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When Inefficiency Begets Efficiency

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The participants in a September 2002 Workshop on *Topics in Mathematical Economics* in honor of Birgit Grodal decided to have a series of papers appear on Birgit Grodal's 60'th birthday, June 24, 2003.

The Institute of Economics suggested that the papers became Discussion Papers from the Institute.

The editor of *Economic Theory* offered to consider the papers for a special Festschrift issue of the journal with Karl Vind as Guest Editor.

This paper is one of the many papers sent to the Discussion Paper series.

Most of these papers will later also be published in a special issue of *Economic Theory*.

Tillykke Birgit

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Head of Institute — Guest Editor

WHEN INEFFICIENCY BEGETS EFFICIENCY*

Hans Gersbach† and Hans Haller‡

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Abstract

Collective consumption decisions taken by the members of a household may prove inefficient. The impact of such inefficient household decisions on market performance is investigated. At one extreme, market efficiency can occur even when household decisions are inefficient, namely when household inefficiencies are merely due to inefficient net trades with the market. At the other extreme, market efficiency is bound to fail, if household inefficiencies are solely caused by an inefficient distribution of a household's aggregate consumption to its individual members. This leads us to consider competitive forces as a disciplinary device for households. When households compete for both resources and members then household stability requires efficient or not too inefficient internal distribution.

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1 Introduction

Conventional economic terminology uses "consumer" and "household" as synonyms and with few exceptions, both theoretical and empirical economics have treated households as if they were single consumers. Both from a normative and a positive perspective, this prevailing practice raises the question whether distinguishing between a household and its members makes any difference. Chiappori (1988, 1992) who is primilary interested in testable implications regarding household demand, presents a model of collective rationality of households as an alternative to the model where households are treated like single consumers. Our main focus here and elsewhere is normative. It lies on the impact that the nature of collective household decisions has on market performance. The issue at hand is to what extent competitive exchange among multi-member households leads to a Pareto-optimal allocation, i.e. an efficient market outcome. The classical answer is in the affirmative: market outcomes are efficient. Beyond a matter of belief, this welfare conclusion obtains as a formal result known as the first theorem of welfare economics in the traditional model of competitive exchange among optimizing individuals: competitive equilibrium allocations are efficient. Obviously, the welfare conclusion persists if multi-member households are treated like single consumers. But what if they are not, if each household member has her own preferences and efficiency, both at the household and the economy level, is defined in terms of these individual preferences? According to the formal analysis of Haller (2000), the answer still is in the affirmative as long as each household makes an optimal (efficient) choice subject to its budget constraint and, by doing so, exhausts its budget.

Following in the footsteps of Haller (2000), we are going to elaborate further on the normative issue of efficiency, whether and when (in)efficiency at the household level translates into (in)efficiency at the economy level. Obviously, efficiency at the houshold level need not always imply efficiency at the economy level. This can occur even in economies consisting exclusively of one-person households, provided that some consumers possess satiation points in the interior of their budget sets whereas other consumers have non-satiated preferences and exhaust their budgets. With multi-person households, however, this phenomenon is more likely. A household with negative intra-household externalities may have a bliss point despite the fact that each household member has monotonic preferences with respect to her individual consumption. Just imagine a household composed of two smokers. Each household member may individually prefer to always smoke more, since the additional nicotine intake more than compensates for the deterioration of air quality it causes. Nevertheless, the negative externalities due to air pollution can be such that the two smokers agree on an unconstrained "optimum" consumption for the household. As a consequence, efficient household decisions can lead to inefficiency at the economy level.

In view of the smoker example, the main contribution of Haller (2000) consists in identifying externalities such that efficient household decisions beget efficiency at the economy level. Here we start from the opposite assumption that collective household decision-making could be prone to severe frictions and, as a consequence, to inefficiencies. Then a new and, perhaps, more challenging question arises: How is market performance affected by inefficient household decisions? One intriguing possibility is that inefficiencies at the micro level neutralize each other so that the resulting market allocation is efficient.[1] The more likely scenario is that inefficiencies at the micro level cause global inefficiency. In the sequel, two specific types of inefficient household decisions will be isolated. While in general one would expect the two types of inefficiency to coexist, it turns out that considerable insight can already be gained from investigating each type in isolation. The first type of household inefficiency results from an inefficient net trade with the market and does not rule out global efficiency. The second type of household inefficiency results from an inefficient distribution of the household's aggregate consumption to individual household members and always causes global inefficiency. Both types of inefficiencies are considered in Section 4, after introducing a model with fixed household structure in Section 2 and restating the first welfare theorem in Section 3.

In Section 5, we investigate more sytematically if and how inefficient net trades by one household can be compensated by inefficient net trades of other households so that an efficient equilibrium allocation results. In Section 6 we address the question to what extent inefficient internal distribution — which always leads to an inefficient equilibrium allocation — will be eliminated if

[1]Incidentally, a similar scenario is frequently invoked to counter the objection that individual consumers lack full rationality as postulated by neoclassical economic theory: Individual deviations from full rationality may offset each other and, thus, not affect aggregates.

households compete for resources and members. The latter requires choice of household affiliation and, therefore, a variable household structure. We extend the model so that an allocation consists of an allocation of commodities plus a household structure, that is a partition of the population into households. In the absence of externalities, the threat of leaving a multi-person household and forming a single-person household eliminates inefficient internal distribution in the prevailing households. In the presence of externalities this threat is not enough to prevent inefficient internal distribution. However, the threat to form a new household that is similar to the old one but makes better consumption decisions proves effective.

2 Model of Competitive Exchange

To model competitive exchange among multi-member households, consider a pure exchange economy composed of finitely many households $h = 1, \ldots, H$. The commodity space is $\mathbb{R}^\ell$ with $\ell \geq 1$. Household h is endowed with a commodity bundle $\omega_h \in \mathbb{R}^\ell$, $\omega_h > 0$.

Each household h consists of finitely many members $i = hm$ with $m = 1, \ldots, m(h)$ and $m(h) \geq 1$. Put $I = \{hm : h = 1, \ldots, H; m = 1, \ldots, m(h)\}$. A generic individual $i = hm \in I$ has:

- consumption set $X_i = \mathbb{R}^\ell_+$;
- preferences $\succsim_i$ on the allocation space $\mathcal{X} \equiv \prod_{j \in I} X_j$ represented by a utility function $U_i : \mathcal{X} \longrightarrow \mathbb{R}$.

This general formulation allows for economy-wide externalities. The latter promises to be a fertile topic of research even in the traditional context of competitive exchange among individuals. But in accordance with the main focus of the current paper, we propose to restrict attention to externalities that are of particular interest for an inquiry into competitive exchange among households. In the sequel, condition (E1) will be imposed which requires that consumption externalities, if any, exist only between members of the same household. Some more notation is needed for an explicit formulation of such intra-household externalities.

Let $\mathbf{x} = (x_i)$, $\mathbf{y} = (y_i)$, $\mathbf{z} = (z_i)$ denote generic elements of $\mathcal{X}$. For $h = 1, \ldots, H$, define $\mathcal{X}_h = \prod_{n=1}^{m(h)} X_{hn}$ with generic elements $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)})$.

If $\mathbf{x} \in \mathcal{X}$ is an allocation, then for $h = 1, \ldots, H$, household consumption is $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)}) \in \mathcal{X}_h$. Now we are ready to define the kind of intra-household externalities which will be assumed hereafter.

(E1) Intra-Household Externalities: $U_i(\mathbf{x}) = U_i(\mathbf{x_h})$
for $i = hm$, $\mathbf{x} \in \mathcal{X}$.

We shall also refer to the special case of no externalities, i.e.

(E2) Absence of Externalities: $U_i(\mathbf{x}) = U_i(x_i)$
for $i = hm$, $\mathbf{x} = (x_i) \in \mathcal{X}$.

The first theorem of welfare economics asserts that any competitive equilibrium allocation in the sense of Walras is Pareto-optimal. Here, like in Haller (2000), we want to allow for the possibility of a household composed of several members who arrive at a collective decision on household consumption. For the economy with social endowment $\omega = \sum_h \omega_h$ and consumers $i = hm$ $(h = 1, \ldots, H; m = 1, \ldots, m(h))$, an efficient or Pareto-optimal allocation is defined in the standard fashion based on individual preferences:

DEFINITION 1 *An allocation* $\mathbf{x} = (x_i) \in \mathcal{X}$ *is* **efficient or Pareto-optimal**, *if*

(i) $\sum_i x_i = \omega$, *i.e.* $\mathbf{x}$ *is feasible and*

(ii) *there does not exist a feasible allocation* $\mathbf{y} = (y_i) \in \mathcal{X}$ *with* $U_i(\mathbf{y}) \geq U_i(\mathbf{x})$ *for all* i *and* $U_i(\mathbf{y}) > U_i(\mathbf{x})$ *for some* i.

