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The Poverty of Linear Nations: Lessons from Taking an AK Model to the Data.*

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Abstract

This paper takes an AK model to the PWT data. In the model, intratemporal and intertemporal shocks are reduced forms for different technologies, and determine the variation of the growth rate. Using the policy functions of the model we recover time series for the unobserved technology shock for a panel of countries. We can then evaluate both how well the model fits the data and what the contribution of the different shocks to the variation of growth rates is. We find that the data is largely inconsistent with the AK structure. However, we isolate what we believe are pervasive patterns in macroeconomic models: a negative correlation between intra and intertemporal shocks, and an ever increasing level of technology matched with ever cheaper consumption relative to investment.

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1 Introduction

Endogenous growth implies eventual linearity in the fundamental dynamic equation of the economy, which is the law of motion for the accumulable state variable. One approach to investigate the nature of growth relies on testing whether investment rates (or savings rates) are systematically related to output growth rates. If they are one concludes in favor of endogenous growth, as McGrattan (1998), Bernanke and Gurkaynak (2001), and Kocherlakota and Yi (1996), among others, do.[1] If they are not one concludes in favor of diminishing returns, as Mankiw, Romer and Weil (1992) and more recently Jones (1995) do.

This paper takes an AK model to the data. The prior is that if endogenous growth theory is correct, then a very stylized linear model should do well against the raw data, just as the early stylized concave models did against log detrended data. We assume that countries are always sufficiently close to the balanced growth path to make transitional dynamics of second order in explaining movements in growth rates. We restrict our analysis to a few countries, but Chari, Kehoe and McGrattan (2001) take this reasoning to all countries in a concave model:

> "The richest countries are typically thought of as being approximately on a balanced growth path. Since the poorest countries grow approximately at the same rate as the richest, this suggest that the poorest countries are as well." [Chari, Kehoe, and McGrattan (2001), p 3][2,3]

Finally, we allow growth rates to fluctuate by making the balanced growth path stochastic. Fatás (2000) looks at a stochastic AK model to reproduce

[1] Bernanke and Gurkaynak redo the exercise of MRW, but instead of doing two separate regressions - one for income per capita in steady state and another for the growth rate as a function of initial income - they estimate both simultaneously.

[2] This idea supports their approach of using a large panel of countries to estimate a unique stochastic process, determining individual country experiences as simple idiosyncratic draws from it. We view their paper as studying positions off the BGP, assuming that convergence to this BGP is a concave process. The issue is then, whether the AK model is in the right spirit by ignoring positions outside the stochastic balanced growth path. The assumption is that even in a multi stock economy with transitional dynamics, as long as the shocks to the model are not too large at any moment, the economy will remain close enough to its BGP for practical purposes.

[3] Caselli (2001), page 61, also has a suggestive view of the data: "The most dramatic feature of cross country income data is of course the enormous dispersion of per capita income. ... as a first approximation this enormously dispersed distribution has been roughly stable over time, at least since 1960. This stability is at least in part a consequence of largely serially uncorrelated growth rates."

the positive correlation between long term growth rates and the persistence of output fluctuations.[4]

Our exercise has a first aim of following the challenge of Klenow and Rodriguez-Clare (1997) by being more explicit in taking model implications to the data. The linearity associated with endogenous growth has powerful implications on model predictions. Here we test these predictions by taking the intertemporal optimality condition of the model and the resulting optimal decision to the data. This contrasts with exploring the more general prediction that investment rates are related to growth rates.

Our second goal is to identify the main sources explaining movements in growth rates. Our model economy contains two stylized mechanisms that affect growth outcomes which are summarized by two different shocks: an intratemporal technology shock and an intertemporal technology shock. These shocks are a reduced form for a variety of economic theories. The intertemporal shock is a shock that affects the technology that transforms current savings into future productive capital.[5] Chari, Kehoe and McGrattan (2001) consider an intertemporal shock in the same spirit and interpret it as "investment distortions". The shock is proxied by the relative price of consumption to investment. These authors consider only the intertemporal shock and show that it is an important determinant of the variability of relative income levels across countries. In a closely related paper Restuccia and Urrutia (2001) look at the cross country patterns of the relationship between the investment shock and the investment to output ratio.[6] The intratemporal shock is a technology shock as in the RBC literature, and integrates the mechanism of growth and cycles in the spirit of Jones, Manuelli and Siu (2000). Arguably, the main implication of Solow (1956) is that total factor productivity is the essencial source of growth.[7] In this light the omission of the technology shock can lead to biased inferences. The bias is hard to assess: Ingram, Kocherlakota and Savin (1994) show that it is impossible to attribute a precise share

[4]This reconciles Jones (1995) with McGrattan (1998): McGrattan (1998) defends AK models using a process (smoothing moments over several periods) that cannot distinguish AK models from endogenous growth models with transitional dynamics. Jones (1995) rejects AK models by looking at short run deviations from the balanced growth path.

[5]It is a proxy for a random financial intermediation technology, a random production function for capital in a multisector economy, or even for a world where new technologies appear embodied in capital goods. Greenwood, Hercovitz and Huffman (1988) is a classic reference.

[6]Both of these models have diminishing returns to broad capital accumulation.

[7]Mankiw, Romer and Weil (1992) took the Solow model to the data and concluded that factor accumulation explains most of the difference in income per capita levels. This triggered a large literature reexamining the evidence and we are back to searching for the determinants of TFP.

of the explanatory power to any particular shock.

We extract the technology shock - which is unobserved - using the policy functions implied by dynamic optimality, and the available data from the Penn World Tables (version 6.1) on consumption, output and relative prices for a variety of countries. Then we study the relationship between the two shocks, and provide a quantitative illustration of the bias when one tries to assign explanatory power to the different shocks.

We point out that this exercise is atheoretical apart from the imposition of linearity and endogenous growth. The data, by implying the relative sizes, degree of correlation, and relative variabilities in the shocks will be the judge of whether theories which have a prominent role for intertemporal shocks are more likely than others which emphasize intratemporal ones. Of course this is conditional on the procedure adopted, but we are looking for implications that are robust to the different approaches.

The paper proceeds with the description of the model and the data. Then we take model implications to the data. We compare some of our work with similar exercises in related literature, and revisit the problems in judging which of the two mechanisms generating growth is the dominant one. We finish by discussing the maintained assumption of linearity.

2 Model

All endogenous growth models with a Balanced Growth Path amount to some microfoundation leading to a linear differential or difference equation. We look at a planner's problem where utility of the representative agent is maximized subject to a budget constraint where aggregate output is divided between consumption and savings, $y_t = A_t k_t = c_t + s_t$. Production is of the AK form, where A_t is the intratemporal technology shock. There is also an intertemporal technology that transforms current savings into investment, $I_t = \theta_t s_t$, and is summarized by the shock (θ_t).[8] The data counterpart of θ is $\frac{p_c}{p_I}$ as may be seen from writing $y_t = c_t + I_t/\ \theta_t$.[9] Thus an increase in θ constitutes an increase in the efficiency of the intertemporal technology or a decrease in investment "distortions". Finally, capital depreciates at rate δ. Capital accumulation is given by, $k_{t+1} = (1-\delta)\, k_t + I_t$.

[8] As in the "R&D accelerator" of Barlevi (2001) and in the financial intermediation literature. In the RBC literature an early reference is Greenwood and Huffman (1984).

[9] Chari, Kehoe and McGrattan (2001) use the same time series to represent their investment distortion shock. Their notation is $1 + \theta_t = p_I/p_c$. They have a concave model $(y = Ak^{\alpha}, \alpha < 1)$ and estimate a time-varying switching process between two distributions, one with low variance and another with high variance, for the relative price process, using the entire PWT dataset.

The problem of the planner is

$$Max \left\{ E_{t=0} \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$
$$s.t.\ k_{t+1} = (\theta_t A_t + 1 - \delta) k_t - \theta_t c_t$$

Solving with respect to k_{t+1} we obtain the Euler equation of this economy, where β is the discount factor,

$$u'(c_t) = \theta_t \beta E_t \left\{ u'(c_{t+1}) \left[A_{t+1} + \frac{1-\delta}{\theta_{t+1}} \right] \right\}$$

There is no steady state in this model but rather a balanced growth path. With a utility function given by $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$, and using $\frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} = (1+g)^{-\gamma}$, the equivalent to the unconditional steady state here is (where (A, θ) denote unconditional expectations of these variables)

$$\frac{1}{\beta \theta} = (1 + g_c)^{-\gamma} \left[A + \frac{1-\delta}{\theta} \right]$$

Typically, one solves for the long run growth rate $g_c(A, \theta)$ and performs comparative statics on this variable. For example, here the more inneficient the intertemporal technology (lower θ), the lower the growth rate of consumption, g_c. But in this paper we want to do the inverse inference: from knowledge of g_c we want to infer the properties of (A, θ).

