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Roots: The Case of Yugoslavia

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# High Inflation, Hyperinflation and Explosive Roots. The Case of Yugoslavia.

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## Abstract

The focus is on 'explosive root VAR' modelling of money, prices, wages, and exchange rates applied to the Yugoslav high inflation/hyperinflation transition period from a centrally planned economy to a more market oriented economy. The I(2) model, which has previously been used to estimate the Cagan model for hyperinflation, is shown to yield incorrect inference when there are explosive roots in the data. The paper develops an econometric framework for the empirical analysis of hyperinflationary episodes and illustrates the importance of exploiting the system dynamics of all the variables in the system for a full understanding of the hyperinflationary mechanisms. The empirical results suggest that excessive nominal wage claims, inflationary expectations and the rate of currency depreciation were the main causes to the Yugoslav hyperinflation rather than the financing of government debt by money printing.

Keywords: Explosive roots, Hyperinflation, Polynomial Cointegration, Transition Economies

## 1. Introduction

The economic mechanisms generating hyperinflation has been subject to much interest among economists all since Cagan's seminal work (1956) on the demand for

money in periods of hyperinflation. His model predicted a negative expected inflation elasticity,  $\alpha$ , in the demand for money relation in periods of hyperinflation. He also showed that the coefficient  $\alpha$  could be used to derive the 'optimal' rate of inflation ( $1/\alpha$ ) associated with maximum seignorage that could be achieved by printing money. Cagan applied his model to a variety of hyperinflation episodes and found that the average inflation rates observed widely exceeded the derived 'optimal' rate. Thus, given the assumptions of the model that excess money supply was the cause of inflation and controlled by the central bank authorities, the results suggested that central banks had expanded money stock in a non-optimal manner. This result was challenged by Sargent (1977) who argued that the non-optimality finding was a result of Cagan assuming adaptive inflationary expectations instead of rational expectations. Sargent recalculated  $\alpha$  assuming the latter and obtained estimates of 'optimal' inflation rates which to some extent were more in line with observed average inflation rates. Nevertheless, the  $\alpha$  coefficients were imprecisely estimated and, thus, the observed hyperinflation behavior was not convincingly shown to conform with the predictions from the Cagan model.

Taylor (1991) pointed out that none of the above papers addressed the question of nonstationarity of the variables and showed that if inverse velocity ( $m - p - y$ ) and inflation ( $\Delta p$ ) were nonstationarity of order one then the Cagan model implied cointegration between actual inflation and inverse velocity. In this case the coefficient  $\alpha$  became uniquely identified using the cointegration property and this was the case regardless of the way expectations were formed as long as the forecast error, i.e. the deviation between expected and realized inflation was a stationary process. This assumption, i.e.  $\Delta p \sim I(1)$  and, hence,  $p \sim I(2)$ , is consistent with the cointegrated VAR model for  $I(2)$  data. Thus, Taylor's article started a renewed interest in the Cagan model which was re-estimated for a number of high/ hyperinflation episodes within the  $I(2)$  cointegration framework. See for example, Engsted (1993), Petrovic and Vuojesevic (2000), Taylor (1991).

This paper argues that the  $I(2)$  behavior is not an adequate description of price behavior in periods of hyperinflation. The assumption of a unit root in the inflation rate implies a linear stochastic trend in which the stochastic increments (permanent shocks) cumulate with equal weights, whereas inflation under hyperinflation episodes is not characterized by linear but exponential growth. The purpose here is to demonstrate that the VAR analysis can be applied when the data contain an explosive root and that it can provide a powerful tool for both estimating the Cagan elasticity coefficient  $\alpha$  and, more importantly, for investigating the mechanisms that have generated hyperinflation.

Estimation and test results for the cointegrated VAR model have been derived for  $I(1)$  and  $I(2)$  vector processes (Johansen, 1995), whereas explosive roots have

only recently been considered in Nielsen (2002a, 2002b). We first discuss the problem of how to detect such roots and then suggest a reformulation of the cointegrated VAR model that explicitly takes account of this problem.

In spite of the strong belief that hyperinflation is related to with excess expansion of money supply, the empirical support of the Cagan model has not been very convincing (Cagan, 1955, Engsted, 1993). In the case of the former Yugoslavia Petrovic (1995) and Petrovic and Vujosevic (2000) found that the mechanisms behind the hyperinflation were primarily related to state-subsidized credit financing of excessive wage increases in socially owned firms and that money supply was accommodating wage inflation instead of the other way around. Thus, hyperinflationary mechanisms may or may not be different from the ones described by the Cagan model.

The question we ask in this paper is whether we can arrive at more conclusive results by properly accounting for the explosive root in the data and by exploiting the dynamics of the adjustment process for all the variables of the system. Therefore, instead of assuming from the outset that only money expansion is important for the accelerating inflation spiral we treat wage inflation and currency depreciation as equally important from an empirical point of view.

We illustrate the explosive root case with an VAR analysis of prices, wages, exchange rates, and money and how they interact through the dynamics of the short-run adjustment process in the high and hyperinflation period of the former Yugoslavia. Previous studies of the Yugoslav hyperinflation (Lahiri, 1991, Petrovic, 1995 and Petrovic and Vujosevic, 1996) have not considered the possibility of explosive roots in the data.

The paper is organized as follows. Section 2 presents the institutional background for the high and hyperinflation period of the former Yugoslavia and Section 3 provides an introduction to the econometrics of explosive root process. Section 4 discusses the cointegrated VAR model with an explosive root and Section 5 relates the cointegration results to dynamic adjustment behavior under hyperinflation episodes. Section 6 discusses the role of long-run price homogeneity. Section 7 introduces the data and the empirical model and discusses the choice of cointegration rank indices. Section 8 describes the empirical long-run results, Section 9 the medium-run results and Section 10 the short-run adjustment results. Section 11 concludes.

## 2. Institutional background

The former Yugoslavia introduced extensive market-based reforms as early as in the mid-60s but, nevertheless, retained a dominant social sector. State-owned enterprises were frequently replaced by firms with labor management, i.e. by firms

with an incentive to maximize wages instead of productivity and profits. Because the losses of these firms were not directly covered by the government they did not add to the public deficit but the state performed an informal pressure on the banking system to cover these losses by forwarding soft loans to labor-managed firms. These credits were given in domestic currency at the current official rate and were mainly associated with external borrowing and domestic savings held in foreign currency <sup>1</sup>.

The inflation rate was quite low during the 1970s but started to build up at the beginning of the 1980s. This resulted in large currency depreciation which largely annihilated the debt of the socially owned firms. In fact credits were essentially given at negative real interest rates. The foreign currency deposits were left in the Central Bank which had to bear the burden of the debts. Therefore, the expansion of base money in the eighties as a result of the high inflation rate was not directly related to the government deficit. Nevertheless, a major part of the fiscal deficit in this period can be related to the losses of the low-cost credits to socially owned enterprises. Thus, as Lahiri (1991) and Petrovic (1995) pointed out, even if no open fiscal deficit was recorded a quasi fiscal deficit emerged.

It is informative to see how the growth rates of money base, wages, prices, and nominal exchange rates have developed in this period. Table 2.1 reports a ranking (highest growth = 1, lowest = 4) of average monthly growth rates per annum in the eighties. It appears that the highest average nominal growth rates were associated with the black market exchange rates,  $s$ , in 1980-1984 and 1987-1988 and with wages,  $w$ , in 1985-1986 and 1989. In the whole decade (with the exception for 1984) price inflation,  $\overline{\Delta p}$ , has steadily been on the second place, whereas annual money growth,  $\overline{\Delta m}$ , has been on the third or the fourth place. This is a first indication that price inflation has adjusted either to accelerating depreciation rates in the black market or to excessive wage increases and that money growth has accommodated the accelerating price inflation.

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<sup>1</sup>Foreign currency deposits were introduced in early 1960s to make domestic banking system more attractive.

Table 2.1: Average monthly growth rates of prices, wages, exchange rates and money from 1980-1989.

Year	$\overline{\Delta p}$	ranking	$\overline{\Delta w}$	ranking	$\overline{\Delta s}$	ranking	$\overline{\Delta m}$	ranking
1980	0.0269	2	0.0261	3	0.0353	1	0.0163	4
1981	0.0276	2	0.0252	4	0.0296	1	0.0253	3
1982	0.0221	2	0.0174	4	0.0335	1	0.0179	3
1983	0.0386	2	0.0256	3	0.0582	1	0.0136	4
1984	0.0350	3	0.0341	4	0.0435	1	0.0386	2
1985	0.0490	2	0.0562	1	0.0325	4	0.0447	3
1986	0.0544	2	0.0678	1	0.0316	4	0.0520	3
1987	0.0813	2	0.0615	4	0.0834	1	0.0624	3
1988	0.1041	2	0.0979	3	0.1193	1	0.0925	4
1989	0.2764	2	0.2862	1	0.2614	3	0.2507	4

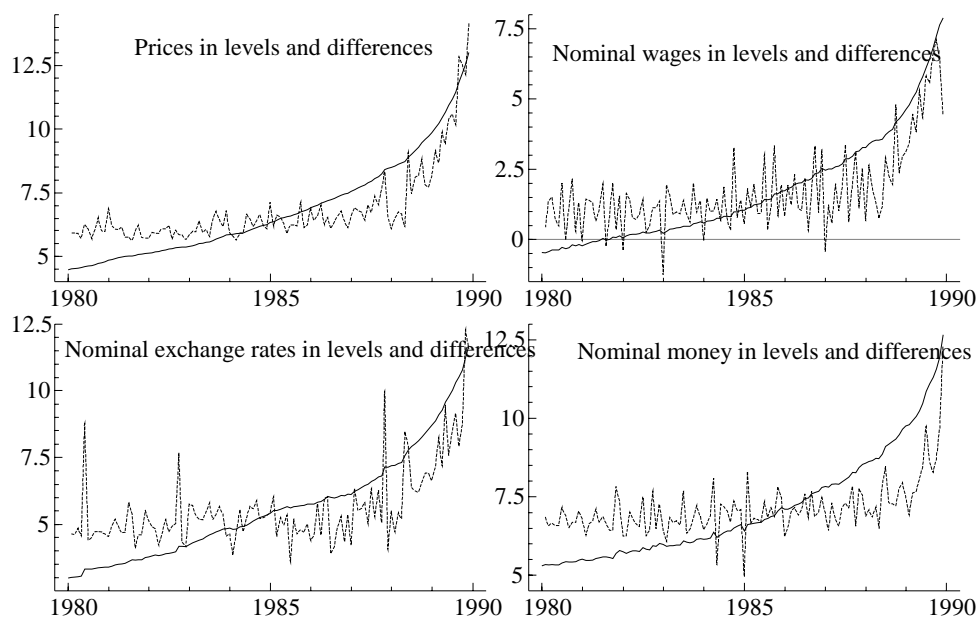


Figure 1. Mean and variance adjusted graphs of prices, wages, spot exchange rates and money stock in levels and differences.