To complete the modeling of competitive exchange among households, one has to specify how households interact with the market. Haller (2000) assumes efficient bargaining within households. The latter means that a household h chooses an allocation at the Pareto frontier of its budget set, i.e. an element of its efficient budget set $EB_h(p)$ as defined below. In contrast, the present paper is aimed at investigating the impact of inefficient household decisions on market performance. This extended research agenda necessitates a more general definition of a competitive equilibrium among households than the one adopted in Haller (2000). To this end, consider a household h and a price system $p \in \mathbb{R}^\ell$. For $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)}) \in \mathcal{X}_h$, denote

$$p * \mathbf{x_h} = p \cdot \left(\sum_{m=1}^{m(h)} x_{hm} \right).$$

Then h's **budget set** is defined as $B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} \leq p \cdot \omega_h\}$.

For future reference, we also define household h's **binding budget set** or **balanced budget set** as $BB_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} = p \cdot \omega_h\}$.

Demand correspondences describe the possible outcomes of collective household decision-making. A (possibly empty-valued) correspondence

$$D_h : \mathbb{R}^\ell \Longrightarrow \mathcal{X}_h$$

is called a **demand correspondence for household** h, if $D_h(p) \subseteq B_h(p)$ for all $p \in \mathbb{R}^\ell$. How households form their demands is a key component of the definition of a competitive equilibrium among households.

DEFINITION 2 *Given a profile $D = (D_1, \ldots, D_H)$ of demand correspondences for households, a* **competitive D-equilibrium** $(p; \mathbf{x})$ *is a price system p together with a feasible allocation $\mathbf{x} = (x_i)$ satisfying*

(iii) $\mathbf{x_h} \in D_h(p)$ *for* $h = 1, \ldots, H$.

Thus in a competitive equilibrium, each household makes a collective choice under its budget constraint and markets clear. At this general level, the concept of a competitive equilibrium among households is flexible enough to accommodate all conceivable collective decision criteria of households. Of course, additional restrictions on the profile D could and should be imposed whenever warranted by the objective of the research effort. Occasionally, it may be opportune to replace the market clearing condition (i) by a free disposal condition: $\sum_i x_i \leq \omega$. However, such an occasion will not arise during the course of this investigation.

3 Efficient Household Decisions

Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household. Such a notion of efficient household decision is captured by the concept of an efficient budget set.

Given a price system p, define consumer h's **efficient budget set** $EB_h(p)$ as the set of $\mathbf{x_h} \in B_h(p)$ with the property that there is no $\mathbf{y_h} \in B_h(p)$ such that

$$U_{hm}(\mathbf{y_h}) \geq U_{hm}(\mathbf{x_h}) \text{ for all } m = 1, \ldots, m(h);$$

$$U_{hm}(\mathbf{y_h}) > U_{hm}(\mathbf{x_h}) \text{ for some } m = 1, \ldots, m(h).$$

Classical versions of the first theorem of welfare economics are based on the crucial property that each consumer's demand lies on the consumer's budget line or hyperplane — which implies Walras' Law. This property follows from local non-satiation, for instance monotonicity of consumer preferences. With the possibility of multi-person households and intra-household externalities, the crucial property needs to be adapted. The modified property is called budget exhaustion and stipulates that each household's choice lies on the household's "budget line". For example, monotonicity in own consumption combined with non-negative externalities yields budget exhaustion. The formal definition is as follows.

(BE) Budget Exhaustion: For each household $h = 1, \ldots, H$, and any price system $p \in \mathbb{R}^\ell$, $EB_h(p) \subseteq BB_h(p)$.

Notice that $EB \equiv (EB_1(\cdot), \ldots, EB_H(\cdot))$ is an example of a profile of demand correspondences for households. Therefore, a key result of Haller (2000) can be rephrased as follows.

Proposition 1 (First Welfare Theorem) *Suppose (E1) and (BE). If* $(p; \mathbf{x})$ *is a competitive* EB*-equilibrium, then* $\mathbf{x}$ *is a Pareto-optimal allocation.*

In other words, efficiency at the household level implies efficiency at the economy level, if each household has to exhaust its budget in order to put into effect an efficient consumption decision for its members. For the existence of such an equilibrium see Gersbach and Haller (1999).

4 Inefficient Household Decisions

On purely analytic grounds, it is fruitful to treat the household decision as a two-step decision, although the household need not perceive it that way. In the first step, the household chooses an aggregate or total consumption bundle for the household subject to its budget constraint. In more technical terms, the household determines its net trade with the market. In a more graphic description, the household fixes the dimensions of an Edgeworth Box for the household. In the second step, the household distributes its total consumption bundle among its members. More graphically, the household picks a point (an allocation) within its previously chosen Edgeworth Box. To arrive at an efficient consumption decision under its budget constraint, the household has to first choose the right Edgeworth Box and then pick a point on the contract curve in that Edgeworth Box. Therefore, one can identify two sources of inefficiencies committed by the household:

a) inefficient net trade with the market;
b) inefficient internal distribution.

Of course, the two types of inefficient decision-making can be compounded. But it is analytically convenient to consider each of them separately. More importantly, this sort of piecemeal analysis renders interesting results already.

To formalize the two types of household inefficiency, it is convenient to introduce yet another distinguished subset of a household's budget set. For each household h and every price system p, we define the **potentially efficient budget set** $PEB_h(p)$ as the set of $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)}) \in B_h(p)$ for which there exists $\mathbf{y_h} = (y_{h1}, \ldots, y_{hm(h)}) \in EB_h(p)$ such that

$\sum_{m=1}^{m(h)} y_{hm} = \sum_{m=1}^{m(h)} x_{hm}$ and

$U_{hm}(\mathbf{y_h}) \geq U_{hm}(\mathbf{x_h})$ for all $m = 1, \ldots, m(h)$.

When choosing an element from its potentially efficient budget set, the household makes an efficient net trade, but may not achieve efficient internal distribution.

4.1 Inefficient Net Trades

Suppose that a household performs an inefficient net trade with the market which means that the household could improve (in a weak sense) the welfare of its members by making a different choice under its budget constraint, but in order to achieve that would have to change its net trade with the market. If the household wants to correct its mistake after market clearing, then the net trades of some other households would have to be altered as well, possibly to the detriment of the welfare of the other households' members. This line of argument suggests that inefficient net trades might lead to an efficient market allocation. The following formal result obtains:

Proposition 2 (Accidental Welfare Theorem)
Let $\ell \geq 2$ and consider a non-empty population I partitioned into households $h = 1, \ldots, H$. Then there exist

1. *household endowments and consumer preferences satisfying (E1),*
2. *a profile of demand correspondences D for the associated exchange economy and*
3. *a competitive D-equilibrium $(p^*; \mathbf{x}^*)$ for that economy*

with the property that

4. *each household h performs an inefficient net trade with the market in the sense that $\mathbf{x}_\mathbf{h}^* \notin PEB_h(p^*)$, and*
5. *the allocation $\mathbf{x}^*$ is Pareto-optimal.*

SKETCH OF PROOF. It suffices to outline the argument for the simplest case of two commodities, $\ell = 2$, and a single household, $H = 1$, with a single member denoted i. Consequently, (E1) amounts to (E2). Let the consumer be endowed with the commodity bundle $\omega_i = (1, 1)$ and his preferences be represented by the Cobb-Douglas utility function

$$U_i(x_i) = x_{i1}^{1/2} x_{i2}^{1/2}$$

for $x_i = (x_{i1}, x_{i2}) \in \mathbb{R}^\ell_+$. For each price system $p = (p_1, p_2) \in \mathbb{R}^\ell_{++}$, this consumer has a Marshallian demand

$$x_i(p) = \left(\frac{p_1 + p_2}{2p_1}, \frac{p_1 + p_2}{2p_2} \right).$$

Conversely, at each consumption bundle $\mathbf{x} \in \mathbb{R}^{\ell}_{++}$, this consumer's inverse demand or supporting price system is given, up to normalization, by grad $U_i(\mathbf{x})$, the gradient of U_i at $\mathbf{x}$.

Let us assume that instead of realizing his Marshallian net trade $x_i(p) - \omega_i$ with the market, the consumer always chooses zero net trade with the market which corresponds to the constant demand function $D(p) = D_i(p) \equiv \omega_i$. Now consider the price system $p^* = (1, 2)$. Then $(p^*; \omega_i)$ is a competitive D-equilibrium and ω_i is a Pareto-optimal allocation for this economy. But under his budget constraint, the consumer performs an inefficient net trade with the market, because his actual demand $\omega_i = (1, 1)$ differs from his Marshallian demand $x_i(p^*) = (3/2, 3/4)$. However, the former is Pareto-optimal whereas the latter is socially infeasible. This proves the assertion. Q.E.D.

