mathsf{c}\in\mathsf{C}} \mathsf{x}(\mathsf{c}) = 0 \right\}$$

it is easy to check that

$$\nu(F(E)) = F(\mathsf{E}).$$

A feasible state x and a price $p \in \mathbb{R}^{IL} \setminus \{0\}$ is a Walras equilibrium with coordination in E if there does not exist a

$$\mathsf{y} \text{ and a } \mathsf{c} \text{ such that } e_{\mathsf{c}}(\mathsf{x}, \mathsf{y}) \in \mathsf{P_c}(\mathsf{x}) \text{ and } p\mathsf{y}(\mathsf{c}) = 0$$

The set of equilibria will denoted $Eq(\mathsf{E})$. It is clear that

$$(x, p) \in Eq(E) \Rightarrow (\nu(x), p) \in Eq(\mathsf{E})$$

and conversely if $p_i \neq 0$ for all $i \in I$. The function ν is linear and the sets $\sum_{i\in I_c} \nu(M_i)$ are closed and convex (linear subspaces) so all the properties of $(X_c, \mathcal{P}_c)_{c\in C}$ are inherited by $(\mathsf{X_c}, \mathsf{P_c})_{\mathsf{c}\in\mathsf{C}}$ This again means that all the properties assumed in theorem 25 page 9 will hold . So the economy E with the assumptions inherited from E is a special case of the E economy with the assumptions from theorem 25 page 9. The assumption that there is local non-satiation in all markets means that no p_i can be non-zero, so the existence of equilibria in E gives equilibria in E ■

8 Indifference

The special case of indifference gives back Walras equilibrium if the market structure is complete.

In order to define indifference we define for all B the map

$$\sum\nolimits_I : \mathbb{R}^{BI} \to \mathbb{R}^B \text{ by } \sum\nolimits_I (x) = \sum_{i\in I} x(i) \in \mathbb{R}^B \qquad (\textstyle\sum_I)$$

Definition 22 (Indifference) *A consumer i in the economy E is indifferent (between the institutions in I) if*

$$\sum\nolimits_I^{-1} \sum\nolimits_I (X_c \times gr\mathcal{P}_c) = X_c \times gr\mathcal{P}_c \subset \mathbb{R}^{3LIC} \qquad (Indifference)$$

The interpretation is that only the sum over all institutions matters for whether a trade is in the feasible set for a consumer or for whether a trade is preferred to another. $\sum_I X_c \subset \mathbb{R}^{LIC}$ will be the net trade space for consumer c and $\sum_I gr\mathcal{P}_c = gr \sum_I \mathcal{P}_c \subset \mathbb{R}^{2LIC}$ will be the preferences $\sum_I \mathcal{P}_c$ for net trades. With the independence assumption

$$y \in \mathcal{P}_c(x) \Leftrightarrow \sum\nolimits_I y \in \sum\nolimits_I \mathcal{P}_c\left(\sum\nolimits_I x\right)$$

All properties of $(X_c, \mathcal{P}_c)$ in theorem 18 page 6 will be inherited by $(\sum_I X_c, \sum_I \mathcal{P}_c)$. If $\mathcal{P}_c$ is monotone, $\sum_I \mathcal{P}_c$ is monotone.

Theorem 23 *Assume in the economy E that all consumers are indifferent, and that preferences are monotone and convex valued. Then any equilibrium will be Walras equilibrium, i.e.*

$$W(E) = Eq(E)$$

Proof. Define the economy

$$\sum\nolimits_I E = \left(L, C, \left(\sum\nolimits_I M_i\right), \left(\sum\nolimits_I X_c, \sum\nolimits_I \mathcal{P}_c\right)_{c \in C}\right) \qquad (\textstyle\sum_I E)$$

as the same economy, except that there is only one premarket, namely the sum in $\mathbb{R}^L$ over all the premarkets in E. Because of the indifference assumption, for any equilibrium (x, p) in $E, \sum_I x$ will be equilibrium in $\sum_I E$ in the market $\sum_I M_i(p)$. $\sum_I M_i(p)$ is a linear subspace containing $\sum_I x(c)$ for all $c \in C, \sum_I \mathcal{P}_c(\sum_I x)$ is convex and disjoint from $\sum_I M_i(p)$, so a separating hyperplane argument implies that $\sum_I M_i(p)$ is contained in a hyperplane in $\mathbb{R}^L$, so there exist a $q \in \mathbb{R}^L_+$ such that

$$M_i(p) \subset \{z \in M_i \,|qz = 0\}$$

for all $i \in I$ ■

Theorem 24 *Assume in the economy E that all consumers are indifferent, and that preferences are monotone and convex valued. Then any equilibrium in an economy with a complete market structure will be a complete Walras equilibrium, i.e*

$$CW(E) = Eq(E)$$

Proof. Trivial from the preceding theorem and the definition of complete market ■

9 Conclusion

Transaction costs for the consumers are not explicit in this paper, but are of course implicit in the preferences on the state space. Transaction costs could be one reason for not making the indifference assumption. Transaction costs can also be used to justify the compactness assumption.

Letting the set of premarkets be very large (for example including subspaces spanned by coordinates and diagonals) we would with realistic assumptions on preferences get that only very few markets will be used in an equilibrium, and only under special assumptions would the market structure used in equilibrium be complete.