Figure 1 illustrates graphically the increasingly high inflation rates in the first half of the 1980s and the strongly accelerating rates in the second half. In the last quarter of 1989 inflation reached the hyperinflation monthly rate of 50%. During this period there were several attempts by the government to put inflation under control, but they were mostly unsuccessful. The stabilization package at

the beginning of 1990, containing restrictive monetary and fiscal measures as well as wage control, appeared to be more effective as it stopped inflation within three months. A new convertible dinar was instituted and the exchange rate fixed. Throughout the year 1990 there was essentially no inflation, but in the following year inflation reappeared due to serious structural problems and political instability of the country.

The subsequent empirical analysis is based on the sample period 1980:1-1989:10 which is slightly shorter than the one in Petrovic and Vujosevic (2000). The latter included the stabilization year 1990 and the first seven months of 1991 when inflation reappeared as a prelude to the disintegration of the former Yugoslavia.

### 3. Hyperinflation and explosive roots

Before addressing the question of stationary cointegration relations between 'explosive' variables it is useful to discuss the notion of a unit root versus an explosive root trend. In both cases the concept of a shock to the variables of system and the permanence of its effect are essential ingredients. As an illustration we model inflation,  $\Delta p_t$  as a simple first order autoregressive process with parameter  $\lambda$  :

$$\begin{aligned}\Delta p_t &= \lambda \Delta p_{t-1} + \varepsilon_t, \quad t = 1, \dots, T, \\ &= \lambda^{t+1} \Delta p_0 + \lambda^t \varepsilon_1 + \lambda^{t-1} \varepsilon_2 + \dots + \lambda^1 \varepsilon_{t-1} + \varepsilon_t\end{aligned}\tag{3.1}$$

where the initial value  $\Delta p_0$  is assumed fixed. Although simple (3.1) can illustrate three important cases:

- $\lambda < 1$  implies stationary growth rates, i.e. shocks to inflation tend to die out gradually. In this case  $\varepsilon_t$  is a shock with a transitory effect over the period 1 -  $T$ .
- $\lambda = 1$  implies first order nonstationary growth rates, i.e. shocks to inflation have a long-lasting effect. In this case  $\varepsilon_t$  is a shock with a permanent effect over the period 1 -  $T$ .
- $\lambda > 1$  implies exponential growth rates, i.e. past shocks have an increasingly large effect on the present growth rate. In this case  $\varepsilon_t$  is a shock with an explosive effect over the period 1 -  $T$ .

In the explosive root case ( $\lambda > 1$ ) a past shock to the variable has not only a lasting but an ever-increasing effect. This makes intuitive sense in periods of

hyperinflation: when the inflation spiral has gathered momentum the cost of not anticipating future inflationary changes becomes increasingly high. Therefore, agents quickly learn that a shock (for example an expansion in money base or a wage increase) is likely to cause further acceleration of inflation rate and they adjust their expectations accordingly. When expectations have become self-fulfilling it is extremely hard to stop an accelerating inflation rate without very drastic measures like, for example, a price and a wage freeze. Therefore, the Yugoslav hyperinflation episode, like most other similar episodes, ended abruptly with the wage and price freeze in 1990.

When the growth rates exhibit explosive behavior, i.e. when  $\lambda > 1$ , differencing the data once or several times cannot remove the stochastic trend. This can be demonstrated by applying the difference operator  $(1-L)$  to (3.1):

$$\begin{aligned} (1-L)\Delta p_t &= (1-L)(1-\lambda L)^{-1}\varepsilon_{1t}, \quad t = \dots, 1, \dots, T \\ \Delta^2 p_t &= (1 + (\lambda - 1)L + \lambda(\lambda - 1)L^2 + \dots)\varepsilon_{1t} \end{aligned} \quad (3.2)$$

The graphs in Figure 1 illustrate that the levels and the differences of prices, wages, exchange rates, and money exhibit essentially the same exponential behavior. Since, exponential growth data cannot be made stationary by increasing the order of differencing, the VAR model for I(2) data is not the right econometric model in this case. Nevertheless, the VAR model with explosive roots is based on a similar logic as the I(2) model. Both models are defined by two reduced rank matrices, of which one (the 'usual'  $\Pi$  matrix) is the same for both models but the other is differently defined.

#### 4. The VAR model with explosive roots

The VAR model has previously been usually defined for the case where all roots of the characteristic polynomial are either outside or on the unit circle, hence excluding explosive roots. Nielsen (2001) showed that for univariate unit root testing the asymptotic test results hold even for the case of an explosive root. Nielsen (2002a and 2002b) extended the results to the multivariate case showing that the cointegrated VAR approach can be used to estimate long-run dynamic steady-state relations even when there are explosive roots in the data. Note, however, that hyperinflation episodes almost by definition are short and using the concept of 'long-run' relations might give the wrong connotation. From the perspective of an agent risking to lose his wealth unless acting immediately a month is already the 'long run' and the concept of a long-run relation will be interpreted with this in mind.



We will first demonstrate that explosive roots, similarly as double unit roots, can be annihilated by polynomial cointegration, and then formally define the explosive root VAR model. The baseline model is the VAR with a constant term,  $\mu$ , seasonal dummies,  $S_t$ , and intervention dummies,  $D_t$ , given by:

$$\begin{aligned} \Delta x_t &= \Gamma_1 \Delta x_{t-1} + \dots + \Gamma_k \Delta x_{t-k} + \Pi x_{t-2} + \mu_0 + \Phi_1 S_t + \Phi_2 D_t + \varepsilon_t, \\ \varepsilon_t &\sim N_p(0, \Sigma), \quad t = 1, \dots, T \end{aligned} \quad (4.1)$$

In (4.1) all parameters  $\{\Gamma_1, \dots, \Gamma_k, \Pi, \mu_0, \Phi_1, \Phi_2, \Sigma\}$  are unrestricted and ML estimates can be obtained by OLS equation by equation. The reduced rank hypothesis of  $\Pi$  is formulated as:

$$\Pi = \alpha \beta' \quad (4.2)$$

where  $\alpha, \beta$  are  $p \times r$  (Johansen, 1991). Thus, the cointegration rank,  $r$ , defines the number of polynomially stationary cointegrating relations independently of whether the data are I(1), I(2), or explosive. The intuition behind this result can be shown using the well-known method of concentrating the likelihood function (see for example Johansen and Juselius, 1990 and Johansen, 1995).

We define the OLS residuals  $R_{0t}$  and  $R_{1t}$  as:

$$\Delta x_t = \hat{B}'_0 Z_t + R_{0t} \quad (4.3)$$

and

$$x_{t-2} = \hat{B}'_1 Z_t + R_{1t} \quad (4.4)$$

where  $Z'_t = [\Delta x'_{t-1}, \dots, \Delta x'_{t-k}, S'_t, D'_t, \mu]$ ,  $\hat{B}'_0 = [\hat{B}'_{01}, \dots, \hat{B}'_{0k}, \hat{B}'_{0s}, \hat{B}'_{0d}, \hat{B}'_{0\mu}]$ , and  $\hat{B}'_1 = [\hat{B}'_{11}, \dots, \hat{B}'_{1k}, \hat{B}'_{1s}, \hat{B}'_{1d}, \hat{B}'_{1\mu}]$ . Section 4 demonstrated that  $\Delta x_t$  contains an explosive root but no unit root, whereas  $x_t$  contains both a unit and an explosive root. Thus,  $\Delta x_t$ ,  $x_{t-2}$ , and  $Z_t$  share a common explosive trend which is cancelled in the regression of  $\Delta x_t$  on  $x_{t-2}$  and  $Z_t$  so that  $R_{0t} \sim I(0)$  and  $R_{1t} \sim I(1)$  in (4.3) and (4.4). The concentrated cointegration model:

$$R_{0t} = \alpha \beta' R_{1t} + \varepsilon_t \quad (4.5)$$

corresponds to the standard cointegrated VAR model in I(0) differences and I(1) levels. It is useful to insert the expression for  $R_{1t}$  based on (4.4) into (4.5):

$$\begin{aligned} R_{0t} &= \alpha \beta' \{x_{t-2} - \hat{B}'_{11} \Delta x_{t-1} - \dots - \hat{B}'_{1k} \Delta x_{t-k}\} + \varepsilon_t \\ &= \alpha \{\beta' x_{t-2} - \beta' \hat{B}'_{11} \Delta x_{t-1} - \dots - \beta' \hat{B}'_{1k} \Delta x_{t-k}\} + \varepsilon_t \\ &= \alpha \{\beta' x_{t-2} - \hat{C}'_{11} \Delta x_{t-1} - \dots - \hat{C}'_{1k} \Delta x_{t-k}\} + \varepsilon_t \end{aligned} \quad (4.6)$$

where  $\hat{C}_{1j} = \beta' \hat{B}_{1j}$  and the effects of the dummy variables have been disregarded for simplicity. It is now easy to see that (4.6) defines a polynomially cointegrating relation between  $\beta' x_t$  and  $\Delta x_{t-k}$ .