Furthermore, we are not interested in comparative statics, but in understanding the nature of variations in the growth rate. Therefore we now impose logarithmic utility because it allows us to solve the dynamic programming problem analitically. To the defense of this shortcut we put forth that logarithmic utility is widely used, and also that here only the particular utility shape is a strong assumption because the linearity of the model is a building block of the entire exercise. Our model therefore retains some generality and the explicit policy functions we derive will prove extraordinarily useful.[10]

Before writing the dynamic programming problem, we define our random variables as Markov processes. The state space for the two shocks is a vector index and the state is defined as one realization for the pair (A, θ), and there are n possible pairs. The Markov transition matrix for this composite random

[10] Recently Jovanovic (2002) uses also log utility in an AK model to derive the negative correlation between growth and volatility found by Ramey and Ramey (1991).

variable is defined as $\Pi = [\pi_{ji}]$, where the current state is j and the future state is i.[11] The dynamic programming problem is then

$$V(k,j) = \max_{k'} \left\{ \log\left(B_j k - \frac{1}{\theta_j} k' \right) + \beta \sum_{i=1}^{n} \pi_{ji} V(k', i) \right\}$$

where $B_j = \left[A_j + \frac{(1-\delta)}{\theta_j} \right]$. This problem has a solution for the value function:

$$V(k,j) = a_j + b_j \log(k)$$

which implies a policy function of the type $k'(k,j) = \lambda_j k$, where the b_j are functions only of (β, Π), and the λ_j are functions of $\left(\beta, \Pi, A_j^j, \theta_j^j\right)$. The slope (b_j) of the value function is the solution to $[I(n) - \beta\Pi] \times [b] = [1]$, and $b(\beta, \Pi)$ is the same for all states j, and in fact it is simply $b = 1/(1-\beta)$. Using the first order condition in the above problem, $u_c \frac{1}{\theta_t} = \beta E V_{k_{t+1}}$, the policy function is given by

$$k_{t+1} = [B_t \theta_t] \left[\frac{\beta b}{1 + \beta b} \right] k_t = \beta \theta_t \left[A_t + \frac{(1-\delta)}{\theta_t} \right] k_t$$

where $\frac{\beta b}{1+\beta b} \equiv \beta$. This policy function has a significant property: it contains no parameters of the Markov probability matrix $\Pi = [\pi_{ji}]$. This will be important below. We can now write current optimal consumption as

$$c_t = \left[A_t + \frac{(1-\delta)}{\theta_t} \right] k_t - \left[\frac{1}{\theta_t} \right] k_{t+1} = \left[A_t + \frac{(1-\delta)}{\theta_t} \right] (1-\beta) k_t$$

and this is completely determined by the current values of the state variables. It is therefore not necessary to have information on c_t, A_t, θ_t, *and* k_t, to analize this economy. One variable is redundant, and since measures of capital are the least reliable, that is the data we eliminate.

We can further work this expression to get consumption growth only as a function of the stochastic processes[12]

$$\frac{c_{t+1}}{c_t} = \left[A_{t+1} + \frac{(1-\delta)}{\theta_{t+1}} \right] \theta_t \beta$$

[11] This specification captures the process estimated by CKMG, also a process with a trend and a random shock around it, and finally a unit root process. For existence in a linear problem with a unit root process see Deaton (1991).

[12] If we have $E(\theta_t) \approx 1$ (roughly what we see in the data) to match models where this technology is absent, the mean of A_t will have to be somewhat bigger than δ. The *size* of the shocks (their mean) matters to get the model in line with the observed magnitudes of both $\frac{c_t}{y_t}$ and $\frac{c_{t+1}}{c_t}$, unlike in concave models.

We can also eliminate capital by using the consumption to output ratio:

$$\frac{c_t}{y_t} = (1-\beta) + (1-\beta)(1-\delta)\left[\frac{1}{A_t\theta_t}\right]$$

where we know all of (θ_t, c_t, y_t) separately. Other equations can be derived from the policy function:

$$\begin{aligned}\frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t}\beta\left[\theta_t A_t + (1-\delta)\right] \\ \frac{y_{t+1}}{y_t} - \frac{c_{t+1}}{c_t} &= \beta(1-\delta)\left[\frac{A_{t+1}}{A_t} - \frac{\theta_t}{\theta_{t+1}}\right]\end{aligned}$$

This last equation simply states that if the growth rate of the technology shock is strong, this makes output grow faster than consumption, and if the growth rate of θ_t is stronger this is also the case (because in that case the relative price of consumption is rising so agents respond by consuming less and investing more). Furthermore, it clearly shows that the certainty model would have the same growth rates for consumption and output but that in a stochastic environment this need not happen at any point.

What now? We want to see how well this model performs. Using the data and the equations above we back out a time series for the technology shock for each country. But now we note that the different equations we derive allow us to use for example data on $\frac{c_t}{y_t}$ and $\frac{c_{t+1}}{c_t}$ to recover two time series for (A_t) which may not be identical. We want to see how close these two series are to each other, and also study their relationship with θ_t since the characteristics of the shocks and their relative contribution to the behaviour of the growth rate are key economic issues. But first we look at the data.

3 Data

All the data used in the paper come from the PWT 6.1. The data are in real terms, in 1996 prices. We use data for 24 countries and for the years 1950 trough 2000 in our analysis. Since some countries in our restricted sample lack the observation for 1950, we actually used the sample only from 1951 to 2000, resulting in 50 observations for each of the 24 countries. In what follows in parenthesis are the labels in the PWT dataset. As a measure of the intertemporal shock we use the price of investment goods (PI) and the price of consumption goods (PC). This is the PPP index for consumption and for investment divided by the exchange rate. Their ratio $\frac{p_c}{p_i}$ is the time series proxy for the intertemporal shock. We also extract the consumption and investment shares of GDP (KC,KI), and a GDP measure (RGDPL) to go with them.

Government

Because the model does not have government, we must remove government expenditure from our data. The cleanest procedure is to impose balanced budget with income taxes and assume expenditure is an exogenous additive shock. This is common in the RBC literature, and is done by Chari, Kehoe and McGrattan (2001).[13] Given $G = \tau Y$, we have:

$$Y - G = Y - \tau Y = (1-\tau)Y = (1-\tau)Ak = \tilde{A}k$$

and we note here that Canton (2001) explores an RBC model with random tax rates as a driving mechanism, very much in this spirit. However, the microeconomic structure of the intratemporal shock is not an issue in this paper.

Prices

According to the PWT we can write $pY = p_c C + p_I I + p_G G$, and if we interpret the investment data as showing $k' - (1-\delta)k = I$, the shock θ described below in the model is actually $\frac{p_c}{p_I}$. The technology shock includes more terms now:

$$\begin{aligned} \frac{p}{p_c}Y - \frac{p_G}{p_c}G &= \left(\frac{p}{p_c} - \frac{p_G}{p_c}\tau\right)Y = \left(\frac{p}{p_c} - \frac{p_G}{p_c}\tau\right)Ak = \tilde{A}k \\ \tilde{A}k &= C + \frac{p_I}{p_c}\left[k' - (1-\delta)k\right] \end{aligned}$$

External Balance

Finally, in the data the three shares of consumption (KC), investment (KI) and government expenditure (KG) do not add up to 1. The missing element is the difference between exports and imports (E=X-M). Here we assume this object is an independent component of aggregate expenditure proportional to output (at a random factor e) so that we can subtract it as another shock. We have

$$\frac{p}{p_c}Y - \frac{p_G}{p_c}G - \frac{p_E}{p_c}E = \left(\frac{p}{p_c} - \frac{p_G}{p_c}\tau - \frac{p_E}{p_c}e\right)Ak = \tilde{A}k$$

and this completes our procedure to get the model in line with the data.

Shares

[13] The data, however, suggest this may not be the best reduced form: only 11 out 22 countries have the same sign on the correlations $\rho(c,g)$ and $\rho(i,g)$, which is what we expect if government expenditure works as an additive shock.

The consumption and investment shares must be recomputed. New output is now equal to $\tilde{Y} = Y\left[\frac{C}{Y} + \frac{p_I}{p_c}\frac{I}{Y}\right]$, which removes the government component and the external balance. The corresponding consumption share of this measure of output is $\frac{[C/Y]}{\frac{C}{Y}+\frac{p_I}{p_c}\frac{I}{Y}}$.

Data facts

The short sample characteristics of our data are important. We performed a variety of unit root tests and their outcome points to stationarity in consumption (and output) growth (c_{t+1}/c_t), and to a unit root in the consumption share (c_t/y_t) and the relative price (p_c/p_I). Our unit root testing and data treatment follows Baxter, Jermann and King (1998) who also investigate the stationarity of some NIPA ratios for eleven countries and find mixed evidence of non stationarity.[14] This feature would clearly condition the inference regarding the relative importance of the two shocks, and their relationship, but we again follow the reasoning of Baxter, Jermann and King (1998) and proceed with our analysis assuming the data are draws from stationary distributions.[15] We explore the implications of the presence of unit roots in separate work and provide here only passing reference to them. With the data we can now examine a first implication of our model.

4 A preliminary test of the model

We use the expressions for the rate of growth of consumption and income and the expression for the share of consumption to obtain a simple implication we can take to the data. After some algebra we are able to eliminate first the technology shocks and then β and δ, obtaining the following identity (which the model implies is verified at every point):

$$Z_t \equiv \frac{y_{t+1}}{y_t} - \frac{c_t}{c_{t-1}}\frac{\theta_t}{\theta_{t+1}}\frac{\theta_t}{\theta_{t-1}} \equiv 0$$

We construct a time series of the left hand side of this equation (Z_t) for each country in our panel. If the model is correct any divergences between the data and zero are due to measurement error. We assume this measurement error is iid normally distributed with mean zero, and test whether the sample means for each country are significantly different from zero. As it

[14] The tests were a variety of univariate Dickey-Fuller tests. We computed also 95% confidence intervals for the autoregressive root, following Stock (1991).