Obviously, this trivial example generalizes to arbitrary numbers of consumers ($|I| \geq 1$) and goods ($\ell \geq 2$), to arbitrary household structures and a wide variety of consumer characteristics including instances of competitive equilibria with active trade like in cases of self-inflicted rationing with net trades $\frac{1}{2}[x_i(p) - \omega_i]$. Why then the attribute "accidental"? The reason is that the phenomenon of inefficient household decisions consistent with market efficiency is frequent in some sense and rare in some other sense. In support of this assertion, let us revisit the case $\ell = 2$. Let there be $H \geq 2$ single-person households, with both households and consumers labelled $i = 1, 2, \ldots, H$. Furthermore, let each consumer i be endowed with a strictly positive commodity bundle $\omega_i = (\omega_{i1}, \omega_{i2}) \in \mathbb{R}^{\ell}_{++}$ and have preferences of the Cobb-Douglas type,

$$U_i(x_i) = x_{i1}^{\alpha_i} x_{i2}^{1-\alpha_i} \text{ for } x_i = (x_{i1}, x_{i2}) \in X_i,$$

with $0 < \alpha_i < 1$.

Now fix $\omega_i, i \in I$, and some $\lambda > 0$. Then there exist unique exponents $\alpha_i, i \in I$, and coefficients $\mu_1 > 0, \ldots, \mu_H > 0$ such that

$$\mu_1 \cdot \text{grad } U_1(\omega_1) = \ldots = \mu_H \cdot \text{grad } U_H(\omega_H) = (\lambda, 1). \tag{1}$$

Namely, $\alpha_i = \frac{\omega_{i1}}{\omega_{i2}} \cdot \lambda \Big/ \left(1 + \frac{\omega_{i1}}{\omega_{i2}} \cdot \lambda\right), i \in I$, is necessary and sufficient for (1). Equation (1) in turn is necessary and sufficient for Pareto-optimality of the initial allocation of resources. Hence, whenever (1) holds, the essence of the above one-consumer example is preserved: Choose again $D_i(p) \equiv \omega_i$ for each i and set $p^* = (\lambda, 2)$. Then $(p^*; (\omega_1, \ldots, \omega_H))$ is a competitive D-equilibrium with inefficient net trades, but an efficient market outcome. This shows that in a specific sense, the phenomenon of inefficient household decisions consistent with market efficiency is a frequent one: Given the endowments ω_i, $i \in I$, variation of λ yields a continuum of corresponding examples. On the other hand, validity of (1) or, equivalently, Pareto-optimality of the initial allocation is not robust with respect to small perturbations of the preference parameters $\alpha_1, \ldots, \alpha_H$. In fact, the no trade allocation given by the endowments ω_i, $i \in I$, is not Pareto-optimal for most choices of preference parameters. But if the initial allocation of resources is not Pareto-optimal, then the foregoing construction of inefficient net trades leading to an efficient market outcome easily collapses. This suggests that in a certain sense, the phenomenon of inefficient household decisions compatible with market efficiency is a rare one.

4.2 Inefficient Internal Distribution

Suppose that a household performs an efficient net trade with the market which means that the household can achieve an efficient choice under its budget constraint by suitably dividing its aggregate consumption bundle among its members. But the actually chosen internal distribution of commodities may be inefficient in the sense that redistribution within the household can improve the welfare of its members. If so, the mistake can be rectified simply by internal reallocation without affecting the welfare of members of other households. This leads to the conclusion that inefficient internal distribution, a particular type of inefficient household decision, always begets global inefficiency. Indeed, the following formal result holds true where $PEB \equiv (PEB_1(\cdot), \ldots, PEB_H(\cdot))$ denotes the profile of potentially efficient budget correspondences.

Proposition 3 (Anti-Welfare Theorem) *Suppose (E1).*
If $(p; \mathbf{x})$ is a competitive PEB-equilibrium and $\mathbf{x_h} \notin EB_h(p)$ for some household h, then $\mathbf{x}$ is not a Pareto-optimal allocation.

PROOF. Assume (E1). Let $(p; \mathbf{x})$ be as hypothesized and h be a household with $\mathbf{x_h} \notin EB_h(p)$. Since $\mathbf{x_h} \in PEB_h(p)$, there exists $\mathbf{z_h} \in EB_h(p)$ with $\sum_{m=1}^{m(h)} z_{hm} = \sum_{m=1}^{m(h)} x_{hm}$ and

$U_{hm}(\mathbf{z_h}) \geq U_{hm}(\mathbf{x_h})$ for all $m = 1, \ldots, m(h)$.

Since $\mathbf{z_h} \in EB_h(p)$, but $\mathbf{x_h} \notin EB_h(p)$, $U_{hm}(\mathbf{z_h}) > U_{hm}(\mathbf{x_h})$ has to hold for some $m = 1, \ldots, m(h)$. Now set $\mathbf{y_h} = \mathbf{z_h}$ and $\mathbf{y_k} = \mathbf{x_k}$ for households $k \neq h$. This defines a feasible allocation $\mathbf{y} = (y_i)_{i \in I}$. Because of (E1),

$U_i(\mathbf{y}) > U_i(\mathbf{x})$ for certain members i of household h and

$U_j(\mathbf{y}) = U_j(\mathbf{x})$ for all other consumers j.

Hence as asserted, $\mathbf{x}$ is not Pareto-optimal. Q.E.D.

5 Compensation Across Households

After having identified how inefficient household decisions may or may not beget efficiency, we consider (in)efficient decisions across households and ask whether inefficient net trades by one household can be compensated by inefficient net trades of other households. This question is irrelevant in the case of the specific consumer characteristics we have used to derive and discuss the Accidental Welfare Theorem, because of special auto-corrective features of that case. Notice that in that case, consumers would always pick the right allocation, $\omega = (\omega_1, \ldots, \omega_H)$, though possibly for the wrong reasons. Namely, let $D_i(p) \equiv \omega_i$ and $p^* = (1, 2)$ as before and further $p^0 = (1, 1)$. Then $(p^*; \omega)$ is a D-equilibrium and, up to price normalization, $(p^0; \omega)$ is the EB-equilibrium. In fact, if $\widehat{D}_i = D_i$ for some but not all i and $\widehat{D}_i = EB_i$ for all others, then $(p^0; \omega)$ is the $\widehat{D}$-equilibrium. Thus there are only two possibilities: If all consumers exhibit totally inelastic demands D_i, equilibrium prices may be distorted away from the Walrasian equilibrium prices, yet still the Walrasian equilibrium allocation obtains. If some consumers exhibit Marshallian demands and the rest exhibits totally inelastic demands, then both equilibrium prices and equilibrium quantities turn out to be Walrasian.

Now let us consider instead a situation where an inefficient net trade made by one household leads to an inefficient equilibrium allocation, unless it is compensated by an inefficient net trade of another household. It suffices to focus on the simplest case of two commodities, $\ell = 2$, and two one-person households $h_1 = \{i\}$ and $h_2 = \{j\}$. Consequently, $(E1)$ amounts to $(E2)$.

Let consumers be endowed with the strictly positive bundles $\omega_i = (\omega_{i1}, \omega_{i2})$ and $\omega_j = (\omega_{j1}, \omega_{j2})$. Preferences are represented by the respective Cobb-Douglas utility functions

$$U_i(x_i) = x_{i1}^{\alpha_i}\, x_{i2}^{1-\alpha_i},$$

$$U_j(x_j) = x_{j1}^{\alpha_j}\, x_{j2}^{1-\alpha_j}$$

for $x_i, x_j \in \mathbb{R}^2_+$. Finally, let us assume that the initial endowment allocation $\omega = (\omega_i, \omega_j)$ is not Pareto-optimal, contrary to our previous assumption, so that there are potential gains from trade. Let us normalize the price system p by setting $p_1 = 1$. Then the consumers' Marshallian demands are given by

$$\begin{aligned} x_i(p) &= \Big(\alpha_i(\omega_{i1} + p_2\,\omega_{i2}), (1-\alpha_i)\frac{\omega_i + p_2\omega_{i2}}{p_2}\Big), \\ x_j(p) &= \Big(\alpha_j(\omega_{j1} + p_2\,\omega_{j2}), (1-\alpha_j)\frac{\omega_j + p_2\omega_{j2}}{p_2}\Big). \end{aligned}$$

With respect to these demand functions, there exists a competitive equilibrium $(p^0; \mathbf{x^0})$ with the price system $p^0 = (1, p_2^0)$ given by

$$p_2^0 = \frac{(1-\alpha_i)\omega_{i1} + (1-\alpha_j)\omega_{j1}}{\alpha_i\omega_{i2} + \alpha_j\omega_{j2}}.$$

Suppose now that consumer i chooses his demand according to

$$\widetilde{x}_i(p) = \Big(\alpha_i(\omega_{i1} + p_2\omega_{i2}) + \Delta_i, \frac{(1-\alpha_i)(\omega_{i1} + p_2\omega_{i2})}{p_2} - \frac{\Delta_i}{p_2}\Big)$$

with some mistake $\Delta_i \neq 0$ that is independent of p_2 and satisfies $|\Delta_i| < \min\{\alpha_i, 1-\alpha_i\} \cdot \omega_{i1}$. While consumer i now performs an inefficient net trade under her budget constraint, consumer j is assumed to behave according to his Marshallian demand. Consider the resulting competitive equilibrium $(p^*; \mathbf{x}^*)$ with

$$p_2^* = \frac{(1-\alpha_i)\omega_{i1} + (1-\alpha_j)\omega_{j1} - \Delta_i}{\alpha_i\omega_{i2} + \alpha_j\omega_{j2}}.$$

The allocation $\mathbf{x}^*$ will be inefficient regardless of the equilibrium price p_2^*.