By inserting (4.3) into (4.5) it is easy to see how the short-run adjustment dynamics are influenced by the explosive roots:

$$\Delta x_t = \hat{B}'_{01} \Delta x_{t-1} + \dots + \hat{B}'_{0k-1} \Delta x_{t-k+1} + \alpha \beta' R_{1t} + \varepsilon_t \quad (4.7)$$

Given that  $\varepsilon_t \sim I(0)$  and  $\beta' R_{1t} \sim I(0)$  equation (4.7) implies that there exist stationary linear combinations between current and lagged differences of the process even if the differences of the process are explosive by themselves<sup>2</sup>. To summarize: When data contain explosive roots  $\Delta x_t \sim I(\lambda)$ ,  $\beta' x_t \sim I(\lambda)$  but  $\beta' R_{1t} \sim I(0)$  and  $R_{0t} \sim I(0)$ . The VAR model contains two types of cointegration relations, one between levels and differences described by (4.6) and another between differences described by (4.3).

It is useful to formulate the VAR model with an explosive root as:

$$\begin{aligned} \Delta_\lambda \Delta_1 x_t &= \Pi_\lambda \Delta_1 x_{t-1} + \Pi_1 \Delta_\lambda x_{t-1} + \Gamma_{\lambda 1} \Delta_\lambda \Delta_1 x_{t-1} + \dots + \Gamma_{\lambda k-2} \Delta_\lambda \Delta_1 x_{t-1} + \\ &+ \mu_0 + \Phi_1 S_t + \Phi_2 D_t + \varepsilon_t \end{aligned} \quad (4.8)$$

where  $\Delta_\lambda = 1 - \lambda L$ ,  $\lambda > 1.0$ . The two reduced rank problems can now be defined by:

$$\Pi_1 = \alpha_1 \beta'_1, \quad (4.9)$$

where  $\alpha_1, \beta_1$  is of rank  $r$  defining the number of polynomially cointegrated relations in the system, and by:

$$\Pi_\lambda = \alpha_\lambda \beta'_\lambda, \quad (4.10)$$

where  $\alpha_\lambda, \beta_\lambda$  is of reduced rank  $p - s_\lambda$  defining the number of stationary relations between growth rates, and  $s_\lambda$  is the number of common 'explosive root' trends in the system.

The association between the unrestricted VAR(2) and (4.8) is as follows:

$$\begin{aligned} \Pi &= \lambda \alpha_1 \beta'_1 \\ \Gamma_1 &= \lambda I + \lambda \alpha_1 \beta'_1 + \alpha_\lambda \beta'_\lambda. \end{aligned}$$

Noting that  $1 - \lambda L = (1 - \lambda) + \lambda(1 - L)$  we can rewrite:

$$\alpha_1 \beta'_1 \Delta_\lambda x_{t-1} = (1 - \lambda) \alpha_1 \beta'_1 x_{t-1} + \lambda \alpha_1 \beta'_1 \Delta x_{t-1} = \tilde{\alpha}_1 \beta'_1 x_{t-1} + \omega' \Delta x_{t-1} \quad (4.11)$$

where  $\tilde{\alpha}_1 = (1 - \lambda) \alpha_1$  and  $\omega = \lambda \alpha_1$ . The explosive root VAR model subject to the reduced rank restrictions is given by:

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<sup>2</sup>By subtracting  $\lambda \Delta x_{t-1}$  from both sides of (4.7) we notice that this implies stationary cointegration relations between the growth rates.

$$\begin{aligned} \Delta_\lambda \Delta_1 x_t &= \alpha_\lambda \beta'_\lambda \Delta_1 x_{t-1} + (\tilde{\alpha}_1 \beta'_1 x_{t-1} + \omega' \Delta x_{t-1}) + \Gamma_{\lambda 1} \Delta_\lambda \Delta_1 x_{t-1} + \dots + \Gamma_{\lambda k-2} \Delta_\lambda \Delta_1 x_{t-k+2} \\ &\quad + \mu_0 + \Phi_1 S_t + \Phi_2 D_t + \varepsilon_t \end{aligned} \quad (4.12)$$

The solution of (4.12) is given in Nielsen (2002):

$$x_t = C_1 \sum_{i=1}^t \varepsilon_i + C_\lambda \lambda^t \sum_{i=1}^t \lambda^{-i} \varepsilon_i + Y_t + A_1 + \lambda^t A_\lambda \quad (4.13)$$

where  $A_1$  and  $A_\lambda$  are functions of the initial values and

$$C_1 = -\beta_{1\perp} (\alpha'_{1\perp} \dot{A}(1) \beta_{1\perp})^{-1} \alpha'_{1\perp}, \quad (4.14)$$

$$C_\lambda = -\lambda \beta_{\lambda\perp} (\alpha'_{\lambda\perp} \dot{A}(\frac{1}{\lambda}) \beta_{\lambda\perp})^{-1} \alpha'_{\lambda\perp}, \quad (4.15)$$

where  $\dot{A}(1)$  and  $\dot{A}(\frac{1}{\lambda})$  is the first derivative of the characteristic function  $A(z)$  of (4.12) evaluated at  $z = 1$  and  $z = \frac{1}{\lambda}$ , respectively.

## 5. Dynamic adjustment behavior and hyperinflation

The previous section demonstrated that there are three different levels of adjustment behavior in the VAR model with explosive roots:

1. Long-run adjustment described by  $r$  stationary polynomially cointegrated relations,  $\beta'_1 x_t + \omega \Delta x_t$ , combining a linear relation between levels of variables,  $\beta'_1 x_t \sim I(\lambda)$  with a linear relation of growth rates,  $\Delta x_t \sim I(\lambda)$ , or, equivalently, by  $r$  stationary relations  $\beta'_1 \Delta_\lambda x_t \sim I(1)$ , between first order non-stationary growth rates,  $\Delta_\lambda x_t \sim I(1)$ .
2. Medium long-run adjustment described by  $p-s_\lambda$  stationary relations,  $\beta'_\lambda \Delta x_{t-1}$ , between growth rates,  $\Delta x_t \sim I(\lambda)$ .
3. Short-run adjustment in stationary acceleration rates,  $\Delta \Delta_\lambda x_t \sim I(0)$ , reacting on deviations from the steady-state relations defined in 1 and 2.

Here we will primarily discuss the polynomially cointegrated relations described in terms of dynamic steady-state relations based on the assumptions that hyperinflation has been generated either by excess supply of money, excess nominal wage claims, or excess currency depreciation. The dynamics of the short-run adjustment behavior will be discussed at the end of this section.

When discussing the long-run steady-state relations it is useful to distinguish between the following three cases:

1.  $r = 0, p - s_\lambda \neq 0$ , i.e. no long-run dynamic adjustment relations between levels and differences. There are  $p - s_\lambda$  stationary relations between the growth rates.
2.  $r = 1 = s_\lambda$ , i.e. there is one linear combination between the levels of the variables which cancels the I(1) trend, but unless  $\omega = 0$ , not the 'explosive root' trend.
3.  $r > 1 > s_\lambda$ , i.e. there exists one linear combination between the levels of the variables which cancels both the I(1) and the 'explosive root' trend and another which cancels the I(1) trend but not, in general, the 'explosive root' trend.

### 5.1. Hyperinflation and excess supply of money

The Cagan model for money demand in hyperinflationary episodes contains the expected inflation rate as a crucial determinant. As explained in Section 2 foreign currency deposits became a very dominant component of liquid assets in this period. Therefore, not only expected inflation rate, but also movements in real exchange rates are likely to have been crucial for the demand for real balances. A modified version of the Cagan money demand model becomes:

$$(m - p - y)_t + a_1(s - p)_t + a_2\Delta p_{t+1}^e = u_{1t} \quad (5.1)$$

where  $m_t$  is money stock at time  $t$ ,  $y_t$  is a measure of real income,  $\Delta p_{t+1}^e = p_{t+1}^e - p_t$  where  $p_{t+1}^e$  denotes the expectation of  $p_{t+1}$  at time  $t$ ,  $u_{1t}$  is a stochastic disturbance term, and  $a_1 < 0$ ,  $a_2 < 0$ , is consistent with money demand.

The case  $u_{1t} \sim I(0)$  is consistent with money supply having accommodated money demand. If  $a_1 < 0$ ,  $a_2 < 0$ , then the negative effects from inflationary expectations and currency depreciation implies that money stock has increased less than prices. Generally,  $u_{1t} \sim I(0)$  is econometrically feasible when  $r > s_\lambda$ .

The case  $u_{1t} \sim I(\lambda)$  with  $\lambda > 1$ , implies that money supply has deviated from the money demand steady-state path by an 'explosive root' trend. This could, for example, be the result of the central bank having expanded money supply to provide the government with seignorage revenue. In this case we would not expect the rate of money supply to exceed the 'optimal' rate that maximizes the inflation tax revenue. Another example could be that the central bank had 'excessively' expanded money base in anticipation of future currency devaluation to meet the demand for subsidized credit by socially owned firms.

Note that the case  $u_{1t} \sim I(1)$  is generally not consistent with the explosive root model. This is because in this case real money balances would have deviated from

the money demand steady-state path by an I(1) trend, which would be consistent with the I(2) model<sup>3</sup>, but not with the explosive root model.

## 5.2. Hyperinflation and excess nominal wage claims

Long-run steady-state behavior in the labor market presumes real wages,  $w_t - p_t$ , to follow productivity trend,  $c_t$ . In hyperinflation periods we expect inflationary expectations to have strongly influenced nominal wage claims:

$$w_t - p_t - c_t = a_3 \Delta p_{t+1}^e + u_{2t} \quad (5.2)$$

where  $u_{2t}$  is a stochastic disturbance term.

The case  $u_{2t} \sim I(0)$  implies that wages have followed the long-run path given by (??). If  $a_3 = 0$ , then nominal wages have followed prices one to one. If  $a_3 > 0$ , then nominal wages have grown more than prices corrected for productivity and it seems likely that excess wage claims based on inflationary expectations have been part of the hyperinflationary behavior. On the other hand, if  $a_3 < 0$ , then nominal wages have not been able to catch up with the accelerating prices. This could be the case if nominal wage claims have been based on actual instead of expected inflation and prices change more frequently than wages. Note that the case  $u_{2t} \sim I(0)$  is generally consistent with  $r > s_\lambda$ .

The case  $u_{2t} \sim I(\lambda)$  with  $\lambda > 1$  implies that nominal wages deviate from the 'long-run' path (??) by an explosive stochastic trend and this deviation is related to the accelerating inflation rate, i.e.  $u_{2t} = f(\Delta p_t)$ .