[15] The unit root in θ, actually helps us rationalize the AK model as a model of broad capital, since the source of the unit root is probably capital embodied technological progress.

turns out, they are not, and the test statistic (the sample mean over its standard deviation) lies comfortably inside the usual confidence intervals for the normal distribution. Table 1 below contains the statistic to test whether a sample mean differs from zero, assuming each observation of the statistic (Z_t) is iid normally distributed with mean zero and some standard deviation. We must multiply the sample standard deviation by $\sqrt{T}$, to obtain the standard deviation of the mean. So $MET_j = \bar{Z}_j / \sigma\left(\bar{Z}_j\right)$, where $\sigma(x)$ is the standard deviation of x and $\bar{Z}_j$ is the sample mean of Z_{jt} for each country j:

Table1	MET		MET		MET
AUS	0.14	FRA	0.13	MEX	-0.14
AUT	-0.07	GBR	0.37	NLD	0.26
BEL	0.48	GRC	-0.03	NOR	-0.22
CAN	0.24	IRL^{*}	0.70	NZL	-0.19
CHE^{**}	0.23	ISL	-0.06	PRT^{*}	0.13
DNK	0.09	ITA	-0.70	SWE	-0.52
ESP	-0.03	JPN^{**}	-0.02	TUR	-0.82
FIN	-0.16	LUX	-0.56	USA^{*}	0.31

All countries have a statistic well inside any usual (± 1.96) confidence interval of the standard normal distribution, implying a non rejection of the null hypothesis that Z_t is not statistically different from zero. An average, however, tells us only so much. We could have a zero mean with a trend intercepting zero at the sample midpoint. This would be troublesome. But it is not the case. A plot of the time series of the Z_t for all countries in Figure 1 reveals a clear noise around an almost perfect zero. The one caveat is that a regression of Z_t against a constant and Z_{t-1} shows that 19 countries in Table 1 have significant negative autocorrelation. The countries marked with an asterisk have a T statistic on the first lag below 1.96, and two stars indicate well below 1.96.

These are encouraging outcomes since this test largely does not reject the model. Given the well known fragility of this model when confronted with the data, this outcome and its robustness is surprising.[16]

[16] This test yields the same qualitative results if we use the data straight from the PWT without extracting government expenditure or external balance. This test also yields the same results with the previous versions of the PWT data.

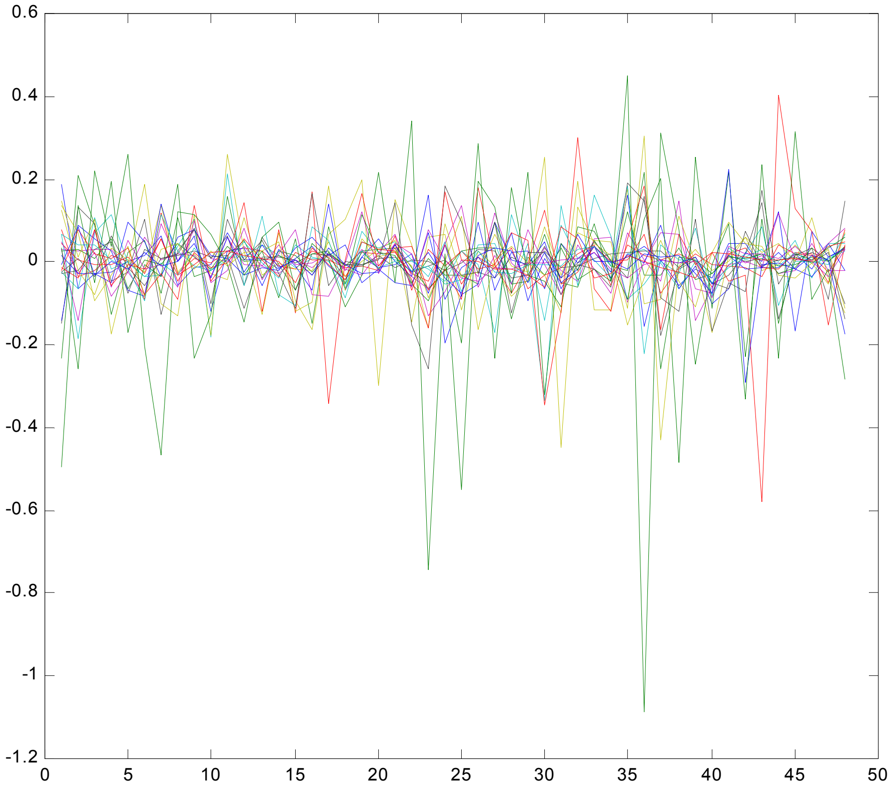

Figure 1:

5 Goodness of Fit

Now that we are encouraged we proceed with recovering the A's and seeing how close they are. Then we look at the correlation between A and theta, and at how well the model fits the data. Finally we investigate what the relative contribution of the two shocks is.

5.1 Recovering the shocks

In order to proceed we need values for our parameters. To this effect we follow the standard procedure in the literature and impose a common value of (β, δ), for all countries. These values are 0.94 and 0.1 respectively, and are not estimated but rather follow a common benchmark in macroeconomic models.

We invert two of the equations derived to obtain two series for the intratemporal shock. From the consumption growth expression we obtain

$$A_{t+1} = \frac{c_{t+1}}{c_t}\frac{1}{\theta_t \beta} - \frac{(1-\delta)}{\theta_{t+1}} \Rightarrow A_t\left(\frac{c_t}{c_{t-1}}, \theta_{t-1}, \theta_t\right)$$

and from the consumption output ratio we get:

$$A_t = \frac{1}{\theta_t}\frac{(1-\beta)(1-\delta)}{\frac{c_t}{y_t} - (1-\beta)} \equiv A_t\left(\frac{c_t}{y_t}, \theta_t\right)$$

We note that from this algebra we recover the *exact* time series of A. Of course, this is conditional on the model being true, on the parameter values (β, δ) and on having $\frac{c_{t+1}}{c_t}$, $\frac{c_t}{y_t}$, and θ_t, measured without error. We will look at measurement error in θ later.

Now, one issue is whether these two series are significantly different from each other. According to the model the difference

$$DA_t = A_t\left(\frac{c_t}{c_{t-1}}, \theta_{t-1}, \theta_t\right) - A_t\left(\frac{c_t}{y_t}, \theta_t\right) \equiv 0$$

should be identically zero, and therefore we assume as before that any divergences between the two series are due to iid measurement error normally distributed with mean zero.[17]

[17] Output is defined as $Y \times \left[\frac{C}{Y} + \frac{p_I}{p_c}\frac{I}{Y}\right]$, removing the government component and the external balance. The new consumption share on the left hand side is defined as $cy = \frac{[C/Y]}{\frac{C}{Y} + \frac{p_I}{p_c}\frac{I}{Y}}$, where the right hand side has all variables in the original.

Table2	*Test*		*Test*		*Test*
AUS	18	*FRA*	20	*MEX*	5
AUT	16	*GBR*	28	*NLD*	17
BEL	25	*GRC*	10	*NOR*	20
CAN	24	*IRL*	20	*NZL*	9
CHE	21	*ISL*	9	*PRT*	12
DNK	11	*ITA*	14	*SWE*	11
ESP	16	*JPN***	14	*TUR*	4
FIN	15	*LUX*	7	*USA*	26

and clearly the test on whether DA is different from zero shown in Table 2 emphatically rejects it (the tests are above 1.96) for every country.[18] Curiously, only three countries display significat trend or autocorrelation in this difference indicator. Japan is the most significant of those. Now, the difference between the two shocks is hardly surprising and the key issue is whether we can still learn something from the AK model given that we are naturally going to reject its most drastic implications. We will, however, return to the issue of the divergence between the two technology series at the end of the paper.

5.2 Correlation between the shocks

It is natural to expect the correlation between $\{A_{jt}\}$ and $\{\theta_{jt}\}$ to be positive. This amounts to the idea that good times are marked by higher efficiency of all technologies and vice versa. If the idea is of an indicator of quality of institutions, then across countries we should also expect country averages of these two shocks to be positively correlated.

Testing the correlation coeffcient

The time series $\theta_{j,t}$ for country j is treated as an exogenous shock in the model, so that we do not need to investigate its relationship with the endogenous variables. We disregard cross country connections, unlike Acemoglu and Ventura (2001). We test whether there is significant linear correlation between θ_t and A_t. Assuming both variables are stationary, the estimated correlation coefficient is given by $\hat{\rho} = \frac{Cov(A,\theta)}{\sigma(A)\sigma(\theta)}$, and this variable is distributed

[18] This was predictable given that the mean of $A_t\left(\frac{c_t}{y_t}, \theta_t\right)$ is 0.409 times the mean of $A_t\left(\frac{c_t}{c_{t-1}}, \theta_{t-1}, \theta_t\right)$, and the factor for the standard deviation is 0.389. These numbers are computed as cross section mean of (mean(A1)/mean(A2)) for each country. and cross section mean of (std(log(A1))/std(log(A2))).

with mean ρ, and standard deviation $\sqrt{\left(1-\hat{\rho}^2\right)/\left(T-2\right)}$, leading to the test statistic:

$$t_{(T-2)} = (\hat{\rho}-\rho)\sqrt{\frac{T-2}{1-\hat{\rho}^2}}$$

and under the null hypothesis that they are uncorrelated we just set the true correlation at $\rho = 0$.

We recall that the choice of data $\left(\frac{c_t}{y_t}, \frac{c_{t+1}}{c_t}\right)$ used to generate the technology shock is not inoccuous. Table 3 below shows the correlation between the series (A, θ). There are two sets of correlations, the first one (first column) uses the A series generated using the $\frac{c_t}{y_t}$ equation, and the second one uses the A generated by the $\frac{c_{t+1}}{c_t}$ equation.[19]. As a rule of thumb, a correlation with an absolute value above 0.28 is statistically significant at 95%

Table3 $\rho(A,\theta)$	$\frac{c_t}{y_t}$	$\frac{c_{t+1}}{c_t}$	$\rho(A,\theta)$	$\frac{c_t}{y_t}$	$\frac{c_{t+1}}{c_t}$	$\rho(A,\theta)$	$\frac{c_t}{y_t}$	$\frac{c_{t+1}}{c_t}$
AUS	-0.01^*	-0.05^*	FRA	0.49	-0.56	MEX	0.82	0.18^*
AUT	0.49	0.14^*	GBR	0.17^*	-0.06^*	NLD	-0.07^*	-0.16^*
BEL	0.65	-0.13^*	GRC	0.47	0.31	NOR	0.49	-0.13^*
CAN	0.16^*	-0.27^*	IRL	0.14^*	0.17^*	NZL	0.63	-0.01^*
CHE	0.60	0.02^*	ISL	0.30	-0.17^*	PRT	0.18^*	0.04^*
DNK	-0.65	0.07^*	ITA	-0.43	0.22^*	SWE	0.67	-0.01^*
ESP	0.29	-0.29	JPN^{**}	-0.00^*	-0.67	TUR	-0.18^*	-0.07^*
FIN	0.62	-0.32	LUX	0.21^*	0.05^*	USA	-0.33	-0.09^*

There are several things to take from this exercise. First, the choice of data matters: A test on the null that the mean of the difference between the values in the two columns is zero yields the value 3.3191 which is a rejection. Second, there is no uniformity of results. When correlations are significant using both data sources (four countries), they often come with opposite signs (Spain, Finland and France, the exception being Greece), and often the correlations are not significantly different from zero (when marked with an asterisk). Also, the correlations are mainly positive if we use c_t/y_t but mainly negative if we use c_{t+1}/c_t.