Suppose now that consumer j makes an inefficient net trade as well according to

$$(2) \qquad \widetilde{x}_j(p) = \Big(\alpha_j(\omega_{j1} + p_2\omega_{j2}) - \Delta_j, \frac{(1-\alpha_j)(\omega_{j1} + p_2\omega_{j2})}{p_2} + \frac{\Delta_j}{p_2}\Big)$$

for some $\Delta_j \neq 0$ independent of p_2 and such that $|\Delta_j| < \min\{\alpha_j, 1-\alpha_j\} \cdot \omega_{j1}$. The resulting competitive equilibrium is $(p^{**}; \mathbf{x}^{**})$ with prices $p_1^{**} = 1$ and

$$(3) \qquad \begin{aligned} p_2^{**} &= \tfrac{(1-\alpha_i)\omega_{i1} + (1-\alpha_j)\omega_{j1} - \Delta_1 - \Delta_2}{\alpha_i \omega_{i2} + \alpha_j omega_{j2}} \\ &= p_2^0 - \tfrac{\Delta_1 + \Delta_2}{\alpha_i \omega_{i2} + \alpha_j \omega_{j2}} \end{aligned}$$

The resulting allocation $\mathbf{x}^{**}$ is Pareto-optimal if and only if the marginal rates of substitution for the two consumers coincide. The latter condition amounts to

$$(4) \qquad \frac{\omega_{i1} + p_2^{**}\omega_{i2} - \Delta_i}{\omega_{i1} + p_2^{**}\omega_{i2} + \Delta_i} = \frac{\omega_{j1} + p_2^{**}\omega_{j2} + \Delta_j}{\omega_{j1} + p_2^{**}\omega_{j2} - \Delta_j}$$

or

$$(5) \qquad F(\Delta_i, \Delta_j) \equiv \frac{\omega_{i1} + p_2^{**}\omega_{i2} - \Delta_i}{\omega_{i1} + p_2^{**}\omega_{i2} + \Delta_i} - \frac{\omega_{j1} + p_2^{**}\omega_{j2} + \Delta_j}{\omega_{j1} + p_2^{**}\omega_{j2} - \Delta_j} = 0.$$

Now at the Walrasian outcome, $F(0,0) = 0$ and $\partial F / \partial \Delta_j(0,0) \neq 0$. Therefore, by the implicit function theorem, there exists an open neighborhood $N_i(0)$ such that for all $\Delta_i \in N_i(0)$, there is a unique $\Delta_j(\Delta_i)$ with $F(\Delta_i, \Delta_j(\Delta_i)) = 0$. That is to each small "mistake" Δ_i corresponds exactly one "compensating mistake" $\Delta_j(\Delta_i)$ that leads to an optimal equilibrium allocation. $\Delta_j(\Delta_i)$ can be explicitly determined by solving the quadratic equation in Δ_j associated with (4).

The analysis of this example shows among other things:

Proposition 4 (Compensating Inefficient Net Trades)
There exist economies with intra-household externalities and at least two households, labelled h_1 and h_2, with two profiles of demand correspondences, $D^* = (D_1^*, \ldots, D_H^*)$ and $D^{**} = (D_1^{**}, \ldots, D_H^{**})$, with a competitive D^*–equilibrium $(p^*; \mathbf{x}^*)$ and a competitive D^{**}–equilibrium $(p^{**}; \mathbf{x}^{**})$ such that:

1. $D_{h_1}^* = D_{h_1}^{**}$ *and* $D_h^* = D_h^{**} = EB_h$ *for all* $h \notin \{h_1, h_2\}$.
2. $D_{h_1}^* \cap EB_{h_1} = \emptyset$, $D_{h_2}^* = EB_{h_2}$, *and the allocation* $\mathbf{x}^*$ *is Pareto inefficient.*
3. $D_{h_1}^{**} \cap EB_{h_1} = \emptyset$, $D_{h_2}^{**} \cap EB_{h_2} = \emptyset$, *and the allocation* $\mathbf{x}^{**}$ *is Pareto efficient (Pareto-optimal).*

Let us add a few more observations to the last example and the implied proposition. First, the example exhibits single-person households and absence of externalities. Second, given $\Delta_i \in N_i(0)$, consumer j has to make the "right mistake" $\Delta_j(\Delta_i)$ in order to achieve a Pareto-optimal outcome; otherwise, the ensuing equilibrium allocation is not Pareto-optimal. This observation parallels the "accidental" nature of the conclusion of the Accidental Welfare Theorem. But different parameters of the model are allowed to vary in the two situations. Now the mistakes have to match whereas before consumer characteristics had to match. Third, a suitable pair of mistakes, $\Delta = (\Delta_i, \Delta_j)$ with $\Delta_j = \Delta_j(\Delta_i)$ determines a unique Pareto-optimal allocation $\mathbf{x}^{**}(\Delta)$. Conversely, select any point $\mathbf{x}^{**}$ on the contract curve such that (in the Edgeworth Box) the straight line L through $\mathbf{x}^{**}$ and ω is negatively sloped. For instance, a core allocation will do. Choose $p_2^{**} > 0$ so that $p^{**} = (1, p_2^{**})$ is a normal vector to this line, i.e. L is the budget line with respect to the price system p^{**}. Set $\Delta_i = x_{i1}^{**} - x_{i1}(p^{**})$ and $\Delta_j = -[x_{j1}^{**} - x_{j1}(p^{**})]$. Then $\Delta_j = \Delta_j(\Delta_i)$ and $\mathbf{x}^{**} = \mathbf{x}^{**}(\Delta)$. Finally, observe that the Marshallian demands of Cobb-Douglas consumers exhibit fixed expenditure shares. Therefore, the "mistakes" Δ_i and Δ_j can be interpreted as mistakes in the determination of the expenditure shares.

Next let us consider a different situation where an inefficient net trade made by one household cannot be compensated by an inefficient net trade of another household and necessarily leads to an inefficient equilibrium allocation. To this end, we focus again on the simplest case of two commodities, $\ell = 2$, and two one-person households $h_1 = \{i\}$ and $h_2 = \{j\}$.

Let consumer i be endowed with the commodity bundle $\omega_i = (\omega_{i1}, \omega_{i2}) = (2, 1)$ and consumer j be endowed with $\omega_j = (\omega_{j1}, \omega_{j2}) = (6, 12)$. Preferences are represented by the respective utility functions

$$U_i(x_i) = \min\{x_{i1}\,, x_{i2}\},$$

$$U_j(x_j) = x_{j1}^{1/2}\, x_{j2}^{1/2}$$

for $x_i, x_j \in \mathbb{R}^2_+$. Then the consumers' Marshallian demands are given by

$$\begin{aligned} x_i(p) &= \Big(\frac{2p_1 + p_2}{p_1 + p_2}, \frac{2p_1 + p_2}{p_1 + p_2}\Big), \\ x_j(p) &= \Big(\frac{6p_1 + 12p_2}{2p_1}, \frac{6p_1 + 12p_2}{2p_2}\Big). \end{aligned}$$

With respect to these demand functions, there exists a competitive equilibrium $(p^0; \mathbf{x^0})$ with the price system $p^0 = (1, p_2^0)$ given by $p_2^0 = (\sqrt{19} - 1)/6$. This shows that there are potential gains from trade and the initial endowment allocation is not Pareto-optimal.

Suppose now that consumer i chooses her demand according to

$$\widetilde{x}_i(p) = (x_i(p) + \omega_i)/2.$$

Then consumer i performs an inefficient net trade under her budget constraint, by means of self-inflicted rationing, realizing only half of her efficient net trade. The corresponding individual excess demand function is continuous, bounded, and satisfying Walras Law on the price simplex.