## 5.3. Hyperinflation and exchange rate expectations

The fundamental real exchange rate,  $s_t - p_t + p_t^*$  is generally assumed to describe long-run steady-state behavior in the external market. In hyperinflation periods we expect inflationary expectations to have strongly influenced the nominal exchange rate:

$$s_t - p_t + p_t^* = a_4 \Delta p_{t+1}^e + u_{3t} \quad (5.3)$$

where  $p^*$  denotes a foreign price variable,  $u_{3t}$  is a stochastic disturbance term.

The case  $u_{3t} \sim I(0)$  implies that nominal exchange rates have followed the long-run path given by (5.3). If  $a_4 = 0$  then nominal exchange rates and relative prices have grown equally fast. If  $a_4 > 0$  then nominal exchange rates have

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<sup>3</sup>Most of the more recent empirical models of hyperinflation have used the I(2) model to test the Cagan model. This was also the assumption made by Sargent (1977) for the case when money supply has exceeded agents's desired quantities as a consequence of maximizing seignorage.

grown faster than relative prices, resulting in a steady depreciation of the domestic currency. If  $a_4 < 0$  then the domestic currency has steadily appreciated. The latter case would be consistent with the central bank having controlled exchange rates as a means to control inflation. Note that the case  $u_{3t} \sim I(0)$  is generally associated with  $r > s_\lambda$ .

The case  $u_{3t} \sim I(\lambda)$  with  $\lambda > 1$ , implies that the real exchange rate deviates from the long-run path (5.3) by an explosive stochastic trend and this deviation is related to the accelerating inflation rate, i.e.  $u_{3t} = f(\Delta p_t)$ .

#### 5.4. Hyperinflation and the dynamics of the system

Hypothetically we expect accelerating price inflation to be associated with excess nominal wages,  $(w - p - c)$ , excess expansion of money stock,  $(m - p - y)$ , excess depreciation,  $(s - p - p^*)$ , and inflationary expectations. But we also know that inflationary expectations based on the observed price behavior are likely to have important effects on the money growth rate, the wage growth rate, and the depreciation rate. Thus, dynamic feed-back and interaction effects are likely to be important in the system. This can be described by the following vector error-correction model based on the assumption of one 'explosive root' trend and one 'unit root' trend:

$$\begin{aligned} \begin{bmatrix} \Delta\Delta_\lambda m_t \\ \Delta\Delta_\lambda w_t \\ \Delta\Delta_\lambda s_t \\ \Delta\Delta_\lambda p_t \end{bmatrix} &= \begin{bmatrix} -\gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & -\gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & -\gamma_{33} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} \end{bmatrix} \begin{bmatrix} (\Delta m - b_1 \Delta p)_{t-1} \\ (\Delta w - b_2 \Delta p)_{t-1} \\ (\Delta s - b_3 \Delta p)_{t-1} \end{bmatrix} + \\ &+ \begin{bmatrix} -\alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & -\alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & -\alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \end{bmatrix} \begin{bmatrix} (m - m^*)_{t-1} \\ (w - w^*)_{t-1} \\ (s - s^*)_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{m,t} \\ \varepsilon_{w,t} \\ \varepsilon_{s,t} \\ \varepsilon_{p,t} \end{bmatrix} \end{aligned} \quad (5.4)$$

where  $(m - m^*)$ ,  $(w - w^*)$  and  $(s - s^*)$  correspond to the polynomially cointegrated relations defined by (4.11). A minus sign indicate error-correction behavior. The coefficients  $b_i$  indicate whether money stock, wages and exchange rates have grown less, more, or similarly as prices.

The empirical question is how price inflation have adjusted to these imbalances and how inflationary expectations have fed into the system dynamics. If the underlying cause of hyperinflation is related to the expansion of money supply then we would expect  $\gamma_{21} > 0$ ,  $\gamma_{31} > 0$ ,  $\gamma_{41} > 0$  and  $\alpha_{21} > 0$ ,  $\alpha_{31} > 0$ ,  $\alpha_{41} > 0$ , or to excess nominal wages, then  $\gamma_{12} > 0$ ,  $\gamma_{32} > 0$ ,  $\gamma_{42} > 0$  and  $\alpha_{12} > 0$ ,  $\alpha_{32} > 0$ ,  $\alpha_{42} > 0$ , or, finally, to excess currency depreciation, then  $\gamma_{13} > 0$ ,  $\gamma_{23} > 0$ ,  $\gamma_{43} > 0$  and

$\alpha_{13} > 0, \alpha_{23} > 0, \alpha_{43} > 0$ . The last equation, the price adjustment equation, is of particular interest as it describes how price inflation has dynamically adjusted to departures from steady-state and to the lagged growth rates of the other nominal variables in the system. It is, therefore, reproduced below:

$$\begin{aligned} \Delta\Delta_{\lambda}p_t = & \gamma_{41}(\Delta m - b_1\Delta p)_{t-1} + \gamma_{42}(\Delta w - b_2\Delta p)_{t-1} + \gamma_{43}(\Delta s - b_3\Delta p)_{t-1} + \\ & + \alpha_{41}(m - p - y)_{t-1} + \alpha_{42}(w - p - c)_{t-1} + a_{43}(s - p - p^*)_{t-1} + a_5\Delta p_{t+1}^e + \varepsilon_t \end{aligned} \quad (5.5)$$

where  $a_5 = \alpha_{41}a_2 + \alpha_{42}a_3 + \alpha_{43}a_4$ . If  $\gamma_{4i} > 0, a_{4i} > 0, i = 1, 2, 3$ , consistent with equilibrium-correcting behavior in prices and the first three equations of (5.4) are similarly equilibrium-correcting, then  $a_5 > 0$  would generally be consistent with accelerating price inflation. The inflationary spiral can be described by wages, money and exchange rates adjusting to accelerating prices, then by higher wages, money and exchange rates resulting in increasing cost-push inflation and in higher inflationary expectations. But the inflationary spiral has to start at some point in time, before which  $a_5 = 0$ . Therefore, it seems likely that inflationary expectations have become significant in (5.5) after some initial disturbance to the system such as excess expansion of money supply, extraordinary increases in nominal wages, or excess depreciation of the foreign currency. Some information of this can be found in (5.5). For example, if the expansion of money supply was the underlying cause of hyperinflation we would expect lagged values of money growth to be significant in (5.5). Thus the relative weights of the estimated coefficients  $\gamma_{4i}$  and  $a_{4i} i = 1, 2, 3$  are indicative of how price inflation has dynamically adjusted to the lagged growth rates of the other nominal variables in the system and, thus, of the importance of each of the potential inflationary sources.

## 6. Long-run price homogeneity

Long-run price homogeneity is a crucial property of an economic system. This is even more so in a situation when inflation is threatening to run out of control. We will first briefly discuss the conditions under which the VAR model with explosive roots is consistent with price homogeneity and then focus on the more plausible case of no price homogeneity.

For simplicity we will assume that  $s_{\lambda} = 1$  and there are no double unit roots in the VAR. There are two conditions for long-run price homogeneity:

$$R\beta_{\lambda} = 0 \quad (6.1)$$

and

$$R\beta_1 = 0 \quad (6.2)$$

where  $R = [1, 1, 1, 1]$ . The first condition implies that  $m - p \sim I(1)$ ,  $w - p \sim I(1)$ ,  $s - p \sim I(1)$ , i.e. each of the nominal variables have the same explosive trend but not necessarily the same  $I(1)$  trend. The second condition implies that there exists  $r$  homogeneous relations between the levels of the variables  $\beta_1' x_t$  which cancel the  $I(1)$  trend but not necessarily the explosive root trend. If one of the conditions (6.1) - (6.2) holds the other would not be expected to hold. Therefore,  $\beta_1' x_t$  need not be homogeneous in prices and we need to understand the reason why.

We consider the case  $\beta_1' x_t = \tilde{\beta}_1' x_t + bp_t$ , where  $\tilde{\beta}_1' x_t$  is homogeneous and  $bp_t$  is a measure of the deviation from long-run price homogeneity. By assuming the following time series model for prices:

$$\begin{aligned} \Delta \Delta_\lambda p_t &= v_t \\ p_t &= \lambda p_{t-1} + \Sigma_i v_i + \text{deterministic comp.} \\ p_t &= -1/(1 - \lambda) \Delta p_{t+1} + \Sigma_i v_i + \text{deterministic comp.} \end{aligned} \quad (6.3)$$

where  $v_t \sim I(0)$ , it will be possible to interpret the deviation from long-run price homogeneity (as we find in the empirical analysis of Section 8) in terms of inflationary expectations.

## 7. Misspecification tests and choice of rank<sup>4</sup>

During periods of hyperinflation real growth is of an altogether different order of magnitude compared to nominal growth. Therefore, as already argued in Cagan (1956) and later in Taylor (1991) and Petrovic (1995), the empirical analysis can exclusively focus on the interactions of the crucial nominal determinants and their effect on the inflation spiral. This is the motivation for not including productivity and foreign prices in the empirical study. Similarly as Petrovic and Vuojesevic (2000) we focus on the determination of nominal wages, base money, exchange rates, and consumer prices.

The VAR model is estimated with two lags, an unrestricted constant, seasonal dummies, and three dummy variables to be defined below. The variable vector  $x_t$  is defined by:

$$x_t' = [w - p, s - p, m - p, p]_t, \quad t = 1980:1-1989:10$$

where  $w$  is the log of nominal wages,  $p$  is the log of the consumer price index,  $s$  is the log of black market exchange rate and  $m$  is the log of base money. The price

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<sup>4</sup>The empirical results of this section and the next have been estimated using Cats for Rats (Hansen and Juselius, 1994)



and wage data are collected from various issues of the Yugoslav Federal Statistical Office publication. The black market exchange rate, defined as domestic currency (dinar) per US dollar, has been collected by one of the authors (Mladenovic). Though not officially recognized it was widely used and is considered to be a much more accurate measure of the market price of the Yugoslav dinar than the official exchange rate. The base money data are collected from several issues of National Bank of Yugoslavia Bulletin.

