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Debt, Managerial Incentives and Learning

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Abstract

Using a dynamic model with uncertainty and asymmetric information, we study the impact of debt on managerial compensation and performance targets. In this model, compensation has two roles to play - providing incentives to the manager and learning about his type. We show that debt acts as a substitute of compensation in both dimensions. If uncertainty is not too low, the incentive role of debt dominates the learning role. Thus in the presence of debt, compensation contracts can be more effective in learning about the manager. As debt increases, the pay-performance sensitivity falls and learning increases. We also examine the choice of debt and derive conditions under which a positive level of debt is optimal. We also conduct comparative statics with respect to the degree of asymmetric information and uncertainty.

JEL Classification: D8, G32, J3

1 Introduction

The issues of managerial incentives and agency costs (see Jensen and Meckling (1976)) play an important role in corporate finance. The traditional tool for aligning managerial interests with those of the owners has been compensation contracts (For example, Holmstrom (1979,1999), Harris and Townsend (1981), Freixas, Guesnerie and Tirole (1985), Murphy (1986), Jensen and Murphy (1990), Hirshleifer and Thakor (1992a), Laffont and Tirole (1993), Meyer and Vickers (1997), Jeitschko and Mirman (2001) and Jeitschko, Mirman and Salgueiro (JMS) (2001)). The finance literature has provided important insight into the role of capital structure in providing incentives to the manager to act in the owners' interest. For example, in a pioneering paper, Jensen (1986) argues that debt may provide a useful tool to discipline managers by restricting the amount of 'free cash flow' in their control (see also Stulz (1990) and Hart and Moore (1995)). In addition, it is also well-known that managers incur pecuniary and non-pecuniary costs in bankruptcy states associated with debt. These effects of debt on managerial payoffs suggest that managerial compensation contracts should be different with debt than without.

In this paper, we study the effect of debt on managerial compensation contracts in a dynamic context. That is, we ask how the compensation contract with debt compares with the compensation contract without debt. In particular, is debt a substitute for compensation, implying for example that the presence of debt leads to a lower compensation for a given performance level?[1] If the contracts are repeated over time, what effect does debt have on managerial incentives? Further, if managerial abilities are unknown, how does debt affect the learning process? Finally, how do these effects of debt on compensation feed back into the choice of debt? In particular, if debt had no other benefits and costs, does its effect on compensation justify a positive debt level in equilibrium?

The dynamic aspect of the model allows us to study the effect of debt on learning about the manager's ability/productivity (see Murphy (1986), Holmstrom (1999) and Jeitschko and Mirman (2001) on dynamics of managerial incentives with hidden abilities). The motivation for studying hidden abilities and learning is straightforward. Asymmetric information characterizes contractual relationships among various agents participating in the firm's activities and is at the heart of the agency theory. If adverse selection exists, managers with superior abilities find it easier to shirk or equivalently consume more perquisites if these actions are unobservable and/or there is uncertainty in the outcomes[2]. Thus compensation contracts need to address not only the problem of inducing the optimal effort level for a manager with known ability but also the problem of inducing the optimal effort from the manager given that his ability is unknown. In this paper, we concentrate on the latter problem in a dynamic setting.

In static models, the determination of equilibrium contracts takes the degree of asymmetry as given whereas in dynamic models, learning and thus the degree of asymmetry of information becomes an important

[1] The relationship between leverage and compensation has received some attention in the empirical literature (for example, Mehran (1992), Smith and Watts (1992)and Berger, Ofek and Yermack (1997)). Overall, the results are inconclusive. Our paper provides more insight, especially into the effect of leverage on compensation, rather than the other way around.

[2] A recent paper (see Bernardo, Cai and Luo (2001)) examines managerial compensation in the context of capital budgeting, using a model of adverse selection.

part of the design of contracts. Whether learning occurs or not depends on whether information is valuable. One benefit of learning is increased profits due to more efficient economic decisions in the future. However, it may be costly to learn since further incentives may have to be provided to the manager to reveal his type. Thus in designing the optimal compensation contract, the principal often faces a trade-off between current profits and future profits. We examine the effect of debt on the degree of learning implicit in the design of compensation contracts. The key driver of this effect is bankruptcy associated with debt and the fact that bankruptcy is costly to the manager. We assume uncertainty in the underlying environment in order to allow for incomplete and slow learning, a more realistic scenario in our view. When there is no uncertainty, a separating contract in the first period reveals the type of the manager and thus learning is complete. On the other hand, a pooling contract provides no information. Learning is a non-trivial issue when the underlying environment is random and thus learning need not be complete and immediate.[3]