Suppose further that consumer j chooses his demand according to a demand function $\widetilde{x}_j(\cdot)$ such that the corresponding individual excess demand function is continuous and satisfying Walras Law in the interior of the price simplex; moreover, it satisfies the standard boundary condition. Then by standard arguments, the economy with these demand functions has a competitive equilibrium $(p^*; \mathbf{x}^*)$ with $p^* \gg 0$. Furthermore, $x_i^* \gg 0$, $x_j^* \gg 0$ and $x_{i1}^* > x_{i2}^*$. Therefore, it is possible to find a feasible allocation that strictly Pareto dominates $\mathbf{x}^*$. The important point is that we do not impose any restrictions on the demand function $\widetilde{x}_j(\cdot)$ other than the standard assumptions that guarantee existence of equilibrium. Hence it can differ from Marshallian demand in almost arbitrary ways. Thus we obtain:

Proposition 5 (Lack of Compensation)
There exists an economy with several one-person households and the following property: If household 1 uses a particular inefficient demand function d_1 and each household $h = 2, \ldots, H$ uses any demand function d_h that satisfies standard conditions, then there exists an inefficient competitive d–equilibrium allocation.

Notice that the demand function given by (2) satisfies standard conditions and qualifies for the foregoing impossibility result. On the other hand, the particular demand function chosen for the Leontief consumer does not exhibit a constant shift $\Delta_i \neq 0$ of expenditure shares. The reason for this choice is purely technical: For the Leontief consumer, a constant shift $\Delta_i \neq 0$ would lead to a violation of the non-negativity of demand at certain prices. It is possible, albeit tedious, to recast the example with locally but not globally constant Δ_i and Δ_j.

6 When Outside Options Beget Efficiency

We found that inefficiency can beget efficiency, that inefficient individual or household consumption decisions can lead to Pareto-optimal equilibrium allocations. If an agent's mistake (inefficient net trade to be precise) is suitably compensated by the mistakes (inefficient net trades) of others, then the overall allocation can be efficient. However, an inefficient net trade need not be compensated and — as we demonstrate by example — in some cases cannot be compensated. Furthermore, if the sole source of an inefficient household decision is inefficient internal distribution, then by the Anti-Welfare Theorem, an equilibrium allocation cannot be Pareto-optimal. Elimination or reduction of inefficient internal distribution would improve welfare and obviously would be desirable.

Notice that inefficient internal distribution on the part of households constitutes the analogue of technological inefficiency in the production sector. It is a time honored theme in industrial economics that increased competition among producers reduces both allocative and technical inefficiencies.[2] Moreover, potential competition may suffice to further efficiency. To quote Schumpeter (1975):

> *It is hardly necessary to point out that competition of the kind we now have in mind acts not only when in being but also when it is merely an ever-present threat. It disciplines before it attacks.*[3]

In a similar vein, the concept of contestable markets forwarded by Baumol, Panzer and Willig (1986) postulates that potential hit-and-run competition has the same effect as actual competition.

In this section, we apply the idea that competitive forces can serve as a disciplinary device to the consumption sector. The hope is that competition will cause the elimination or reduction of inefficient internal distribution in households in a similar manner as it causes erosion of managerial slack in

[2] Leibenstein's much acclaimed 1966 article has raised the awareness for technological inefficiencies or X-inefficiencies. Hart (1983) formalizes the idea that competition in the product market reduces managerial slack.

[3] Ibid., p. 85.

firms. Yet we know from the Anti-Welfare Theorem that competition for resources alone will be to no avail in this respect. But it turns out that if household stability is threatened by inefficient internal distribution, if in a sense households are competing for resources and members, then the households which exist in equilibrium must make efficient or not too inefficient decisions. This presumes that dissatisfied household members have the option to leave and that household stability (requiring that nobody wants to exercise the option) is an additional equilibrium condition. Accordingly, we are going to investigate whether and to what extent inefficient household decisions due to inefficient internal distribution are sustainable in equilibrium, if individuals have the option to form new and potentially more efficient households.

The outside options individuals have may vary: an individual may form a single-person household, join another household or form a new household with fractions of existing households. Individuals may leave a household because they dislike its composition. For this reason alone, certain households may not be viable if they are a total mismatch. However, even if a member is content with the household's composition, the member may be dissatisfied with the collective consumption decision and decide to leave. One reason could be that the endowment of the household is such that at the prevailing prices, the household can afford relatively little consumption compared to other households the individual might conceivably join. Another reason could be that the individual gets a bad deal because fellow members get a great deal at his expense. A final reason could be an inefficient consumption decision by the household.

The household cannot do anything about the first reason. It is bound to break up if it lacks sufficient resources to be attractive for its members. The household may be able to preempt the other two causes of a break-up, by not exploiting some of its members for the benefit of others and by making efficient consumption decisions or at least not too inefficient ones. Then the question is how much inefficiency a household can afford without giving a member a reason to leave. Under certain circumstances the answer will be 'none': household stability requires efficiency. In other words, the threat of desertion forces household efficiency. One can also ask how much exploitation a household can afford without giving a member a reason to leave. Under the very same circumstances the answer will be 'none' as well: household stability requires absence of exploitation.

Multi-member households have not been essential for the major results obtained until now. The key arguments can already be made in the traditional context of single-person households and can be readily extended to the broader context of multi-person households. Moreover, so far the household structure was fixed. The new aim necessitates a richer model. The option to leave a household presupposes alternative households and a variable household structure. Imposing stability conditions familiar from the bilateral matching literature[4] makes the household structure endogenous. Consequently, we extend our analysis to a model with endogenous household structure. Inefficient consumption decisions in multi-member households may induce individuals to leave and form new households or join other households, if they have these options. Our basic hypothesis is that competitive exchange across households combined with certain outside options may eliminate or mitigate inefficient internal distribution in the households prevailing in equilibrium.

6.1 Variable Household Structure

To elaborate on the theme of disciplinary capacity of competition, we consider a finite pure exchange economy with variable household structure. There exists a given finite and non-empty set of individuals or consumers, I. A (potential) household is any non-empty subset h of the population I. $\mathcal{H} = \{h \subseteq I | h \neq \emptyset\}$ denotes the set of all potential households. The households that actually form give rise to a **household structure** P, that is a partition of the population I into non-empty subsets. The commodity space, individual consumption sets, household consumption sets and commodity allocations are defined as before.

With a fixed household structure, household membership was part of an individual's identity. Individual $i = hm$ was the m's member of household h. With variable household structure, household membership is an endogenous outcome. An individual may care about household composition and household consumption. Different members may exert different consumption externalities upon others. We maintain the assumption of intra-household externalities. But instead of (E1) it now assumes the form

[4]See the seminal contribution by Gale and Shapley (1962) and the monograph by Roth and Sotomayor (1990).

(HSP) Household-Specific Preferences: $U_i(\mathbf{x}; h) = U_i(\mathbf{x_h}; h)$
for $i \in h, h \in \mathcal{H}, \mathbf{x} \in \mathcal{X}$.

In the following, we are going to consider the special case of

(GSE) Group-Size Externalities: $U_i(\mathbf{x}; h) = V_i(x_i; |h|)$
for $i \in h, h \in \mathcal{H}, \mathbf{x} \in \mathcal{X}$.

In this case, individual i cares only about own consumption and household size. Still, preferences over own consumption may change with household size and, vice versa, preferences over household size can depend on own consumption. In the separable case, $U_i(\mathbf{x}; h) = u_i(x_i) + v_i(|h|)$ and preferences over own consumption and preferences over household size are independent. If $v_i \equiv 0$, then the separable case reduces to (E2), that is absence of externalities.

Every potential household h is endowed with a commodity bundle $\omega_h > 0$. In general, the aggregate or social endowment depends on the prevailing household structure P and equals $\omega_P = \sum_{h \in P} \omega_h$. The social endowment is independent of the household structure if (and only if) the endowment of each household equals the sum of the individual endowments of its members. We call this condition individual property rights.

(IPR) Individual Property Rights: $\omega_h = \sum_{i \in h} \omega_i$ for all $h \in \mathcal{H}$.

After having generalized preferences and endowments to allow for variable household structures, we can define budget sets, efficient budget sets, balanced budget sets, and demand correspondences for arbitrary households accordingly. Define an **allocation** of the economy with variable household structure as a pair $(\mathbf{x}; P)$ where $\mathbf{x} \in \mathcal{X}$ is an allocation of commodities and P is a household structure. The allocation is **feasible**, if $\sum_{i \in I} x_i = \omega_P$. Define a **state** of the economy as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^\ell$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times P$ is an allocation, i.e. $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of commodities and P is an allocation of consumers (a household structure, a partition of the population into households). For a state $(p, \mathbf{x}; P)$ and an individual $i \in I$, let $P(i)$ denote the household in P (the element of P) to which i belongs. We say that in state $(p, \mathbf{x}; P)$,

(a) consumer i can benefit from exit, if $P(i) \neq \{i\}$ and there exists $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i; \{i\}) > U_i(\mathbf{x_{P(i)}}; P(i))$;

(b) consumer i can benefit from joining another household g, if $g \in P$, $g \neq P(i)$ and there exists $\mathbf{y}_{\mathbf{g}\cup\{\mathbf{i}\}} \in B_{g\cup\{i\}}(p)$ such that $U_j(\mathbf{y}_{\mathbf{g}\cup\{\mathbf{i}\}}; \mathbf{g} \cup \{\mathbf{i}\}) > U_j(\mathbf{x}_{\mathbf{P(j)}}; P(j))$ for all $j \in \mathbf{g} \cup \{\mathbf{i}\}$;

(c) a group of consumers h can benefit from forming a new household, if $h \notin P$ and there exists $\mathbf{y}_\mathbf{h} \in B_h(p)$ such that $U_j(\mathbf{y}_\mathbf{h}; h) > U_j(\mathbf{x}_{\mathbf{P(j)}}; P(j))$ for all $j \in h$.