The most serious limitation in the results in this paper, as an explanation of which markets will exist, is the assumption of no transaction costs for the market, and no surplus for a market agent $\left(X_i = \left\{x \in \mathbb{R}^{LIC} \,| \sum_c x(i,c) = 0\right\}\right)$.

If there are no transaction costs for the market agent and the possibility of creating any market is open to more than one agent competition would imply $\sum_c x(i,c) = 0$. But with trancaction costs or cooperation between market agents and producers, the theory of which markets would exist, will be much more complicated.

The way the theorem is proved shows that incomplete markets can be regarded both as a generalization of a complete market and - by a reinterpretation of commodities and consumers - as a special case of an economy with a complete market. This means that economies with incomplete markets can be regarded as special cases of economies with externalities, and more generally that the existing theory of Walras equilibria, optimality, fairness etc. can be used on the special case corresponding to an economy with incomplete markets.

A Walras equilibrium with coordination.

E is the economy in which equilibrium with coordination is defined and existence proved

$$\mathsf{E} = \left(\mathsf{L}, \mathsf{C}, (\mathsf{X}_\mathsf{c}, \mathsf{P}_\mathsf{c}, e_\mathsf{c})_{\mathsf{c}\in\mathsf{C}}, \Delta, g\right)$$

with

L	a finite set of commodities	
C	a finite set of consumers	
$\mathsf{X}_\mathsf{c} \subset \mathbb{R}^{\mathsf{LC}} = \mathsf{Y}$	the feasible set for consumer c	
$\mathsf{P}_\mathsf{c} : \mathsf{X}_\mathsf{c} \to 2^{\mathsf{X}_\mathsf{c}}$,	the preferences of consumer c	(E)
$e_\mathsf{c} : \mathsf{Y} \times \mathsf{Y} \to \mathsf{Y}$	coordination function	
$g : \mathsf{Y} \times \mathsf{Y} \to 2^\mathsf{C}$	approval function	
$\Delta = \left\{\mathsf{p} \in \mathbb{R}^\mathsf{L}_+ \,\middle|\, \sum \mathsf{p}_\mathsf{h} = 1\right\}$	prices	

The state space is $\mathbb{R}^{\mathsf{LC}} = \mathsf{Y}$, the **feasible set** is

$$F(E) = \bigcap_{\mathsf{c}\in\mathsf{C}} \mathsf{X}_\mathsf{c} \cap \left\{ \mathsf{x} \in \mathsf{Y} \,\middle|\, \sum_{\mathsf{c}\in\mathsf{C}} \mathsf{x}(\mathsf{c}) = 0 \right\}$$

A pair (x, p) where $\mathsf{x} \in \mathsf{X}_0$ and $\mathsf{p} \in \Delta$ is **equilibrium with coordination** if there does not exist a $\mathsf{y} \in \mathsf{Y}$ such that

$$e_\mathsf{c}(\mathsf{x}, \mathsf{y}) \in \mathsf{X}_\mathsf{c} \cap \mathsf{P}_\mathsf{c}(\mathsf{x}), \mathsf{py}(\mathsf{c}) = 0 \text{ for } \mathsf{c} \in g(\mathsf{x}, \mathsf{y})$$

Theorem 25 (Equilibrium) *Let E be the economy as described above. Assume for each $\mathsf{c} \in \mathsf{C}$ that $(\mathsf{X}_\mathsf{c}, \mathsf{P}_\mathsf{c}, e_\mathsf{c})$ satisfies*

X_c *is convex,* $0 \in int\mathsf{X}_\mathsf{c}$ *(interior relative to span*X_c*), and* $\left\{\mathsf{x}(\mathsf{c}) \in \mathbb{R}^\mathsf{L} \,\middle|\, \mathsf{x} \in \mathsf{X}_\mathsf{c}\right\}$ *is bounded below.*

P_c *is irreflexive and has open graph and convex value*

$\mathsf{P}_\mathsf{c}(\mathsf{x}) \supset \mathbb{R}^\mathsf{L}_+ \times \overline{\mathbb{R}}^{(\mathsf{C}\setminus\{\mathsf{c}\})\mathsf{L}}_+ + \mathsf{x}$

$e_c(x,y) \in \beta(x,y) \Rightarrow y \in \beta(x,y)$

There exist (X^c, P^c) *such that* $int X^c \supset X_c \cap int \widetilde{X}, X^c \subset \widetilde{X}, X^C = \bigcap_{c \in C} X^c, P_c(x) = P^c(x) \cap X_c$ *for all* $x \in X_0$, *and* (X^c, P^c) *satisfies (i) and (ii)*

Then $Eq(E) \neq \emptyset$

Proof. See(Grodal and Vind 2001) ■

A special case of this is **Walras equilibrium with coordination in a partition.** Coordination in a partition $(C_j)_{j \in J}$ means that

$$e_c(x,y) = \left((x(b))_{b \notin C_j}, (y(b))_{b \in C_j} \right),$$
$$\text{where } (C_j)_{j \in J} \text{ is a partition of } C \text{ and } c \in C_j$$
$$\text{and } g(x,y) = \left\{ c \in C \,\middle|\, (x(d))_{d \in C_j} \neq (y(d))_{d \in C_j}, \text{ for } c \in C_j \right\}$$

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