Two transitory dummy variables were needed to account for an expansion of money stock in 1984:4 followed by a contraction and a contraction in 1985:1 followed by an expansion. The variable,  $Dval$ , captures the effects of several large currency devaluations. The dummy variables are defined by:

$$\begin{aligned}
 Dtr84_04 &= 1, t = 1984:04, \\
 &= -1, t = 1984:05, \\
 &= 0, \text{ otherwise,} \\
 Dtr85_01 &= 1, t = 1985:01, \\
 &= -1, t = 1985:02, \\
 &= 0, \text{ otherwise,} \\
 Dval &= 0.27, t = 1980:06, \\
 &= 0.20, t = 1982:10, \\
 &= 0.34, t = 1987:11 \\
 &= 0.25, t = 1988:05^5 \\
 &= 0.22, t = 1988:06 \\
 &= 0, \text{ otherwise.}
 \end{aligned}$$

The non-zero values of  $Dval$  are approximately equal to the actual devaluations. All three dummy variables are unrestricted in the VAR model.

Table 7.1 reports multivariate tests for residual normality and first and fourth order autocorrelations as well as univariate tests for normality and ARCH. The baseline VAR model performs remarkably well considering the wild fluctuations in the data. There are moderate signs of non-normality in real money stock and real exchange rates, mostly due to excess kurtosis (see also Figure 6 in Section 10). Even more surprisingly there are hardly any evidence of ARCH effects.

The VAR model with an explosive root is defined by two reduced rank conditions. The determination of the first condition (4.9) can be based on the 'usual' trace statistics derived for  $I(1)$  variables even when there is an explosive root in the data (Nielsen, 2001). The second condition determines the number of stationary relations between the growth rates and is related to the number of explosive

Table 7.1: Misspecification tests and the roots of the system

Multivariate tests:					
Residual autoc. LM <sub>1</sub>	$\chi^2(16)$	=	41.25	p-val.	0.00
Residual autoc. LM <sub>4</sub>	$\chi^2(16)$	=	20.71	p-val.	0.19
Test for normality	$\chi^2(8)$	=	30.14	p-val	0.00
Univariate tests:					
	$\Delta(m-p)$	$\Delta(w-p)$	$\Delta(s-p)$	$\Delta p$	
ARCH(8)	0.69	0.75	<b>6.38</b>	0.81	
Jarq.Bera(2)	<b>7.44</b>	0.78	<b>10.54</b>	0.43	
Eigenvalues of the $\Pi$ -matrix					
	0.34	0.18	0.03	0.00	
Trace test					
	73.09	25.75	3.01	0.10	
5 largest roots of the process:					
Unrestricted model:	1.05	1.01	0.94	0.68	0.36
$r = 2$	1.06	1.00	1.00	0.68	0.38

roots in the model. The latter can be found by estimating the roots of the characteristic polynomial for a given choice of  $r$ . From the outset it seems unlikely to find more than one explosive root, which is also what we find.

Even if the asymptotic tables for the trace test are valid for the explosive root VAR some caution is motivated when the sample period is short and the model contains dummy variables. Therefore, we have also used the roots of the characteristic polynomial and the graphs of  $\beta'_1 x$  compared to  $\beta'_1 R_{1t}$  when choosing  $r$ .

The trace test reported in Table 7.1 suggests  $r = 2$  which is consistent with  $p - r = 2$  common unit root trends in the data. Figure 2 shows the eight roots of the characteristic polynomial of the unrestricted VAR. Table 7.1 reports the five largest roots for  $r$  unrestricted and for  $r = 2$ . There is one fairly large explosive root (1.05), another very close to the unit circle (1.01) and a near unit root (0.94). For  $r = 2$  the largest unrestricted root is 0.68 and the explosive root has become 1.06. Note that the smallest explosive root disappeared when restricting the cointegration rank. This is an indication that it was not significantly different from a unit root.

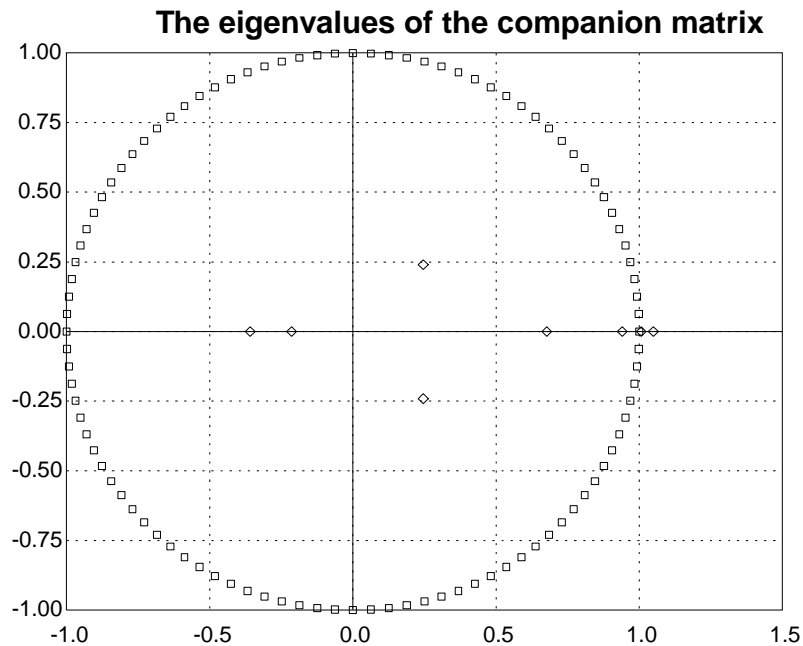


Figure 2. The roots of the unrestricted VAR model

Appendix A shows the graphs of the four eigenvectors defined by  $\beta'_{1,i}x_t$  and  $\beta'_{1,i}R_{1t}$ ,  $i = 1, \dots, 4$ . It appears that the first two relations,  $\beta'_{1,1}R_{1t}$  and  $\beta'_{1,2}R_{1t}$ , exhibit stationary mean-reverting behavior, whereas the third and fourth relation seem to be drifting off. Thus, both the trace test, the characteristic roots and the graphs indicate that the cointegration rank  $r = 2$  and the subsequent analyses are based on this choice. Based on estimated characteristic roots there seem to be just one common explosive trend and the subsequent analyses are based on  $s_\lambda = 1$  with  $\lambda = 1.06$ .

## 8. Empirical long-run results

Table 8.1 reports the two just identified cointegrating vectors together with their corresponding adjustment coefficients<sup>9</sup> as well as the combined effects given by  $\hat{\Pi} = \hat{\alpha}_1 \hat{\beta}'_1$ . The condition for long-run price homogeneity of  $\beta'_1 x_t$  is that  $p_t$  is long-run excludable from the cointegration relations. This was strongly rejected based on a  $\chi^2(2) = 30.8$  test statistic with a p-value of 0.00, consistent with the highly significant coefficients of  $p_t$  in both relations.

Long-run price homogeneity is a crucial property of an economic steady-state relation and the general lack of it in the present period might give some indication

<sup>9</sup>Note that  $\beta'_1 x_t$  does not define a stationary relation without appropriately corrected for the changes of the process. Hence, the significance of the  $\alpha_1$  coefficients are only indicative.

of where to look for the causes to the explosive behavior. It seems plausible that forward looking inflationary expectations and the way they have affected nominal wage setting and price contracts are at the core of hyperinflation. Assuming that  $\Delta p_{t+1}$  is a proxy for  $\Delta p_{t+1}^e$  we can reformulate the first relation  $\hat{\beta}'_1 x_t$  using (6.3):

$$m_t - p_t + 0.31(s - p)_t = -2.7\Delta p_{t+1}^e - 0.16\Sigma v_i + u_{1t} \quad (8.1)$$

where  $\Sigma v_i$  captures the effect of other I(1) left-out effects on money demand such as interest rate effects. It can now be interpreted as a Cagan-type money demand relation in which the demand for real money holdings is negatively related to the expected inflation rate and to the purchasing power of the Yugoslav dinar and with . The coefficient of expected inflation,  $-2.7$ , i.e. Cagan's  $\alpha$ , defines the 'optimal' inflation rate,  $1/\alpha$ , at which the government maximizes seignorage revenue. Using the estimate  $-2.7$  yields an average inflation rate of approximately 0.37 which is the actual values of inflation rate for the last two months of the sample.

The interpretation of the second relation is less straightforward. Assuming again that  $\Delta p_{t+1}$  is a proxy for  $\Delta p_{t+1}^e$  and using (6.3) we can rewrite it as:

$$w_t - p_t = -0.38(s_t - p_t) - 1.5\Delta p_{t+1}^e - 0.09\Sigma v_i + u_{2,t} \quad (8.2)$$

where  $\Sigma v_i$  captures the effect of other I(1) left-out effects on real wages such as unemployment effects. In this form (8.2) essentially describes real wages being negatively related to real exchange rates and expected inflation, implying that nominal wages have grown less than prices and nominal exchange rates. This could be the result of nominal wage increases being based on actual inflation rates in a situation when inflation is accelerating. Table 2.1 shows that until 1984 this seemed to be case. However, Section 5 showed that whether nominal wages have grown less or more than prices depends also on the time-series path of  $u_{2,t}$ . Figure 3 and Figure 4 illustrate this based on the graphs of residuals to the long-run relations with  $\beta'_1 x_t$  in the upper panel and  $\beta'_1 R_{1t}$  in the lower panel. They show that the money demand relation is essentially stationary by itself without the help of the differenced process, whereas the wage relation behaves in an explosive manner. This is an interesting observation as it shows that the deviations from the wage relation,  $u_{2,t}$ , have developed similarly as inflation rate, whereas the deviations from the money demand relation have not. According to (4.11) the explosive residual of the wage relation has to be combined with the inflation rate, or any of the other nominal growth rates to become stationary.