In the spirit of the matching literature (see e.g. Gale and Shapley (1962), Roth and Sotomayor (1990)), a household structure is a "matching" broadly defined and stability of the matching requires that no group of consumers can benefit from forming a new household. It is important to note that in our context stability of a matching depends on household decisions and market conditions, that is the prevailing price system. Next we generalize the notion of competitive equilibrium so that the household structure or matching becomes a constituent part of the equilibrium.

DEFINITION 3 *Let $D = (D_h)_{h\in\mathcal{H}}$ be a profile of demand correspondences for households and $(p, \mathbf{x}; P)$ be a state of the economy. The state $(p, \mathbf{x}; P)$ is a* **competitive D-equilibrium** *if the allocation $(\mathbf{x}; P)$ is feasible and*

(iv) $\mathbf{x}_\mathbf{h} \in D_h(p)$ *for $h \in P$.*

Finally, we generalize the notion of efficient allocation to the current setting where the household structure forms an integral part of an allocation.

DEFINITION 4 *An allocation $(\mathbf{x}; P)$ is* **fully Pareto-optimal** *if it is feasible and there is no other feasible allocation which is weakly preferred to $(\mathbf{x}; P)$ by all consumers and strictly preferred to $(\mathbf{x}; P)$ by some consumer(s).*

6.2 Equilibrium Efficiency of Households

We start with a set of strong assumptions, including absence of externalities, which imply that in equilibrium every multi-member household makes efficient consumption decisions — unless some member benefits from exit. If a multi-member household does make inefficient consumption decisions and its members prefer to stay despite the exit option, then some sort of externality has to be present.

Proposition 6 (Household Efficiency)
Suppose IPR, absence of externalities, continuity and local non-satiation of consumer preferences. Let D be a profile of demand correspondences for households and let $(p, \mathbf{x}; P)$ be a D-equilibrium at which $p \gg 0$ and no consumer benefits from exit. Then $\mathbf{x_h} \in EB_h(p)$ for every multi-member household h in P.

PROOF: Let consumer characteristics, D and $(p, \mathbf{x}; P)$ as hypothesized. Suppose there exists a household $h \in P$ with $|h| \geq 2$ and $\mathbf{x_h} \notin EB_h(p)$. Then there exists $\mathbf{y_h} \in B_h(p)$ with

$$\begin{array}{l} U_j(y_j) > U_j(x_j) \text{ for some } j \in h; \\ U_i(y_i) \geq U_i(x_i) \text{ for all } i \in h. \end{array}$$

Because of continuity and $p \gg 0$, we can choose an $x_i^0 \in EB_{\{i\}}(p)$ for each $i \in h$, that is x_i^0 maximizes the utility of consumer i when the consumer forms a single-person household and is trading from his individual endowment ω_i at prices p. Since no consumer can benefit from exit at state $(p, \mathbf{x}; P)$, $U_i(x_i) \geq U_i(x_i^0)$ for all $i \in h$. Hence,

$$\begin{array}{rcl} U_j(y_j) & > & U_j(x_j^0) \text{ for some } j \in h; \\ U_i(y_i) & \geq & U_i(x_i^0) \text{ for all } i \in h. \end{array}$$

Therefore, $p \cdot y_j > p \cdot \omega_j$ for some $j \in h$ and, by local non-satiation, $p \cdot y_j \geq p \cdot \omega_i$ for all individuals $i \in h$. Summation and IPR yield

$$p * \mathbf{y_h} = p \cdot \left(\sum_{i \in h} y_i \right) > p \cdot \left(\sum_{i \in h} \omega_i \right) = p \cdot \omega_h$$

which, however, is a contradiction to $\mathbf{y_h} \in B_h(p)$. Hence, to the contrary, $\mathbf{x_h} \in EB_h(p)$ for all $h \in P$ with $|h| \geq 2$. Q.E.D.

Let us state some immediate but important consequences of the proposition.

Corollary 1 *Suppose IPR, absence of externalities, continuity and local non-satiation of consumer preferences. Let D be a profile of demand correspondences for households. Consider a D-equilibrium $(p, \mathbf{x}; P)$ at which $p \gg 0$, every household makes efficient net trades and no consumer benefits from exit. Then*

(i) $(p, \mathbf{x}; P)$ *is an EB-equilibrium at which no group of consumers can benefit from forming a new household.*

(ii) $(p, \mathbf{x})$ *is a traditional competitive equilibrium where each agent acts and trades individually.*

PROOF: Let consumer characteristics, D and $(p, \mathbf{x}; P)$ as hypothesized. By assumption, every household makes efficient net trades which means $\mathbf{x_h} \in EB_h(p)$ for every single-person household h in P. By the proposition, $\mathbf{x_h} \in EB_h(p)$ for every multi-member household h in P. By assumption, no consumer benefits from exit. Hence $(p, \mathbf{x}; P)$ is an EB-equilibrium at which no consumer benefits from exit. By the Neutrality Theorem (Proposition 1) of Gersbach and Haller (2002), $(p, \mathbf{x}; P)$ is also an EB-equilibrium at which no consumer can benefit from joining another household. By a similar argument, one can further demonstrate that $(p, \mathbf{x}; P)$ is an EB-equilibrium at which no group of consumers can benefit from forming a new household. Thus (i). By the Neutrality Theorem (Proposition 1) of Gersbach and Haller (2002), (i) implies (ii). Q.E.D.

The corollary depicts circumstances under which households cannot afford any inefficient distribution without giving a member a reason to leave. By assertion (ii) of the corollary, household members cannot fare any better or worse than as single consumers, hence under the same circumstances, a household cannot afford to exploit any of its members without giving them a reason to leave. Thus household stability requires absence of inefficiencies and absence of exploitation. The assumptions also guarantee Pareto-optimality of the equilibrium allocation in addition to equilibrium efficiency of households. Moreover, under these circumstances, the condition that no consumer can benefit from exit implies that no consumer can benefit from joining another household. This implication need not hold in the case of externalities to which we turn next.

6.3 Externalities and (In)efficiency

In the absence of externalities, already the exit option induces efficient distribution of resources within households. When externalities are present it is conceivable that inefficient distribution of resources within households persists despite the opportunities of individuals to explore alternative household affiliations and alternative commodity allocations at the going prices. As will become clear, the three possible outside options: to exit and go single, to join another household, and to form a new household without any restrictions, can differ considerably in their effectiveness in preventing inefficient household decisions when externalities are present.

Our fundamental result is that inefficiencies will be eliminated provided that each household has similar counterparts in society (in the prevailing household structure) with respect to the nature and strength of externalities and consumers are free to form new households. In contrast, the subsequent example shows that the more restrictive options to exit and go single or to leave and join another household prove insufficient to eliminate all inefficiencies. As a rule, however, they are not without consequences as the example also demonstrates. Namely, excessive allocative distortions as well as excessive manifestations of power could cause exit of some household member. This, too, is illustrated by the example.

Proposition 7 *Suppose IPR and GSE and that for each $i \in I$, preferences are continuous, convex and strictly monotone in own consumption. Let D be a profile of demand correspondences for households and let $(p, \mathbf{x}; P)$ be a D-equilibrium at which no group benefits from forming a new household. If g and h are two multi-member households in P of equal size, that is $|g| = |h| > 1$, then $\mathbf{x_g} \in EB_g(p)$ and $\mathbf{x_h} \in EB_h(p)$.*

PROOF: Let g and h as hypothesized. We claim that $\mathbf{x_g} \in EB_g(p)$ and $\mathbf{x_h} \in EB_h(p)$. By symmetry, if suffices to show $\mathbf{x_h} \in EB_h(p)$. Suppose to the contrary that $\mathbf{x_h} \notin EB_h(p)$. Then there exists $\mathbf{x'_h} \in B_h(p)$ with $V_i(x'_i\,; |h|) \geq V_i(x_i\,; |h|)$ for all $i \in h$ and $V_i(x'_i\,; |h|) > V_i(x_i\,; |h|)$ for some $i \in h$. By continuity and strict monotonicity, there exists $\mathbf{x^*_h} \in B_h(p)$ with $\sum_{i \in h} x^*_i = \sum_{i \in h} x'_i$ and $V_i(x^*_i\,; |h|) > V_i(x_i\,; |h|)$ for all $i \in h$.