If we were to take at face value a result of no correlation between the two shocks, we would probably reject the idea of these shocks being a proxy for the functioning of institutions in the economy.[20]

[19] See april2003.m or the old file octob2002.m.

[20] And this of course is ignoring the inference problems created by the possibility of unit

5.3 Which shock is more important?

Here, we perform an exercise that allows us to quantify more clearly the contribution of the different shocks. In this section, due to space considerations, the series for A_t is generated **only** with the c_t/y_t data.

Part 1. Orthogonalizing

We have three sets of measures. In the first measure we use the raw data. For the other two measures, we orthogonalize the shocks to get an independent impact of each shock. We experiment with this orthogonalization because we have no prior on how the two shocks are related, and because we want to isolate the individual contribution of each shock. The process of orthogonalization is not innocuous. In one case we regress by OLS

$$\theta_t = a + bA_t + \epsilon_t$$

and then use the pair $(\hat{\theta}_t = \hat{a} + \hat{b}\bar{A} + \hat{\epsilon}_t, A_t)$ where $\bar{A}$ is the mean of A, thereby removing from θ_t the component that can be explained by A_t. In the other case we just switch the shocks. We do this for every country.

Part 2. Isolating the shocks.

Again we are interested in the impact of each shock on the movement of the different data series. Now we evaluate it by comparing the true data with an adequately generated artificial series. This artificial data is produced by shutting down one of the shocks at its country average. We also do this for different data. For example, regarding the consumption share we compare the true $\frac{c}{y}$ data to the following two alternatives:

$$\begin{aligned} \frac{c_t}{y_t}|_{\bar{A}} &= (1-\beta) + (1-\beta)(1-\delta)\left[\frac{1}{\bar{A}\theta_t}\right] \\ \frac{c_t}{y_t}|_{\bar{\theta}} &= (1-\beta) + (1-\beta)(1-\delta)\left[\frac{1}{A_t\bar{\theta}}\right] \end{aligned}$$

Part 3. Evaluating the impact of each shock.

We run an OLS regression of the true data, first against the artificial series generated with only one shock and the other set to its mean. For example, if we first set the intertemporal shock $(\theta_{j,t})$ to its country specific mean $(\bar{\theta}_j)$ we run:

$$\frac{c_t}{y_t} = \alpha_0 + \alpha_1\left[\frac{c_t}{y_t}|_{\bar{\theta}}\right] + \epsilon_t$$

roots in the relative price and consumption share data.

We then set A to its country specific mean and run

$$\frac{c_t}{y_t} = \alpha_0 + \alpha_1 \left[\frac{c_t}{y_t} |_{\bar{A}} \right] + \epsilon_t$$

to see the difference.[21]

There are a variety of ways to do this experiment. We could simply regress the actual data against θ, or against A, directly. But then we would have problems with mispecification of the regression due to the non linear relationship between the data and the shocks. We choose to do it by running the regression of the actual data against the artificial series because we believe the R squared of this regression is a better measure of what is missing (or not) when we use only one shock.

Part 4. Results

Table 4 shows the R squared of a series of regressions. Column 1 shows the regression of the true $\frac{c_t}{y_t}$ data against $\frac{c_t}{y_t}|_{\bar{A}}$ as defined above, where the technology shock is set at the country specific mean. This produces $R_1^2(\theta_t)$, which is a measure of the explanatory power of θ_t where θ_t is taken raw from the data. Inevitably, $1 - R_1^2$ is then a measure of the explanatory power of A_t.

Column 2 regresses the true $\frac{c_t}{y_t}$ data against data constructed using the projection $\hat{\theta}_t$. We recall here that we obtain $\hat{\theta}_t$ when we regress by OLS $\theta_t = a + bA_t + \epsilon_t$, and then use the pair $(\hat{\theta}_t = \hat{a} + \hat{b}\bar{A} + \hat{\epsilon}_t, A_t)$, thereby removing from θ_t the component that can be explained by A_t (which biases the explanatory power towards A and away from θ). Note that in columns 1 and 2, the artificial $\left[\frac{c_t}{y_t}|_{\bar{A}}\right]$ is always constructed using the true mean of A, and the respective theta series.

Column 3 fixes theta to its country specific mean and uses the original A series, while column 4 uses the orthogonalized series for A.

[21] Chari, Kehoe and McGrattan (2001) and Restuccia and Urrutia (2001) effectively shut down the technology shock.

Table4 c_t/y_t	$R_1^2(\theta_t)$	$R_2^2\left(\hat{\theta}_t\right)$	$R_3^2(A_t)$	$R_4^2\left(\hat{A}_t\right)$
AUS	0.00	0.63	0.36	0.99
AUT	0.25	0.23	0.71	0.73
BEL	0.42	0.24	0.70	0.52
CAN	0.04	0.68	0.13	0.82
CHE	0.35	0.23	0.72	0.57
DNK	0.42	0.83	0.16	0.57
ESP	0.09	0.37	0.65	0.90
FIN	0.38	0.18	0.78	0.56
FRA	0.23	0.27	0.68	0.72
GBR	0.04	0.67	0.33	0.96
GRC	0.15	0.23	0.49	0.38
IRL	0.03	0.61	0.26	0.92
ISL	0.10	0.42	0.53	0.85
ITA	0.19	0.93	0.03	0.77
JPN	**0.00**	0.79	**0.11**	0.91
LUX	0.03	0.43	0.15	0.55
MEX	0.59	0.00	0.80	0.04
NLD	0.01	0.53	0.44	0.99
NOR	0.21	0.27	0.70	0.74
NZL	0.36	0.26	0.60	0.42
PRT	0.04	0.59	0.37	0.91
SWE	0.42	0.14	0.83	0.51
TUR	0.01	0.65	0.01	0.65
USA	0.12	0.86	0.02	0.78
mean	0.186	0.461	0.439	0.698

What this exercise shows us is that we can make different statements about the relative impact of the two shocks. Japan is a good example: If we used $R_1^2(\theta_t) = 0.00$ as an indicator of the explanatory power of θ, we would be lead to believe that technology shocks explain the most. On the other hand, if we decided to use $R_3^2(A_t) = 0.11$ as an indicator, we might conclude the opposite.

5.3.1 Inbreeding

So far this experiment indulges in a cardinal sin. The variable θ and the data are used to generate A, and then A and θ, are used to explain the data we used initially to generate A. Well, we now use a distant cousin, to somewhat reduce the problem. We still use c/y to generate A, but perform the exercise above on output growth. We set A to its country specific mean and run

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t} |_{\bar{A}} \right] + \epsilon_t$$

where $\left[\frac{y_{t+1}}{y_t} |_{\bar{A}} \right] = \beta \left[\bar{A}_j \theta_t + (1 - \delta) \right]$. We first do it with the original θ, series and then repeat it with the orthogonalized one. For the second set of regreassions we set θ its country specific mean and run

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t} |_{\bar{\theta}} \right] + \epsilon_t$$

where $\left[\frac{y_{t+1}}{y_t} |_{\bar{\theta}} \right] = \frac{A_{t+1}}{A_t} \beta \left[A_t \bar{\theta}_j + (1 - \delta) \right]$. Again we repeat it with the orthogonalized A.

The results are in Table 5

Table5 y_{t+1}/y_t	$R_1^2(\theta_t)$	$R_2^2\left(\hat{\theta}_t\right)$	$R_3^2(A_t)$	$R_4^2\left(\hat{A}_t\right)$
AUS	0.22	0.00	0.66	0.69
AUT	0.03	0.01	0.23	0.14
BEL	0.03	0.05	0.53	0.34
CAN	0.01	0.09	0.62	0.78
CHE	0.05	0.00	0.71	0.75
DNK	0.03	0.02	0.00	0.01
ESP	0.06	0.02	0.42	0.08
FIN	0.00	0.06	0.53	0.20
FRA	0.08	0.00	0.36	0.00
GBR	0.05	0.01	0.42	0.60
GRC	0.15	0.03	0.26	0.32
IRL	0.06	0.04	0.49	0.45
ISL	0.00	0.01	0.23	0.14
ITA	0.11	0.02	0.09	0.09
JPN	0.26	0.07	0.62	0.42
LUX	0.02	0.00	0.31	0.08
MEX	0.00	0.01	0.03	0.00
NLD	0.00	0.00	0.39	0.29
NOR	0.01	0.00	0.50	0.25
NZL	0.02	0.15	0.38	0.29
PRT	0.04	0.00	0.21	0.08
SWE	0.01	0.00	0.01	0.01
TUR	0.05	0.01	0.19	0.01
USA	0.00	0.09	0.27	0.80
$mean$	0.054	0.029	0.353	0.285

and casual inspection of this table reveals the same selection bias. These exercises allow us to have a quantitative idea of the problem.[22]

5.4 Model fit

We would like to have a measure of how well the model fits the data. Here we generate the technology shock for every country using both c_t/y_t (A_t^1) and c_{t+1}/c_t (A_t^2). With the time series for both shocks we estimate a joint Markov process (one 9×9 transition matrix and two support vectors each