Table 8.1: The estimated cointegration relations for  $r=2$ 

Just-identified long-run relations				
	$m - p$	$w - p$	$s - p$	$p$
$\hat{\beta}'_{1,1}$	1.00	0.00	<b>0.31</b> (4.4)	<b>0.16</b> (12.0)
$\hat{\beta}'_{1,2}$	0.00	1.00	<b>0.38</b> (7.8)	<b>0.09</b> (9.8)
	$\Delta(m - p)$	$\Delta(w - p)$	$\Delta(s - p)$	$\Delta p$
$\hat{\alpha}'_{1,1}$	<b>-0.38</b> (4.9)	-0.12 (-1.7)	0.01 (0.1)	0.06 (1.6)
$\hat{\alpha}'_{1,2}$	-0.09 (-1.2)	-0.11 (-1.6)	<b>-0.20</b> (3.2)	<b>0.25</b> (6.6)
The combined estimates: The $\Pi$ matrix				
	$m - p$	$w - p$	$s - p$	$p$
$\Delta(m - p)$	<b>-0.38</b> (-4.8)	-0.09 (-1.1)	<b>-0.16</b> (-4.5)	<b>-0.07</b> (-5.3)
$\Delta(w - p)$	-0.12 (-1.7)	-0.11 (-1.6)	<b>-0.08</b> (-2.7)	<b>-0.03</b> (-2.6)
$\Delta(s - p)$	-0.01 (-0.1)	<b>-0.20</b> (-3.1)	<b>-0.07</b> (-2.7)	-0.02 (-1.7)
$\Delta p$	0.06 (1.6)	<b>0.25</b> (6.6)	<b>0.11</b> (6.9)	<b>0.03</b> (5.2)

Table 8.2: The polynomially cointegrated wage relation

	Eq. (1)	Eq. (2)	Eq. (3)
	$\hat{\beta}_{1,2}x_{t-1}$	$\hat{\beta}_{1,2}x_{t-1}$	$\hat{\beta}_{1,2}x_{t-1}$
$\Delta w_t$	- 0.29 (-2.7)	- 0.28 (-2.6)	-
$\Delta s_t$	-0.11 (-1.1)	-	-
$\Delta m_t$	0.23 (2.5)	0.21 (2.4)	-
$\Delta p_t$	1.94 (12.0)	1.86 (13.4)	1.71 (20.2)

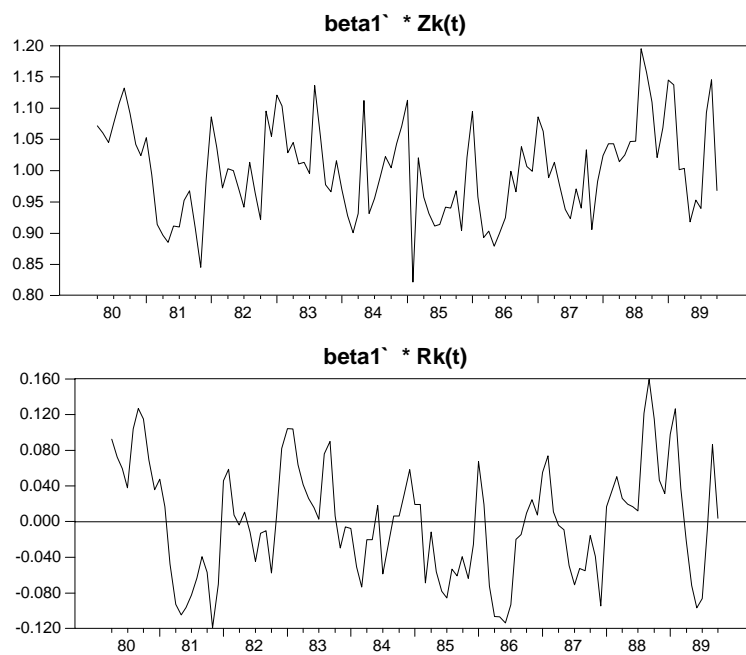


Figure 3. The graphs of the estimated money demand relation given as  $\beta'_{1,1}x_t$  (upper panel) and  $\beta'_{1,1}R_{1t}$  (lower panel).

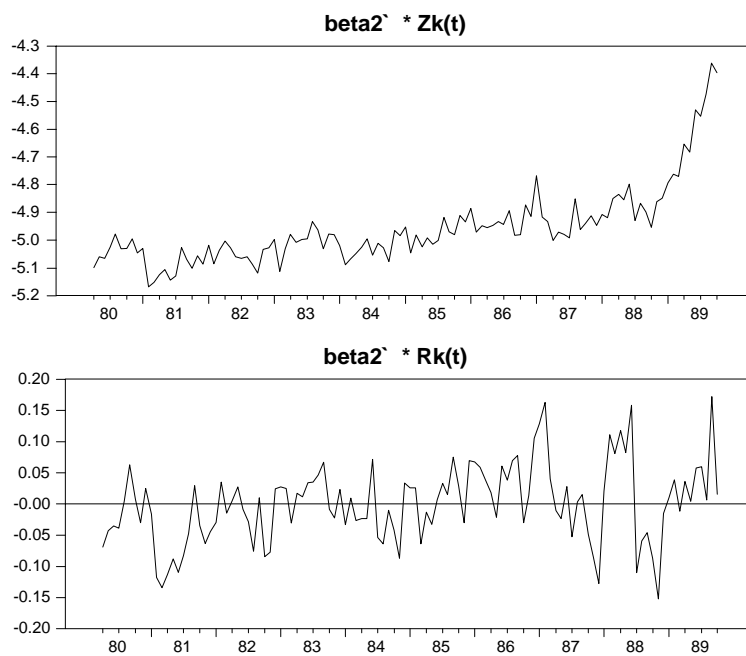


Figure 4. The graphs of the estimated wage relation given as  $\beta'_{1,2}x_t$  (upper panel) and  $\beta'_{1,2}R_{1t}$  (lower panel).

When data contain an explosive root the regression method is likely to produce very precise estimates. Table 8.2 reports the regression of  $\hat{\beta}_{1,2}x_{t-1}$  on current values of all four nominal growth rates. Equation (1) in the table shows that the depreciation rate is not significantly related to the residual  $u_{2,t}$ . Equation (2), which does not include the depreciation rate, shows that  $u_{2,t}$  is strongly related to price inflation and to some extent to the spread between money and wage growth. The latter might be an indication that part of the nominal wage increases was financed by money printing. Equation (3) reports the estimated relationship between excess nominal wages and inflation rate alone.

Based on the above results we have identified two long-run relations:

$$\begin{aligned} (m - m^*)_t &= m_t - p_t + 0.31(s_t - p_t) + 2.7\Delta p_{t+1}^e + \Sigma v_i + u_{1,t} \\ (w - w^*)_t &= w_t - p_t + 0.38(s_t - p_t) + 1.5\Delta p_{t+1} - 1.7\Delta p_{t+1} + \Sigma v_i + e_t. \end{aligned} \tag{8.3}$$

Thus, combining  $\beta'_{1,2}x_t$  with  $\Delta p_{t+1}$  shows that real wages might after all have been able to catch up with the domestic inflation rate, but not in terms of purchasing power relative to a foreign currency.

The  $\hat{\alpha}_{1,1}$  coefficients reported in Table 8.1 provide information about how the system has dynamically adjusted to deviations from the money demand relation. It is noteworthy that no other variables than real money stock have adjusted to deviations from this relation. This is interesting since it implies that the cumulated shocks to the real money equation,  $\hat{\epsilon}_{m-p,t}$ , did not have any long-run impact on the other variables of the system, inclusive price inflation.

The  $\hat{\alpha}_{1,2}$  coefficients show that real wages have only been weakly equilibrium correcting to  $\beta'_{1,2}x_t$ , the wage relation, whereas real depreciation rate (contrary to prices) exhibits significant error-correcting adjustment<sup>10</sup>. This can be seen more clearly from the combined effects measured by the  $\Pi$  matrix, where the diagonal elements are significantly negative for real money and real exchange rate, weakly significant for real wages, but significantly positive for prices (an indication of exponential growth). The last row of the  $\Pi$  matrix, describing the combined effects of the two long-run relations on price inflation, shows that it has been error-correcting towards real wages and real exchange rates but exponentially increasing with price expectations. Thus, inflationary expectations in the goods market seem to have played an important role for the Yugoslav hyperinflation.

Therefore, based on the empirical investigation of the first reduced rank component,  $\alpha_1\beta'_1\Delta_\lambda x_{t-1}$ , we conclude that the empirical evidence so far is more in favor of excess nominal wages than of excess money expansion having been the cause of the Yugoslav hyperinflation.

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<sup>10</sup>The latter result is not robust to small changes in the sample period. If the sample begins a few months earlier or a few month later, the exchange rates becomes weakly exogenous.

Table 9.1: The medium-run cointegrated nominal growth rates

	$\Delta_\lambda \Delta w_t$		$\Delta_\lambda \Delta s_t$		$\Delta_\lambda \Delta m_t$	
	(a)	(b)	(a)	(b)	(a)	(b)
$\Delta p_{t-1}$	1.31 (9.2)	1.43 (11.9)	0.77 (8.0)	0.70 (8.9)	0.67 (4.8)	0.84 (9.0)
$\Delta w_{t-1}$	-1.35 (-14.7)	-1.34 (-15.0)	-0.08 (-1.2)	-	0.07 (0.8)	-
$\Delta s_{t-1}$	0.12 (1.3)	-	-0.85 (-10.2)	-0.86 (-10.7)	0.09 (1.6)	-
$\Delta m_{t-1}$	0.06 (0.8)	-	0.01 (0.2)	-	-1.08 (13.4)	-1.05 (13.7)

## 9. Empirical medium-run results <sup>11</sup>

The second reduced rank problem (4.10) determines  $p - s_\lambda = 3$  stationary cointegration relations between the nonstationary growth rates. We will estimate these based on the following modification of the auxiliary regression in (4.3):

$$\Delta x_t - \lambda I \Delta x_{t-1} = -\lambda I \Delta x_{t-1} + B'_{01} \Delta x_{t-1} + \Phi D_t + \varepsilon_t \quad (9.1)$$

One common explosive root (1.06) gives three stationary relations between the growth rates, implying that they are pairwise cointegrating. Table 9.1, column (a), reports the unrestricted estimates of (9.1) and column (b) the estimates when insignificant coefficients have been set to zero.