Let us turn to household g momentarily. Because of $\mathbf{x_g} \in B_g(p)$ and IPR, there exists a group f of $|g| - 1$ members of g with $\sum_{j \in f} px_j \leq \sum_{j \in f} p\omega_j$. For otherwise, for each group f of $|g| - 1$ members of g, $\sum_{j \in f} px_j > \sum_{j \in f} p\omega_j$ and the one member j_f of g who does not belong to f would have to satisfy

$px_{j_f} < p\omega_{j_f}$. Consequently, every member of j of g would satisfy $px_j < p\omega_j$ and each group f of $|g|-1$ members of g satisfied $\sum_{j\in f} px_j < \sum_{j\in f} p\omega_j$, a contradiction. Choose a group f of $|g|-1$ members of g with $\sum_{j\in f} px_j \leq \sum_{j\in f} p\omega_j$.

Let us now turn to household h again and proceed with an $\mathbf{x}^*_\mathbf{h} \in B_h(p)$ such that $V_i(x^*_i\,; |h|) > V_i(x_i\,; |h|)$ for all $i \in h$. Because of $\mathbf{x}^*_\mathbf{h} \in B_h(p)$ and IPR, there exists $i \in h$ with $px^*_i \leq p\omega_i$. Let us choose such an i and consider the new household $k = f \cup \{i\}$ and the household allocation $\mathbf{y_k} \in \mathcal{X}_k$ given by $y_j = x_j$ for $j \in f$ and $y_j = x^*_i$ for $j = i$. Then $k \notin P$, $|k| = |g| = |h|$ and $\mathbf{y_k} \in B_k(p)$ where g and h are the original households. Because of GSE, $V_j(y_j\,; |k|) = V_j(x_j\,; |g|)$ for $j \in f$ and $V_j(y_j\,; |k|) > V_j(x_j\,; |h|)$ for $j = i$. By continuity and strict monotonicity, there exists $\mathbf{z_k} \in B_k(p)$ with $\sum_{j\in k} z_j = \sum_{j\in k} y_j$ and $V_j(z_j\,; |k|) > V_j(x_j\,; |P(j)|)$ for all $j \in k$. But this contradicts the hypothesis that at the state $(p, \mathbf{x}; P)$ no group can benefit from forming a new household. Hence $\mathbf{x_h} \in EB_h(p)$ has to hold as asserted. Q.E.D.

What drives the argument in the foregoing proof is that in equilibrium, for example a member of a two-person household and member of another two-person household should not benefit from forming a new two-person household, and likewise for larger households of equal size. If a household member can only leave and go single or join another household, then the argument does not go through. This can be seen in the following example which exhibits several other interesting features.

Example. We consider a society of $N = 2n$, $n > 1$ individuals so that $I = \{1, \ldots, N\}$. Assume $\ell = 2$. All individuals are identical. Each $i \in I$ is endowed with the commodity bundle $\omega_i = (1, 1)$ and has preferences represented by the utility function U_i, given as follows for $i \in h \in \mathcal{H}$:

$$\begin{aligned} U_i(\mathbf{x_h}; h) &= (1 + x^1_i)(1 + x^2_i) && \text{if } h = \{i\}, \\ U_i(\mathbf{x_h}; h) &= (1 + x^1_i)(1 + x^2_i) + g && \text{if } |h| = 2, \\ U_i(\mathbf{x_h}; h) &= (1 + x^1_i)(1 + x^2_i) - c && \text{if } |h| > 2, \end{aligned}$$

with $g > 0$ and $c > 0$. Thus individuals would like to form two-person households but dislike larger households.

As a benchmark, let us consider the state $s^* = (\mathbf{x}^*, p; P)$ where $\mathbf{x}^* = (\omega_i)_{i\in I}$, $p = (1, 1)$, and $P = \{\{2\nu - 1, 2\nu\} : \nu = 1, \ldots, n\}$. The state

s^* is an EB-equilibrium at which no group of consumers can benefit from forming a new household. The allocation $(\mathbf{x}^*; P)$ is fully Pareto-optimal. By Corollary 1 (ii), $(p, \mathbf{x}^*)$ is a traditional competitive equilibrium where each agent acts and trades individually. Furthermore, local non-satiation of preferences implies that $(p, \mathbf{x}^*)$ belongs to the core of the traditional pure exchange economy.

Next, let us define for $0 < \epsilon < 1$ the feasible commodity allocation $\mathbf{x}(\epsilon) = (x_i(\epsilon))_{i \in I}$ by setting $x_i(\epsilon) = (1+\epsilon, 1-\epsilon)$ for i odd and $x_i(\epsilon) = (1-\epsilon, 1+\epsilon)$ for i even. Then one can define a profile of household demand correspondences D so that the state $(\mathbf{x}(\epsilon), p; P)$ is a D-equilibrium. Suppose $\epsilon^2 < g$. Then:

(a) $\mathbf{x_h}(\epsilon) \notin EB_h(p)$ for all $h \in P$.

(b) In state $(\mathbf{x}(\epsilon), p; P)$, no consumer can benefit from exit.

(c) In state $(\mathbf{x}(\epsilon), p; P)$, no consumer can benefit from joining another household.

Assertion (a) holds, since $\mathbf{x}^*_\mathbf{h}$ strictly dominates $\mathbf{x_h}(\epsilon)$ for each household $h \in P$. Specifically, each of these households performs an efficient net trade with the market, but an inefficient internal distribution of resources. Assertion (b) holds, since for all $i \in h \in P$, $U_i(\mathbf{x_h}(\epsilon); h) = 4 - \epsilon^2 + g$ which exceeds the maximal utility level $u_i(x_i^*) = 4$ the consumer can achieve as a single person at the prevailing prices. To show assertion (c), suppose a consumer $i \in h \in P$ joins another two-person household $g \in P$ and each member of the three-person household $g \cup \{i\}$ fares better than before. Then because of (b) and the negative group externality $-c$, each $j \in g \cup \{i\}$ must consume a bundle y_j such that $u_j(y_j) > u_j(x_j^*)$. Therefore, $py_j > p\omega_j$ for $j \in g \cup \{i\}$, since $(p, \mathbf{x}^*)$ is a traditional competitive equilibrium where each agent acts and trades individually. Hence $\mathbf{y_{g \cup \{i\}}} \notin B_{g \cup \{i\}}(p)$ and consumer i cannot benefit from joining household g.

This specification of the model satisfies IPR, GSE and continuity, convexity, and strict monotonicity of preferences. At the D-equilibrium $(\mathbf{x}(\epsilon), p; P)$, there are $n \geq 2$ households of size 2. No consumer can benefit from exit or joining another household. In contrast to the case without externalities, these two stability conditions alone do not require households to make efficient decisions. However, by the argument of Proposition 7, some of the existing households would break up, if fractions of existing households could combine into new households and make more efficient consumption decisions.

Instead of making efficient net trades with the market followed by inefficient internal distribution, households could be making different mistakes. For instance, let $n = 2r$ with $r \in \mathbb{N}$ so that $N = 4r$, and set for $0 < \epsilon < 1$ and $i \in I$: $y_i(\epsilon) = (1+\epsilon, 1-\epsilon)$ if $i \leq r$ and $y_i(\epsilon) = (1-\epsilon, 1+\epsilon)$ if $i > r$. Then $\mathbf{y}(\epsilon) = (y_i(\epsilon))_{i \in I}$ is a feasible commodity allocation and the state $(\mathbf{y}(\epsilon), p; P)$ has the same properties as the state $(\mathbf{x}(\epsilon), p; P)$, except that now households are making inefficient net trades with the market followed by efficient internal distribution.