[22] All the numbers in the paper so far are contained in the file april2003.m.

with three elements).[23] Then, we simulate 100 panels of the same dimension of our real panel, initializing each country at its respective element in the support of (A, θ). We simulate the equation that determines **output** growth, and the equation for the consumption output ratio:

$$\begin{aligned} \frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t}\beta\left[\theta_t A_t + (1-\delta)\right] \\ \frac{c_t}{y_t} &= (1-\beta) + \frac{(1-\beta)(1-\delta)}{A_t\theta_t} \end{aligned}$$

For each of the 100 panels, we compute moments for the cross section of $\frac{y_{t+1}}{y_t}$ and $\frac{c_t}{y_t}$. We compute the cross section mean and standard deviation for four separate years. Then we average these moments over the 100 extractions we make, and compare these to the true data moments. We get

$\frac{c_t}{y_t}\,(A_t^1)$	μ_{art}	σ_{art}	μ_{true}	σ_{true}
1960	0.7449	0.0353	0.7579	0.0578
1970	0.7444	0.0354	0.7173	0.0536
1980	0.7435	0.0361	0.7341	0.0408
1990	0.7446	0.0343	0.7426	0.0395

$\frac{y_{t+1}}{y_t}\,(A_t^2)$	μ_{art}	σ_{art}	μ_{true}	σ_{true}
1960	1.0353	0.1586	1.0408	0.0583
1970	1.0441	0.1553	1.0387	0.0578
1980	1.0328	0.1604	0.9945	0.0390
1990	1.0401	0.1625	0.9872	0.0438

Consider the top table. Using this criterion is similar to the exercise done by Restuccia and Urrutia (2001).[24] The apparent fit is misleading because we use c/y to get A and then go back to simulate c/y using the process for A. If instead we use c_t/y_t to get A, and then simulate y_{t+1}/y_t with the

[23] The support vectors have three points. The middle point is the median value over all values for all countries. The high and low points are the median values of the two subsamples separated by the overall median. The transition matrix is computed by simply counting the transitions from cell to cell, while each observation is assigned the cell which is closest in value to it.

[24] See their table 7 on page 114. That is: we take a relationship that we know the model will satisfy, use the necessary data to approximate it, and then compare artificial and real data moments. In their case they uncover a relationship between investment and θ that is quite close - though not exact - in the data. The model then predicts it should be exact. And then the simulations show a small difference between artificial and real data. But this simulation is just a direct reflection of assumptions, and cannot be viewed as a test of the model, just like the tables in our paper are not. Our experiments in this section are contained in the file goodfitjoint.m, also inside the PWT61 folder.

resulting estimated stochastic process *we will be far off in every measure.* On the bottom table we use c_{t+1}/c_t to get A, and then simulate y_{t+1}/y_t, and here, even though we cannot have an almost exact match as in the previous case, we do quite well on the mean, but miss on the standard deviation. If we then try to simulate c_t/y_t, we will be again far off the mark.

We could use this type of experiment to quantify the individual contribution of each shock. But since we know that such an exercise is not informative and we already have a measure of the possible biases in such experiments we do not report them. Instead we move on.

6 Country Ratios

Restuccia and Urrutia (2001) raise an important concern about the use of the relative price data for an individual country. They note that given the way the PWT price data is constructed all the error free information on θ we have is the ratio of thetas between two countries, or as they do, the ratio relative to the United States.[25] In light of this concern we explore what their constraint on using the data implies for the AK model and what exercises we can do under such a constraint. In the end we will investigate if that leads to very different implications from what we get using individual country price data.

6.0.1 Redoing the preliminary test of the model

We had for each country:

$$\frac{y_{t+1}}{y_t} \equiv \frac{c_t}{c_{t-1}} \frac{\theta_t}{\theta_{t+1}} \frac{\theta_t}{\theta_{t-1}}$$

and now for ratios

$$\frac{\frac{y_{i,t+1}}{y_{i,t}}}{\frac{y_{t+1}}{y_t}} \equiv \frac{\frac{c_{i,t}}{c_{i,t-1}} \frac{\theta_{i,t}}{\theta_{i,t+1}} \frac{\theta_{i,t}}{\theta_{i,t-1}}}{\frac{c_t}{c_{t-1}} \frac{\theta_t}{\theta_{t+1}} \frac{\theta_t}{\theta_{t-1}}} \Longleftrightarrow Z_t \equiv \frac{\frac{y_{i,t+1}}{y_{i,t}}}{\frac{y_{t+1}}{y_t}} \frac{\frac{c_t}{c_{t-1}}}{\frac{c_{i,t}}{c_{i,t-1}}} - \frac{\frac{\theta_{i,t}}{\theta_{i,t-1}}}{\frac{\theta_t}{\theta_{t-1}}} \left[\frac{\frac{\theta_{i,t+1}}{\theta_{i,t}}}{\frac{\theta_{t+1}}{\theta_t}} \right]^{-1} \equiv 0$$

where we can easily see that this information will be error free according to the above authors. We construct a time series of the left hand side of this equation for each country in our panel. If the model is correct any divergences between the data and zero are due to measurement error - assumed to be iid normally distributed with mean zero - and test whether the sample means for each country are significantly different from zero. We get for the measurement error test:

Once again all values are well inside the usual 1.96 confidence interval. The time series for (Z_t) is flat around zero. It is zero on average. There is significant (tstatistic $>$1.96) negative serial correlation in 18 countries in the Table1* experiment.[26]

[25] Because the price measure includes an international component, measurement error in this component induces a spurious correlation between relative prices and income (or income growth) or investment rates (RU, page 119).

[26] ISee the file Octob2002.m. The qualitative results of this experiment are unchanged if the data are taken straight from the PWT or corrected only for government expenditure.

Table1$*$	MET		MET		MET
AUS^*	-0.05	FRA	-0.23	MEX	-0.12
AUT	-0.19	GBR	-0.09	NLD^*	0.03
BEL^*	0.06	GRC	-0.09	NOR	-0.40
CAN^*	-0.16	IRL	0.32	NZL	-0.33
CHE^*	-0.12	ISL	-0.09	PRT	0.11
DNK	0.10	ITA	-0.69	SWE	-0.45
ESP	-0.13	JPN^*	-0.25	TUR	-0.84
FIN	-0.30	LUX	-0.59	USA	$**$

Using the data in ratios or using the data for each country individually does not affect the performance of this test. Countries with an asterisk have a not significant first lag in the autoregressive regression.

6.0.2 Technology shocks

We observe without error only

$$R_{i,t} = \frac{\theta_{i,t}}{\theta_{1,t}}$$

where $\theta_{1,t}$ denotes the USA true relative price (unobserved), and $\theta_{i,t}$ denotes the true relative price of country i (also unobserved).

We know also that for each country we have the policy function $k_{t+1} = \beta\left[\theta_t A_t + (1-\delta)\right] k_t$, and we want to recover information about A. If we use the consumption to output ratio equation to generate A we obtain

$$\left[\frac{c_{i,t}}{y_{i,t}}\frac{1}{(1-\beta)} - 1\right]\frac{1}{(1-\delta)} \equiv X_{i,t} = \frac{1}{A_{i,t}\theta_{i,t}} = \frac{1}{A_{i,t}R_{i,t}\theta_t}$$

where we drop the subscript 1 for the USA. If we use $\frac{c_{t+1}}{c_t}$ to infer A we obtain a much less tractable nonlinear expression. Now from this information we can construct from the observables

$$\frac{A_{i,t}}{A_t} = \frac{X_t\theta_t}{X_{i,t}R_{i,t}\theta_t} = \frac{X_t}{X_{i,t}}\frac{1}{R_{i,t}}$$

which is the relative technology ratio for each country at any moment.

In table 6 we write down the mean and standard deviation of the cross section distribution for relative A's and relative $\theta' s$, where $(\mu^1_{cs}, \sigma^1_{cs})$ have the

A's are constructed using c_t/y_t data and $(\mu^2_{cs}, \sigma^2_{cs})$ have the A's constructed with c_{t+1}/c_t data: [27]

Table6	$\mu^1_{cs}\left(\frac{A_j}{A}\right)$	$\sigma^1_{cs}\left(\frac{A_j}{A}\right)$	$\mu_{cs}\left(\frac{\theta_j}{\theta}\right)$	$\sigma_{cs}\left(\frac{\theta_j}{\theta}\right)$	$\mu^2_{cs}\left(\frac{A_j}{A}\right)$	$\sigma^2_{cs}\left(\frac{A_j}{A}\right)$
1960	0.8841	0.1921	**1.2726**	0.2943	0.8142	0.3135
1965	0.9020	0.1571	**1.2353**	0.2284	0.7958	0.1817
1970	0.9719	0.1480	**1.1911**	0.1882	0.7656	0.2987
1975	1.0002	0.1670	**1.1446**	0.1751	1.8153	0.6386
1980	**1.1912**	0.2094	0.9257	0.1443	**0.9765**	0.2671
1985	**1.1078**	0.2771	0.9835	0.1769	**1.0258**	0.2088
1990	**1.1795**	0.1773	0.9371	0.1192	**1.1763**	0.6266
1995	**1.1073**	0.1566	0.9500	0.1105	**0.9927**	0.7277
mean	1.0103	0.1844	1.0987	0.1863	0.9643	0.3883

and from these we learn two things: first, production technology levels are superior (inferior) to the United States in the latter (earlier) part of the sample. Consumption prices ($\theta = \frac{p_c}{p_I}$) are lower (higher) in most countries than the US in the latter (earlier) part of the sample. These cross section means suggest the two shocks are negatively correlated over time.[28]

What about country specific means (over each individual time series)? The country means of relative A are negatively correlated with the country means of relative θ, and the values are -0.904 for A^1, and -0.923 for A^2.[29]

Finally, we also run a cross section regression here in the same spirit as the ones above, but to determine the relationship between technology levels and relative prices. For a given year we run

$$\log\left(\frac{A_j}{A}\right) = \alpha_0 + \alpha_1 \log\left(\frac{\theta_j}{\theta}\right) + \epsilon_j$$

[27] Check files july2002.m and Octob2002.m, april 2003.m.