The test of homogeneity between the nominal growth rates was rejected based on  $\chi^2(1) = 8.52$  with a p-value of 0.00.

The estimates in columns (b) define three medium-run cointegration relations between  $\Delta p$  and  $\Delta w$ ,  $\Delta s$ ,  $\Delta m$ . Normalizing on  $\Delta w$ ,  $\Delta s$ ,  $\Delta m$  yields the following medium-run relations:

$$\begin{aligned} (\Delta w - \Delta w^*)_t &= \Delta w_t - 1.07 \Delta p_t \\ (\Delta s - \Delta s^*)_t &= \Delta s_t - 0.81 \Delta p_t \\ (\Delta m - \Delta m^*)_t &= \Delta m_t - 0.80 \Delta p_t \end{aligned} \quad (9.2)$$

An interesting result is that wage inflation is related to price inflation with a coefficient close to the explosive root  $\lambda$ . Except for the years 1985, 1986 and 1989 this does not seem to be consistent with the ranking of the average growth rates of Table 2.1. But, considering that a shock in the past has an ever-increasing impact on the future in the explosive root model, the large wage increases in 1985 and

<sup>11</sup>The results of this section and the next have been produced in GiveWin (Doornik and Hendry, 1998).



1986 might indeed have given the impetus to the subsequent hyperinflationary growth rates<sup>12</sup>.

The graphs of the three relations (9.2) are shown in Figure 5. All of them exhibit stationary behavior.

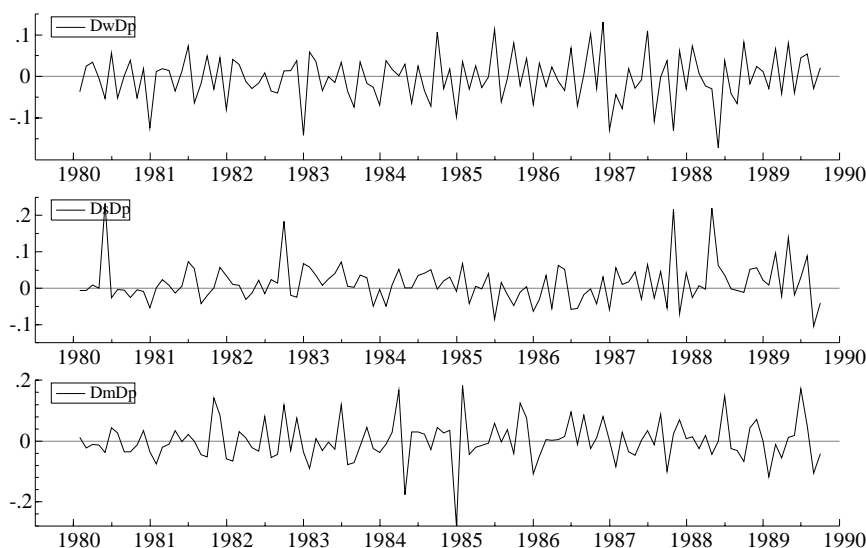


Figure 5. Graphs of the medium-run relations between nominal growth rates.

## 10. The short-run dynamic adjustment model<sup>13</sup>

The cointegration relations in (8.3) and (9.2) describe relations between variables, but do not say anything about the dynamics of the short-run adjustment effects, i.e. how the system variables have reacted to the equilibrium errors defined by the estimated relations.

A unrestricted system of error correction mechanisms similar to (5.4) was first estimated. All of the five cointegration relations given by (8.3) and (9.2) were strongly significant. The graphs of the fitted and actual values, the standardized residuals, autocorrelograms, and the residual histograms are given in Figure 6 for this system. Considering that the monthly growth rates of the variables have varied from 3% to 70 % a month the estimated system performs remarkably well: the residuals are very close to normality with a standard error of around 4% for

<sup>12</sup>This is also consistent with the result (not reported here) that the estimated explosive root was smaller based on the first half of the eighties.

<sup>13</sup>The empirical relationships in this section have been estimated using PcFiml (Doornick and Hendry (1998)).

prices to 5% for wages, and they do not show much evidence of heteroscedasticity or left-out autocorrelations.

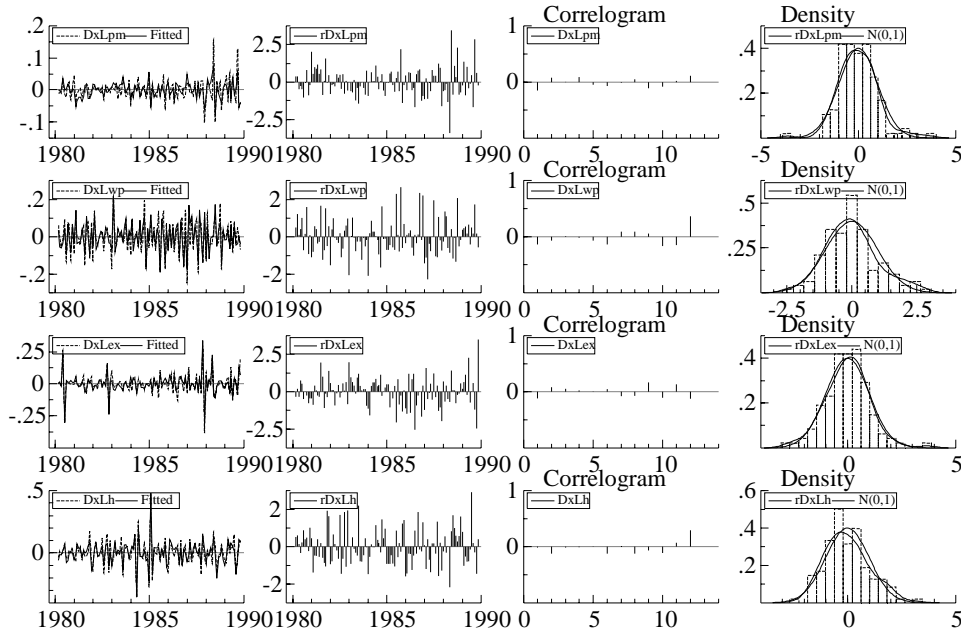


Figure 6. The graphs of actual and fitted values, the residuals, the autocorrelograms, and the residual histograms of the system.

The empirical model was simplified by restricting 20 insignificant coefficients to zero. They were tested with a Likelihood Ratio test approximately distributed as  $\chi^2(20)$  and accepted based on a statistic of 29.7 (p-value 0.08). The estimated results are reported in Table 10.1.

No current effects have been included in the model and the standardized residual covariance matrix,  $\tilde{\Sigma}$ , is, therefore, reported below. The standard errors are reported in the diagonal and the residual cross correlations in the off-diagonal. There is a fairly large correlation coefficient between shocks to nominal wages and prices, indicating that wage and price shocks have followed each other quite closely and a more moderately sized correlation coefficient between wages and money, indicating that some of the wage increases might have been financed by money printing. All other correlation coefficients are very small.

$$\tilde{\Sigma} = \begin{bmatrix} 0.051 & & & \\ 0.01 & 0.038 & & \\ 0.22 & 0.08 & 0.048 & \\ 0.37 & -0.02 & -0.02 & 0.024 \end{bmatrix}$$

Table 10.1: The short-run dynamics

Eq.:	$\Delta_\lambda \Delta w_t$	$\Delta_\lambda \Delta s_t$	$\Delta_\lambda \Delta m_t$	$\Delta_\lambda \Delta p_t$
$(m - m^*)_{t-2}$	-	-	-0.24 (3.7)	-
$(w - w^*)_{t-2}$	-	-	-	0.22 (6.4)
$(\Delta w - 1.07\Delta p)_{t-1}$	-1.45 (-16.5)	-	-	0.12 (2.7)
$(\Delta s - 0.81\Delta p)_{t-1}$	0.35 (3.1)	-0.91 (10.6)	-	0.35 (6.3)
$(\Delta m - 0.80\Delta p)_{t-1}$	-	-	-0.97 (-12.7)	-
$Dval_t$	-	-	-	0.16 (4.0)
$Dval_{t-1}$	-	0.82 (12.2)	-	-0.26 (-4.2)
$Dtr84\_06_t$	-	-0.36 (-4.0)	0.16 (4.8)	-
$Dtr85\_01_t$	-	-	0.17 (4.7)	-

$m^*$  and  $w^*$  are given by (8.3)

The estimated short-run adjustment structure in Table 10.1 describe some interesting features of the Yugoslav hyperinflation transmission mechanism:

1. The long-run money demand relation,  $(m - m^*)_{t-1}$ , appears exclusively in the money equation and the same is true of the medium-run relation,  $(\Delta m - 0.8\Delta p)_{t-1}$ . This implies that neither the cumulated nor the exponentially weighed shocks to money stock have had any long-run impact on the variables of this system. Thus, the Yugoslav hyperinflation does not seem to have been caused by monetary expansion. Instead money supply seems to have been accommodating in the post communist period of the eighties.
2. Nominal wages are positively affected by excess depreciation rates  $(\Delta s - 0.8\Delta p)_{t-1}$  and are significantly error-correcting to the wage-price growth relation  $(\Delta w - 1.07\Delta p)_{t-1}$  but not to the long-run wage relation,  $(w - w^*)_{t-1}$ . The latter suggests that the interpretation of  $w^*$  as a long-run wage relation may not be fully granted.
3. Nominal exchange rates are significantly error-correcting to the depreciation-inflation rate relation  $(\Delta s - 0.81\Delta p)_{t-1}$ , but are not affected by any other equilibrium errors.
4. Prices are significantly affected by lagged deviations from the long-run wage

relation,  $(w - w^*)_{t-1}$ , indicating that it captures some of the imbalances between wages, prices, and exchange rates which have finally led to hyperinflation. Both the wage growth relation  $(\Delta w - 1.07\Delta p)_{t-1}$  and the depreciation rate relation  $(\Delta s - 0.81\Delta p)_{t-1}$  are significant in the price equation, so both lagged wage increases and lagged increases in the depreciation rate have played an important role for the Yugoslav hyperinflation.