Notice that even when inefficiencies within households cannot be ruled out, the weaker stability conditions can have some welfare implications. First, in order to prevent a consumer from leaving, the degree of inefficiency cannot be too large. In case $\epsilon^2 > g$, a consumer would benefit from exit. Therefore, $\epsilon^2 \leq g$ has to hold to prevent exit. Thus the exit option limits the degree of inefficiency a household can afford. Second, in order to prevent a consumer from exit, the gains and losses from internal redistribution cannot be too large. To illustrate this point, consider for $0 < \epsilon < 1$ the feasible commodity allocation $\mathbf{z}(\epsilon) = (z_i(\epsilon))_{i \in I}$ by setting $z_i(\epsilon) = (1 - \epsilon, 1 - \epsilon)$ for i odd and $z_i(\epsilon) = (1 + \epsilon, 1 + \epsilon)$ for i even. Then the allocation $(\mathbf{z}; P)$ is fully Pareto-optimal and the state $(\mathbf{z}, p; P)$ is an EB-equilibrium. But in each household $h \in P$, internal redistribution causes the even numbered member to gain at the expense of the odd numbered person. To prevent the odd numbered consumers to benefit from exit, it has to be the case that $\epsilon < g/3$. Hence excessive allocative distortions as well as excessive manifestations of power would cause exit of some household member. If externalities become smaller, then there is less and less leeway for distortions and exercise of power. In the limit, without externalities, household decisions have to be efficient and there cannot be any gains and losses from household membership. ■■

Both the last proposition and the example can be reformulated in terms of type economies, at the cost of additional notation. Consumer preferences then depend on household profile (number of each type present) rather than household size. In the proposition, the condition of equal household size has to be replaced by equal household profile. In the alternative example, one obtains a simple model of bilateral matching or a "marriage model", if there are two types (male and female) and consumers prefer heterogeneous two-person households to other households.

Whether competitive forces can have an efficiency enhancing influence on household decisions depends on several factors. Above all, household members must not be locked into existing households by legal provisions or social conventions. For if household members are forced or feel obliged to stay, then the outside option is simply non-existent. Further, group preferences must not dominate consumption preferences. For if a household member finds the household extremely attractive relative to alternative households, then the member might stay regardless of consumption decisions. Similarly, if a member considers the household composition very unsatisfactory in comparison to other conceivable households, then the individual may want to leave irrespective of consumption decisions. In either case the outside option is ineffective. In contrast, the conclusion of the last proposition rests on the opportunity to form a new household of equal size or, more generally, of equal or similar composition so that the current and the alternative new household are equally attractive or comparable in terms of membership. Then the consumption decision becomes decisive for the choice to stay or to leave — which puts the less efficient household at a disadvantage. Still, the threat of outside options is empty if new potential households suffer from the same inefficiencies as existing ones. However, it is plausible that in the process of regrouping and reallocation, individuals search for and realize efficiency gains. If the threat of departure is credible, then household stability requires efficient decisions or at least avoidance of severe inefficiencies. The prevailing or, to be precise, stable households find ways to avoid grave mistakes or significant inefficiencies caused, for example, by strategic behavior, coordination failure, slackness or force of habit.

For a fixed household structure, efficient household decisions together with budget exhaustion guarantee Pareto-optimality of a competitive equilibrium allocation. When the household structure is variable and externalities are present, efficient household decisions together with budget exhaustion guarantee full Pareto-optimality in many but not all cases. It is possible that a competitive EB-equilibrium allocation is dominated by another feasible combination of a household structure and a commodity allocation. This can be the case even if at the equilibrium state no consumer can benefit from exit or joining another household — as exemplified in Gersbach and Haller (2002). In contrast, the more demanding stability condition that no group of consumers can benefit from forming a new household has very strong welfare implications. First of all, it induces efficient household decisions under the hypothesis of Proposition 7. Second, a competitive EB-equilibrium $(p, \mathbf{x}; P)$

at which no group of consumers can benefit from forming a new household yields a full Pareto-optimum in the weak sense that it is impossible to make everybody better off by means of another feasible allocation. Third, weak Pareto-optimality can be replaced by a weak core inclusion property. To this end, consider a non-empty subset J of I. We say that coalition J can strictly improve upon the allocation $(\mathbf{x}; P)$, if there exist a partition Q of J into non-empty subsets and household consumption plans $\mathbf{y_h}, h \in Q$, such that $U_i(\mathbf{y_{Q(i)}}; Q(i)) > U_i(\mathbf{x_{P(i)}}; P(i))$ for all $i \in J$ and $\sum_{i\in J} y_i = \sum_{h\in Q} \omega_h$. In other words, a coalition can strictly improve upon the given allocation, if it can make each of its members better off by forming a subeconomy with its own household structure and allocation of available resources.

Proposition 8 (Weak Core Inclusion)
Let $(p, \mathbf{x}; P)$ be an EB-equilibrium at which no group benefits from forming a new household. Then no coalition of consumers can strictly improve upon the allocation $(\mathbf{x}; P)$.

PROOF: Let $(p, \mathbf{x}; P)$ be an EB-equilibrium at which no group benefits from forming a new household. Suppose coalition J can strictly improve upon the allocation $(\mathbf{x}; P)$ by means of a partition Q of J and household consumption plans $\mathbf{y_h}, h \in Q$. Now let $h \in Q$. Then $U_i(\mathbf{y_h}; h) > U_i(\mathbf{x_{P(i)}}; P(i))$ for all $i \in h$. If $h \in P$, then $p * \mathbf{y_h} > p\omega_h$, since $\mathbf{x_h} \in EB_h(p)$. If $h \notin P$, then $p * \mathbf{y_h} > p\omega_h$, since group h cannot benefit from forming a new household. But then $p\sum_{i\in J} y_i = \sum_{i\in J} py_i = \sum_{h\in Q}\sum_{i\in h} py_i = \sum_{h\in Q} p * \mathbf{y_h} > \sum_{h\in Q} p\omega_h = p\sum_{h\in Q}\omega_h$, contradicting $\sum_{i\in J} y_i = \sum_{h\in Q}\omega_h$. Hence no coalition J can strictly improve upon the allocation $(\mathbf{x}; P)$. Q.E.D.

If one assumes in addition the budget exhaustion property and the redistribution property of Gersbach and Haller (2001), then weak core inclusion can be replaced by strong core inclusion, that is the assertion that no coalition of consumers can weakly improve upon the allocation $(\mathbf{x}; P)$.

7 Concluding Remarks

The basic premise of this and our previous work is that the allocation of resources among consumers and the ensuing welfare properties are obviously affected by the specifics of a pre-existing partition of the population into households and the way households make decisions. Conversely, the formation and dissolution of households can be driven in part by economic expectations. Becker (1978, 1993) has explored and popularized this idea. Gersbach and Haller (2001, 2002) study the simultaneous allocation of consumers and commodities in a general equilibrium context where households are assumed to make efficient consumption decisions. In the previous section, we reconsider the simultaneous allocation of consumers and commodities and identify circumstances where household stability requires efficient internal distribution, although in principle households could make inefficient choices.

The occurrence of inefficient household decisions has been emphasized in some of the empirical literature. Perhaps the most compelling evidence that decentralized decision-making within households can be inefficient is provided by Udry (1996). He finds that within Burkina Faso farm households (within certain regions), plots controlled by the women in a household tend to be farmed less intensively than similar plots controlled by the men of the same household. Udry estimates that about 6% of output is lost because of inefficient factor allocation within households, assuming decreasing returns to labor inputs. Our findings in the previous section imply that (the degree of) inefficiency is not merely an inherent property of the household, but is to some extent an endogenous phenomenon and depends on the conditions under which the household operates. This observation applies immediately to the case of inefficiency reported by Udry. Inefficient factor allocation within the household is the kind of mistake that can be rectified at the household level and, therefore, gives rise to a sub-optimal allocation for the economy. Together with other causes — e.g. acquisition and preservation of land property rights through cultivation; non-contractible intra-household allocations — a lock-in situation for married women, that is a lack of outside options can be one of the causes that help perpetuate those inefficiencies.

Giving up the collective rationality postulate amounts to working with fewer structural restrictions, which makes it harder to draw specific conclusions. But some systematic analysis proves possible once one differentiates with respect to the kind of inefficiency. The elementary analysis in Sections 4 and 5 revealed that a particular type of household inefficiency does not rule out market efficiency. Thus inefficiency begets efficiency. The analysis also has identified certain inefficient household decisions that always cause an inefficient market allocation. In either case, the household simply makes a mistake — possibly due to difficulties related to collective decision-making.

Frictions in collective decision-making could manifest themselves in a different form, not analyzed in the present paper: through resources used up in the decision-making process. But then Pareto-optimality in the usual sense might no longer be the appropriate efficiency standard, since very likely resource costs would accrue as well when an outsider tried to interfere in the household's economic affairs. Coase (1990), p. 26, makes a similar observation with respect to production: "... the mere existence of 'externalities' does not, of itself, provide any reason for governmental interventions. ...... The fact that governmental intervention also has its costs makes it very likely that most 'externalities' should be allowed to be continued, if the value of production is to be maximized."

Regarding the original, broader question whether distinguishing between a household and its members makes any difference, Haller (2000) compares the case of efficient collective household decisions and the case where each household member shops on her own with her own interest in mind — after being allotted suitable income or endowment shares. In the absence of any externalities and with standard monotonicity and smoothness conditions, there is no difference. If, however, intra-household externalities are present, then as a rule, there is a significant difference. Individual market participants do not fully internalize intra-household externalities. Thus individual behavior can impede collective rationality. In this paper we have shown that competition for resources and members may restore collective rationality.

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