[28] The correlation coefficient between the cross section mean of A^1 and the cross section mean of θ is -0.9789 across the time sample, and the correlation between the mean of A^2 and the mean of θ is -0.3016. Both correlations are statistically significant. That is, as time passes, if the cross sectional mean of the relative A shock increases, the cross sectional mean of the theta shock falls.

[29] Here we take one value (mean) for the A ratio for each country. We have then 23 observations (because the USA is identically one). We correlate these 23 observations for the A ratio with the 23 observations for the θ ratio.

and the results are in table 7

Table7	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2	
1960	0.0602	−0.9505	2.49	12.0	0.868	A^1
1985	0.0438	−0.9727	4.51	21.1	0.953	A^1
1960	−0.0857	−0.8662	0.58	1.82	0.131	A^2
1985	−0.0264	−0.6982	1.02	5.84	0.608	A^2

so that there is a negative relationship across countries between how cheap investment is (how high θ) relative to the USA and how relatively efficient their technology is.[30]

6.0.3 Unit roots

Although we do not discuss unit roots in this paper we note here the fact that the unit root tests we performed earlier on the data, show for our ratios that the ratio of the $\theta's$ relative to the USA cannot reject a unit root for most countries (21 out of 23), and the ratio of the A shocks, when constructed using C/Y also cannot reject unit roots for 20 out of 23 countries. However, when we construct the A ratio using C_{t+1}/C_t data, we emphatically reject the presence of a unit root for all countries. We conclude this section noting that the main characteristics of the exercises in the rest of the paper are not affected by the ratio correction.

[30] The A ratio is constructed using c/y. This regression indicates a correlation between their ratios relative to the USA. Also, using an estimator for the depreciation rate for each country does not affect these results.

7 One shock approach

We can turn the exercises above on its head. Suppose we take the road of CKMG and eliminate the technology shock. We can then use the data and the policy functions we derive to ask a different question. Take the consumption income ratio for example and invert the expression to get:

$$A_t\theta_t = (1-\delta)\left[\frac{c_t}{y_t}\frac{1}{(1-\beta)} - 1\right]^{-1}$$

Now eliminating the technology shock we can write:

$$\hat{\theta}_t = \frac{(1-\delta)}{A}\left[\frac{c_t}{y_t}\frac{1}{(1-\beta)} - 1\right]^{-1}$$

and thus ask the question: does this estimated series for θ_t, look anything at all like the price ratio we see in the data? We use the parameter $A > 0$ to match the mean for each country. We can then compare many different statistics from the time series for each country. However, for our purposes the correlation between the estimated $\hat{\theta}_t$ and the price ratio from the PWT is enough.

There is one final detail: what should the data for $\frac{c_t}{y_t}$ be? This is not a trivial question. First, if we are recovering θ_t, it is because this variable is assumed to exist both in the data and in the model. So it seems we should use it to construct c/y. A second approach is to use simply the consumption share from the PWT6.1, (KC). A third approach is to use a share more in accordance with the model, KC/(KC+KI). All these possibilities yield different results.

So, first using $\frac{c_t}{y_t} = \frac{KC}{KC+(PI/PC)*KI}$, we get negative correlations between the PWT price ratio and the estimated $\hat{\theta}_t$, for 18 countries:

$\frac{c_t}{y_t}(\theta_t)$	$\rho\left(\hat{\theta}_t,\theta_t\right)$		$\rho\left(\hat{\theta}_t,\theta_t\right)$		$\rho\left(\hat{\theta}_t,\theta_t\right)$
AUS^*	0.00	FRA	−0.49	MEX	−0.81
AUT	−0.49	GBR	−0.18	NLD	0.08
BEL^*	−0.64	GRC	−0.42	NOR	−0.47
CAN^*	−0.17	IRL	−0.17	NZL	−0.62
CHE^*	−0.59	ISL	−0.31	PRT	−0.20
DNK	0.65	ITA	0.43	SWE	−0.66
ESP	−0.30	JPN	−0.03	TUR	0.17
FIN	−0.61	LUX	−0.22	USA	0.34

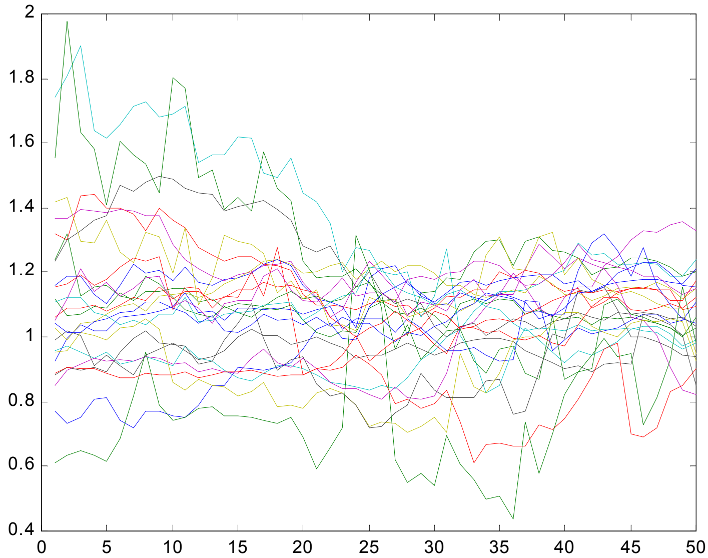

Figure 2:

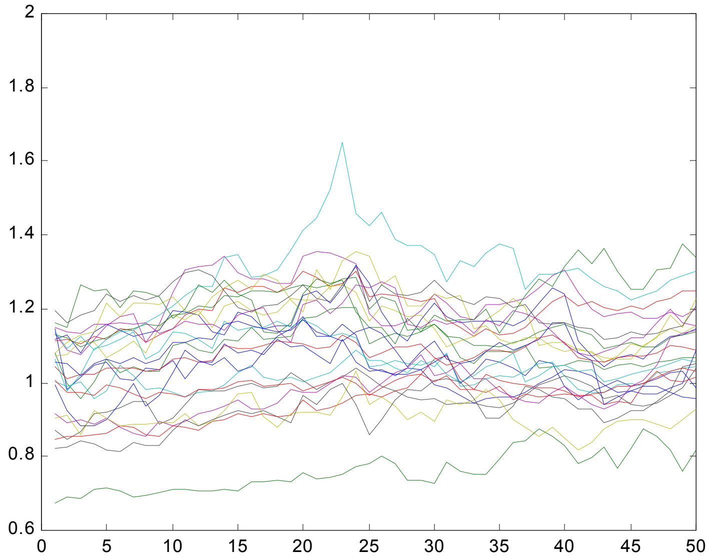

Figure 3:

We can see the two sets of series in these figures. Figure 2 displays the price ratio taken from the PWT61, and Figure 3 displays the estimate above.

Then using $\frac{c_t}{y_t} = KC$, we get 8 negative correlations:

$\frac{c_t}{y_t}(\theta_t)$	$\rho\left(\hat{\theta}_t, \theta_t\right)$		$\rho\left(\hat{\theta}_t, \theta_t\right)$		$\rho\left(\hat{\theta}_t, \theta_t\right)$
AUS^*	0.26	FRA	-0.65	MEX	-0.73
AUT	0.48	GBR	0.22	NLD	0.47
BEL^*	-0.64	GRC	0.29	NOR	0.58
CAN^*	0.63	IRL	0.41	NZL	0.34
CHE^*	-0.26	ISL	0.45	PRT	0.13
DNK	0.45	ITA	0.77	SWE	-0.32
ESP	0.17	JPN	-0.17	TUR	0.55
FIN	-0.45	LUX	0.66	USA	-0.40

and then using $\frac{c_t}{y_t} = \frac{KC}{KC+KI}$, we get 9 negative correlations.[31]

So, what constitutes a match? Clearly strong (how strong?) positive correlations for all countries would be a good start. But we do not obtain that. This can naturally be taken as a failure of the model to match the data. It can also mean that something is missing.

8 More shocks

Here we use a variation of the model to reexamine the Ingram, Kocherlakota, and Savin (1994) problem of singularity.[32] This extension is simple: we just consider that the true intertemporal shock in the model has two components, only one of which we observe in the data as the relative price of consumption to investment. Consider:

$$\theta = \phi_h \theta_k$$

where we use for illustration purposes a subscript labelling human and physical capital. The task then is to recover two shocks from the data, rather

[31] We do not need another table to make the point. The mean difference between $\rho(KC) - \rho\left(\frac{KC}{KC+KI}\right)$ is -0.0903 with a standard deviation of 0.4296.

[32] This can also adress the problem of the broad definition of capital and its data counterpart.

than just one. We rewrite:

$$\begin{aligned}
\frac{c_{t+1}}{c_t} &= \left[A_{t+1} + \frac{(1-\delta)}{\theta_{t+1}\phi_{t+1}}\right]\theta_t\phi_t\beta \\
\frac{c_t}{y_t} &= (1-\beta) + (1-\beta)(1-\delta)\left[\frac{1}{A_t\theta_t\phi_t}\right] \\
\frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t}\beta\left[\theta_t\phi_t A_t + (1-\delta)\right]
\end{aligned}$$

and note that we have several equations and two shocks, but that by the data construction only two equations are independent. The economic question is what we can learn from the shocks we are backing out. After some algebra we obtain:

$$\begin{aligned}
\frac{\phi_t}{\phi_{t-1}} &= \frac{\frac{c_t}{y_t}\frac{1}{1-\beta}}{\frac{c_t}{y_t}\frac{1}{1-\beta} - 1}\frac{\beta(1-\delta)}{\frac{c_t}{c_{t-1}}\frac{\theta_t}{\theta_{t-1}}} = \frac{\frac{c_t}{y_t}}{\frac{c_t}{y_t} - (1-\beta)}\frac{\beta(1-\delta)}{\frac{c_t}{c_{t-1}}\frac{\theta_t}{\theta_{t-1}}} \\
A_t &= \frac{1}{\phi_t\theta_t}\frac{(1-\delta)}{\frac{c_t}{y_t}\frac{1}{1-\beta} - 1}
\end{aligned}$$

and to pin down the ϕ_t series we assume that the initial values equal one for all countries.[33]

The outcome is stunning. The shocks we back out show that A_t is growing exponentially, and second, that the true intertemporal price ($\phi_t\theta_t$) is falling also exponentially (θ_t, despite some evidence of unit roots is a model of stability by comparison with ϕ_t). Basically, consumption goods become ever cheaper, and therefore the relative price of consumption is fast approaching zero. This is triggered by fast growth in technology. In order to have stable growth it is necessary for the relative price of investment to rise quickly. For every country.