5. The relative weights between lagged changes in nominal wages and exchange rates in the price equation seem to suggest that the latter has been more crucial for the Yugoslav hyperinflation behavior. This conclusion is strengthened by noticing that  $w - w^*$  also contains a positive effect from real exchange rates. Furthermore, the relation  $(\Delta s - 0.81\Delta p)_{t-1}$  appears with a positive coefficient in the wage and price equations whereas the wage growth rate relation  $(\Delta w - 1.07\Delta p)_{t-1}$  appears only in the price equation.

Thus, the depreciation rate of the black market exchange rate seems to have played a crucial role for both wage and price inflation in this period. Altogether the results point to the importance of excessive nominal wage claims, inflationary expectations and the rate of currency depreciation rather than the financing of government deficit by money printing. This is consistent with the discussion in Petrovic (1995) and Petrovic and Vujosevic (2000).

## 11. Conclusions

Neither Cagan's model nor any of the subsequent empirical applications have properly addressed the issue of hyperinflation in the context of a full system in which the dynamics of the transmission mechanisms can be addressed. This seems mandatory when the question is whether money has caused prices or prices money. In the end it boils down to the question whether money supply was accommodating money demand, or the other way around. Cagan's model is based on the latter case, but does not explicitly allow for the other possibility. We have here used the estimated dynamics of the full VAR model to give a balanced empirical view in favor of one or the other explanation.

In doing so we have developed an econometric framework for the empirical analysis of hyperinflationary episodes based on VAR models with an explosive root. Two types of equilibrium correction mechanisms were defined: one between levels and differences of the variables defining polynomially cointegrated long-run relations the other between growth rates. The analysis of how the system has dynamically adjusted to these relations are crucial for a full understanding of the mechanisms that have generated the Yugoslav hyperinflation in the eighties.

An important result was that only money was equilibrium correcting to the long-run money demand relation and to the medium-run relation between the growth rate of money and inflation rate. None of the other variables, i.e. wages, prices and exchange rates were affected by the two monetary equilibrium errors. Hence, permanent shocks to money supply have not had any long-run impact on the other variables of the system (inclusive prices). This strongly suggests that money was accommodating in this period and, thus, that the Yugoslav hyperinflation was not generated by excess expansion of money stock. Instead, the empirical results point to the role of inflationary expectations for nominal wage contracts and for the depreciation of black market exchange rate. The underlying cause of the ever increasing inflation rate seems to be in the financing of excessive wage claims with cheap bank credit based on currency deposits, with consequent currency depreciation, and increasing prices.

Many East-European countries have experienced similar high/hyperinflation periods in the transition towards market economies so understanding the econometrics of explosive roots data seems important for an adequate empirical analysis of the macro-economic mechanisms of these economies. This is particularly so considering that many of the Eastern European countries are waiting for the next round of EMU enlargement.

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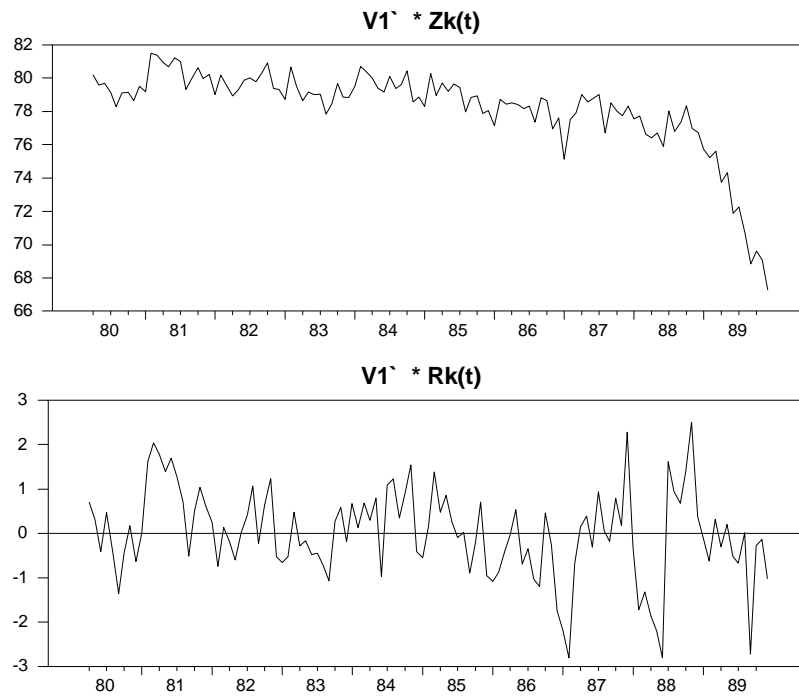
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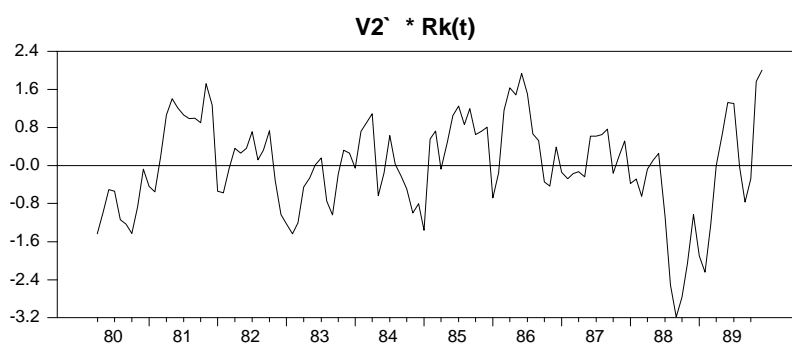
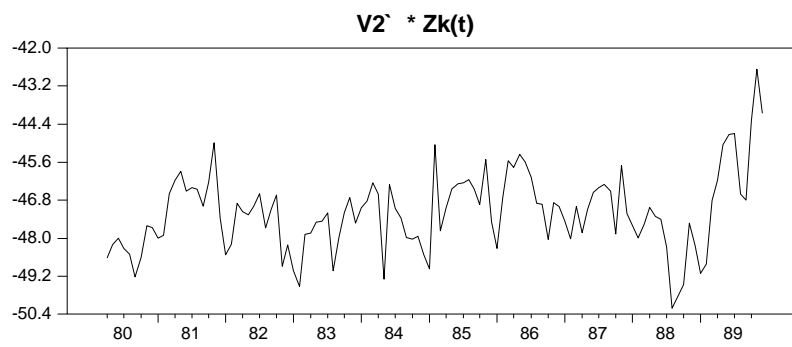
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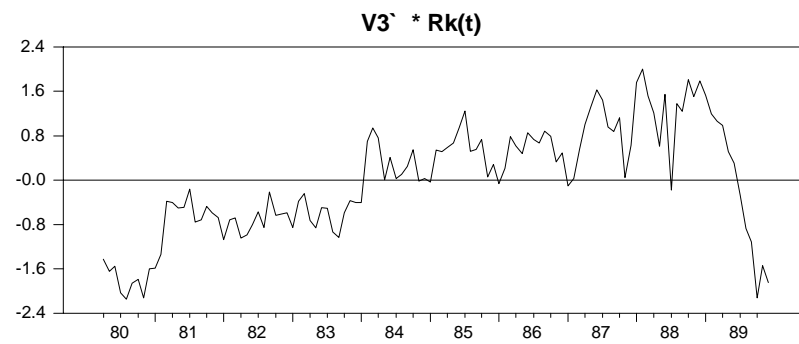
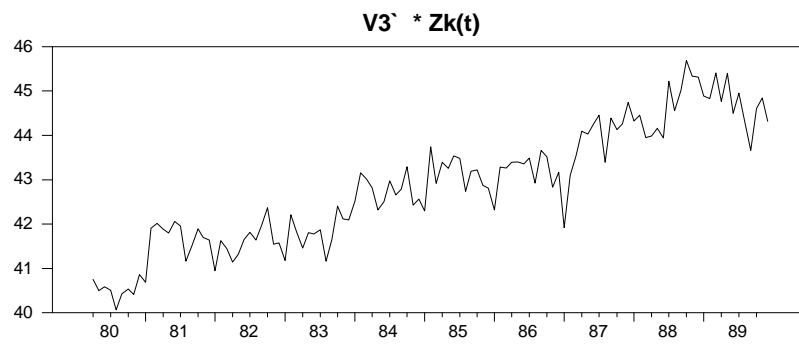
## 12. APPENDIX



The first cointegration relation

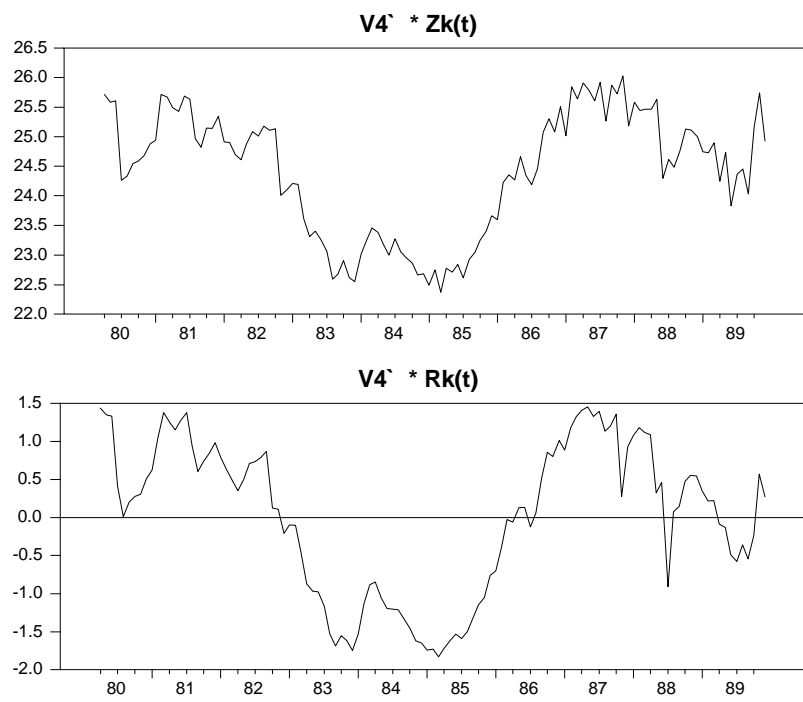


The second cointegration relation



The third cointegration relation





The fourth cointegration relation