The figures below show the time series for ϕ_t (figure 4), then for log detrended ϕ_t (figure 5), then for the implied A_t (figure 6), and its log detrended series (figure 7). The log detrended data is shown only to have a sense of the short term variations in these series. However, the key issue here is that data which we a priori expected to be stationary due to the linearity assumption used to back them out are not stationary.

But the question is: if the AK model has any plausibility at all, what is this outcome then telling us? Is it a theoretical problem that we obtain these non stationary series that seem to cancel each other? Is this related to the exchange rate component studied in Acemoglu and Ventura?

[33] Look for program january2003.m

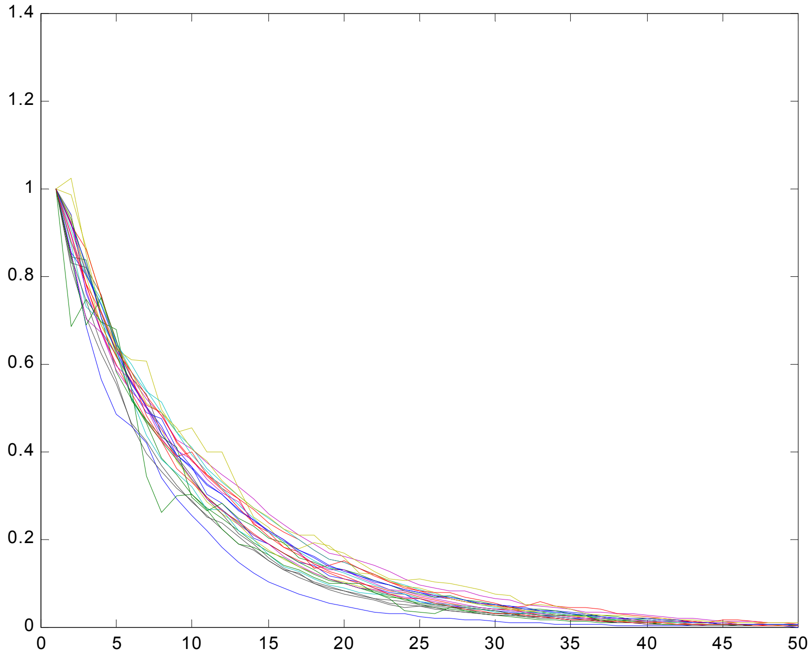

Figure 4:

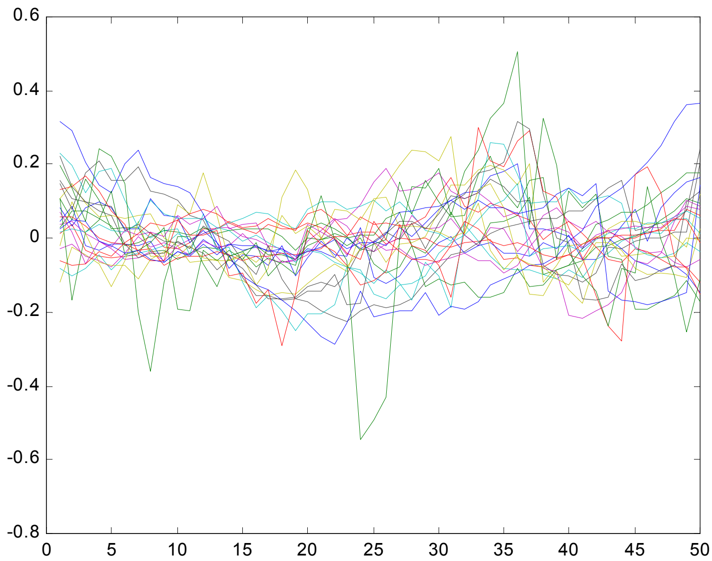

Figure 5:

The shocks we obtain in this way are non stationary and cannot be realizations of a stationary Markov process. However, the expressions we derive are functions of the product $\phi_t A_t$, and this product can be stationary, so that there is no incompatibility with the model. One familiar characteristic also seems to come up again: the negative correlation between the two shocks (here A_t and ϕ_t and also the product $\phi_t \theta_t$). If we take the two shocks (ϕ_t, A_t) and we take logs and remove a linear trend we obtain a flat time series for both. The correlation between these two log detrended series is again mainly negative (18 countries have negative correlation).

We did two robustness checks. First, one can also obtain the shocks using

$$\begin{aligned}
\frac{A_{t+1}}{A_t} &= \frac{y_{t+1}}{y_t} \frac{1}{\beta(1-\delta)} \left[1 + \frac{1-\beta}{\frac{c_t}{y_t} - (1-\beta)} \right]^{-1} \\
\phi_t &= \frac{1}{\theta_t} \frac{\frac{y_{t+1}}{y_t} - \frac{A_{t+1}}{A_t}\beta(1-\delta)}{A_{t+1}\beta}
\end{aligned}$$

and imposing $A_0 = \delta + 0.04$, but it does not change the outcomes. Second, this exercise is done with a common β (0.94) and a common δ (0.1). But maybe carefully adjusting δ for each country will yield white noise series for A and ϕ. But unfortunately that is not the case. There is basically

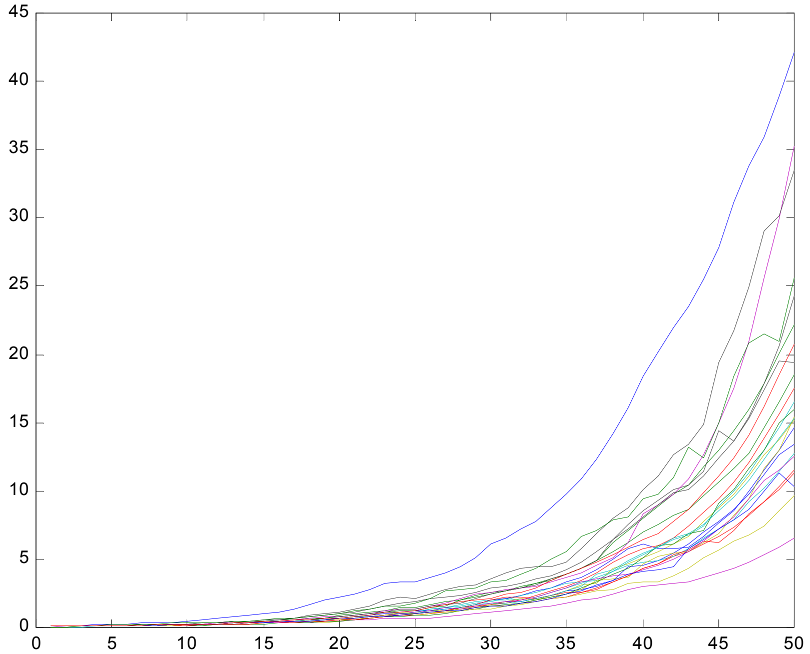

Figure 6:

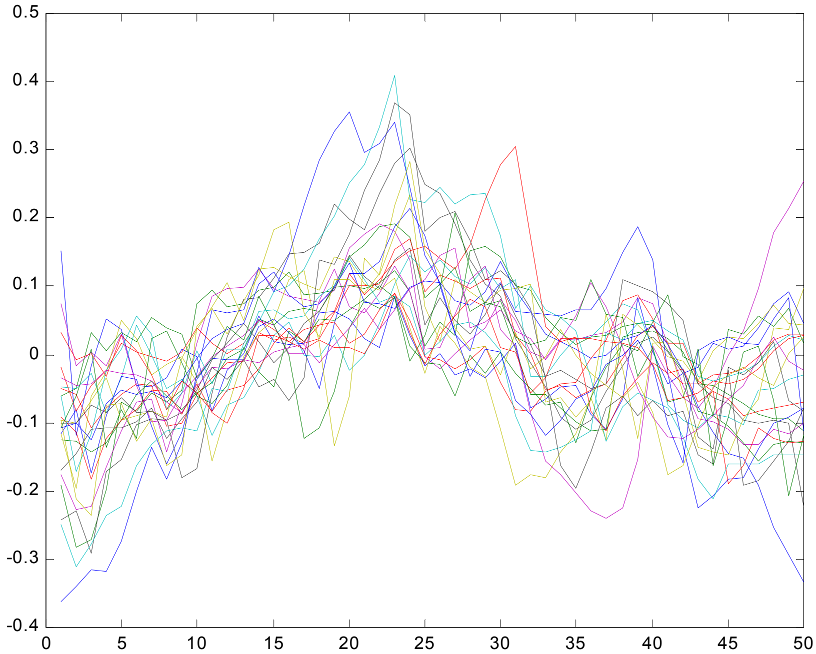

Figure 7:

no plausible positive value of delta that solves the problem (0.01 is already enough to cause damage).

8.1 Another extension

A further extension introduces a preference shock (which amounts here to a stochastic discount factor). Consider the utility function $U(c) = \phi \log(c)$, where ϕ is a random variable. Using the same approach as above we can derive the policy function:

$$k_{t+1} = \beta \left[\frac{\theta_t A_t + (1-\delta)}{\beta + \phi_t (1-\beta)} \right] k_t$$

which of course reduces to the previous function when $\phi = 1$. This produces the expression for the consumption income ratio:

$$\frac{c_t}{y_t} = 1 + \frac{(1-\delta)}{\theta_t A_t} - \frac{\beta}{\theta_t A_t} \left[\frac{\theta_t A_t + (1-\delta)}{\beta + \phi_t (1-\beta)} \right]$$

and one can further develop this model to see if it allows a better fit to the data.

9 Conclusion

We can summarize this paper by the question: "What can we learn by taking a stylized AK model to the (PWT) data?". There is a huge literature on endogenous growth models, and many authors seem to think that developed countries are fairly close to their Balanced Growth Path, which in our opinion suggests that transitional dynamics might be of second order in explaining movements in growth rates. That leaves our stochastic BGP as the key factor driving the economy. What then are the implications of taking these ideas literally?

We consider two sources of fluctuations in the model, an intratemporal shock and an intertemporal shock. The intertemporal shock is observed in the data and proxied by the relative price of consumption over investment. The technology shock is unobserved and one of the tasks of the paper is to recover it.

We solve explicitly for the optimal investment decision in the model and take the exact implications of this optimal decision to the data. This allows us ot recover the *exact* time series for the technology shock. We are then able to investigate the properties of the two shocks and their impact on the growth rate.

We find the the correlation between the shocks is often not significantly different from zero and as likely to be negative as positive. We believe that an "institutional" interpretation of these shocks implies a positive correlation. A negative relationship seems present across countries: the USA is relatively less efficient in terms of total factor productivity in the latter part of the sample where it is also the country with the cheapest investment, and also, when one country is found to be relatively more efficient than the USA in the intratemporal technology, it will tend to have the most expensive investment, or the least efficient intertemporal technology. The reason we cannot assert much more about the correlation is that the model produces several different expressions that can be taken to different data to identify the technology shock with equal legitimacy. This problem is familiar from the literature and of course extends to the determination of the relative impact of the two shocks on the growth rate. Regarding the measurement of this impact we illustrate the fact that one can draw radically different conclusions depending on the way one chooses to do the exercise. It is as possible to find that intratemporal shocks are responsible for most of the movements in the economy, as it is to find that intertemporal shocks dominate.

Finally, we know that the AK model in its stylized form such as the one we use is both open to theoretical criticism and to easy rejection from the data. In fact most of our experiments (with one notable exception) imply a strong rejection of the model. Also, the systematic relationship between growth rates and initial income levels suggests that we cannot treat the data as random movements in the BGP itself. Klenow and Rodriguez-Clare (1997) suggest that one should not try to fit linear models to individual country data. Our point of departure is opposite to theirs: if the theory considers linearity to be a fundamental characteristic of the macroeconomy, then we should take this notion to its extreme. Fundamentally, if linearity is present, then it should be the dominating factor in the time series behaviour of the economy at least for the richer and more stable economies. Somewhere, sometime, we should see AK type behaviour. But this extreme form allows us to study relationships between the shocks that we believe will survive in a more general model. The extraction in the last experiment of a non stationary and ever growing technology and ever cheaper relative price of consumption is interesting in its own right and suggests that we should look more closely at the exercises done by Greenwood, Hercovitz and Krussell (among others) since they perceive the relative price of investment as becoming cheaper. One interesting sucess is the robustness of the initial prediction of the model. One other successfull use of the AK model is Fatás (2000) where he explores particular data features for different countries. So, clearly there is some scope for using the AK model and so we feel the exercise in this paper is warranted.

There are several tasks ahead. One is to investigate non stationarity and its consequences for inference. A second is to obtain measures of depreciation and/or estimate the model. We have actually done structural estimation for the first model in the paper with very poor results reinforcing the idea that linear models cannot fit the data. Finally we should also think hard about the three shock models we briefly explore here and perhaps relate it to other models. Examples of different models with AK features are the work by Acemoglu and Ventura (2001) where they propose a model where all countries have AK technologies, but where trade introduces diminishing returns, and Beaudry and Collard (2002) who propose a model where technological advances imply a move towards more linear technologies, and the adoption phase of this new technology implies the economy behaves as if in a linear model.

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10 Investment rates and relative prices

Here we compare the Restuccia and Urrutia model to our model.[34] For a given country the steady state of their (concave) model implies the investment share[35]

$$\frac{I}{Y} = \theta \frac{\alpha\left[(1+g)(1+n)-(1-\delta)\right]}{\frac{(1+g)^{\sigma}}{\beta}-(1-\delta)}$$

and under the assumption that all countries share the same parameter values except for independent draws of $\theta = \frac{p_c}{p_I}$ from a common distribution[36], the ratio between two countries yields

$$\frac{I^j/Y^j}{I^i/Y^i} = \frac{\theta^j}{\theta^i}$$

which then implies that if country j has a higher θ on average (in steady state), and thus a cheaper investment, its investment rate is correspondingly higher. They show this is true in the data.[37]

But suppose the true model is a stochastic AK model. Then in our

[34] The relative price of investment only measures physical capital in the data and a linear model is supposed to describe a broad notion of capital. Therefore the investment measure and the price measure do not match. This is a well taken and important point. There is no debate about the mispecification. But there are several issues: first, we dont really know what the intertemporal price of human capital is, if the missing stock is human capital - Is it subsidized or taxed? Chances are in poor countries it is taxed even more heavily (with life) than physical capital. Second, their model is also mispecified. Third, in the Arrow model (learning by doing) there is no incompatibility because the source of linearity is not human capital but an externality in the same physical capital.

[35] Here g is the exogenous growth rate of technology. α is the exponent on capital in production. n is the exogenous growth rate of the labor force. δ is capital depreciation. σ is the concavity of CRRA utility, and β is the intertemporal discount factor.

[36] RU estimate a matrix that implies the volatility of the distribution falls with time if we start the time series at the initial distribution. But this is not true for every subsample of countries. Their matrix implies *any* subsample should see the cross section volatility fall secularly.

[37] Note here that RU run a cross section OLS regression of $\frac{I^j/Y^j}{I^{usa}/Y^{usa}}$ against a constant and $\frac{\theta^j}{\theta^{usa}}$, and obtain a coefficient on the theta ratio very close to 1. The R squared however is a bit small. The year that delivers a higher R squared is also the year with the constant further from 1. The constant is also significant in most cases whereas it should not be. So, even though their regression is reasonably successfull, it also can be viewed in the opposite way as showing that a substantial amount is left to be explained in the behaviour of investmnet rates.

framework we have in any given period

$$\frac{I_t^j/Y_t^j}{I_t^i/Y_t^i} = \frac{\theta_t^j\beta - (1-\beta)(1-\delta_j)/A_t^j}{\theta_t^i\beta - (1-\beta)(1-\delta_i)/A_t^i}$$

Here we again just set β equal to 0.94 and use the same δ for all countries at 0.1.

However, the apparent success of the RU analysis hides many caveats. If we take the variable $Z_1 = \frac{\theta^j}{\theta^i}\frac{I^j/Y^j}{I^i/Y^i} - \frac{\theta^j}{\theta^i}$, this variable, for the 23 countries in our sample, produces test statistics that: i) reject that on average Z is zero for 22 countries (exception Ireland), ii) reject that Z is not autocorrelated (all countries), and, iii) reject that a linear trend is not significant (for 17 countries out of 23, so that 6 countries display no significant trend). In fact, the R squared of the first order autoregression is on average 64% implying that the deviation from the model prediction is very high.

If we perform the same tests for the AK expression, with A shut down to 1,[38]

$$Z_2 = \frac{I_t^j/Y_t^j}{I_t^{usa}/Y_t^{usa}} - \frac{\theta_t^j\beta - (1-\beta)(1-\delta)}{\theta_t^{usa}\beta - (1-\beta)(1-\delta)}$$

the characteristics of Z_2 are, for every country, virtually identical to those of Z_1 (as the effect of the nonlinearity caused by $(1-\beta)(1-\delta)$ on the right hand side is very small).

What does this imply for the two models? For the concave model (Z1) this implies that at the very least we reject that all countries have the same parameters. Their model in its simple form fails this test as of course it would. For our model that is not quite the case. What happens is that Z2 will be captured in the implied A shock so this can simply be a statement of what the A shocks look like.

Cross Section Regressions

Now we run a cross section regression of the log of the left hand side against the log of the right hand side of Z1 and Z2.[39] A sample of the cross

[38] If we use the time series for the A shock derived using C/Y we would set the D2 below identically equal to zero. Also, this is what makes the exercise here equivalent to theirs.

[39] The reader is encouraged to consult figure 3 on page 106, and also table 3 on page 107. Restuccia and Urrutia (2001). If we reproduce fig 3 in their paper for our 22 countries, we obtain a clear positive trend (as our theta variable is the inverse of theirs). The regressions are produced in Octob2002.m for early PWT data and April2003.m for the PWT61 data.

section regressions is:

$Z1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2	$T_{\hat{\alpha}_1 \neq 1}$
1960	0.14	1.23	1.85	4.91	0.52	0.73
1985	0.12	1.06	4.28	8.09	0.75	0.04
$Z2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2	$T_{\hat{\alpha}_1 \neq 1}$
1960	0.14	1.17	1.84	4.93	0.52	0.94
1985	0.12	0.99	4.33	8.13	0.75	0.51

This exercise yields coefficients around 1. How close are these values are from the theoretical value of 1? The test $T_{\hat{\alpha}_1 \neq 1}$ in the last column is an indicator that we are indeed close. The main characteristic of this table though, is that there is virtually no difference between the two models. It is our view that the exercise in Restuccia and Urrutia does not constitute a test of their model.