To be specific, in this paper, we ask how debt affects the short-term compensation contracts between managers and owners when managers have private information about their productivity and get disutility from expending effort. We analyze how debt influences the key features of these contracts, in particular, the pay-performance sensitivity and learning. We show that due to the cost of bankruptcy to the manager, compensation is no longer needed as much to induce the good manager to reveal himself and produce the desired output level. A positive probability of bankruptcy also implies that compensation is no longer needed as much to learn about the manager's type. Thus debt substitutes for compensation in both dimensions - incentives and learning. We find that in general, the overall effect of debt on expected compensation and learning depends on the level of debt, the degree of information asymmetry and uncertainty. However, if uncertainty is not too low, the role of debt in substituting for incentives dominates the role in facilitating learning. An implication of this result is that in the presence of debt, shareholders can use compensation con-

[3] Our paper is similar in spirit to Harris and Raviv (1990) in the sense that they also study the role of debt in learning about the firm/manager and they also use debt as a disciplining device. However, the focus of our paper is new- we are interested in the effect of debt on the agency problem between shareholders and the manager, i.e., the extent to which debt and compensation contracts substitute, whereas Harris and Ravis focus on the role of debt in making better operating decisions through generating information about the firm. They do not address the effect of debt on managerial compensation.

tracts more effectively to learn about the manager's ability. Thus debt reduces the current pay-performance sensitivity[4] and increases learning. These effects are stronger, the greater is the information asymmetry and greater is the uncertainty. Thus an empirical prediction of our model is that firms with more debt have lower current pay-performance sensitivity and that this relationship is stronger in firms facing more uncertainty in their profits as well as more variance in the ability of the manager (Smith and Watts (1992) find a negative relationship between leverage and executive compensation.).

We also endogenize the debt level and find that a positive level of debt is optimal, even abstracting from other costs and benefits of debt, only if it is sufficiently likely that the manager has high ability. Intuitively, debt is a costly mechanism for providing incentives. However, we derive conditions under which optimal debt exists. The key benefit of debt is the reduced compensation while the key disadvantage is the negative effect on learning due to lost future profits in the bankruptcy states. The dynamics plays an important role in the existence of optimal debt. Indeed, in the static version, the optimal debt level must be zero. The intuition is that in the static case, the shareholders bear the entire cost that debt inflicts on the manager and thus there is no incentive effect. At the same time, debt introduces distortions in performance targets. In contrast, in the dynamic model, the incentive effect of debt survives since debt lowers the probability of future states in which mimicking is rewarded.

Comparative statics show that optimal level of debt increases with the productivity differential because providing incentives becomes more important. The effects of uncertainty and likelihood that the manager is high-productivity are positive if optimal debt lies in the interior of allowable debt levels, otherwise negative. We provide an example in which higher risk implies a lower optimal debt and thus higher pay-performance sensitivity relative to low-risk firms, consistent with vast empirical evidence (e.g. see Bradley, Jarrell and Kim (1984)), increases for 'moderate' likelihood of high-productivity manager and decreases for 'high' values of this probability. More generally, this paper adds another dimension to the issue of weak pay-performance

[4] The notion of pay-performance sensitivity used here refers to change in average compensation of the manager as his average performance changes.

sensitivity (see Jensen and Murphy (1990)) since debt is shown to serve as a substitute for compensation.

Two recent papers address similar issues as in this paper. Berkovitch, Israel and Spiegel (2000)), while allowing both moral hazard and unknown abilities, focus on a complementary problem, namely the effect of debt on managerial replacement and severance payments. Further, in their model, learning is complete and compensation is determined through bargaining. Calcagno (2000) examines the effect of debt on managerial compensation under moral hazard but does not study unknown abilities and thus learning.

Our work is also similar to John and John (1993) and Brander and Poitevin (1992). These papers address managerial decision making and compensation in the presence of debt, just as we do, but with a different, interesting focus. Both these papers show that managerial compensation contracts can be used to lower agency costs of debt arising from the distorted investment decisions that debt induces shareholders to make (see Green (1984)). Thus in both papers, hiring a manager and compensating him in a particular way is used by shareholders as a commitment device towards the debtholders. In contrast, we study how shareholders can use debt and compensation to provide incentives to the manager to exert the optimal effort and reveal his type. Grossman and Hart (1982) show that managers choose positive debt to commit to maximizing the value of the firm. Although costly bankruptcy is a critical ingredient in their model just as in ours, the focus of their paper is on the role of debt as a commitment device for managers, not the role of debt as a potential tool for shareholders to substitute for compensation in providing incentives and learning about the manager. Dewatripont and Tirole (1994) provide a general model of security design and managerial compensation.[5]

The model in this paper is closely related to JMS (2001). JMS examine a two-period model in which the principal and agent enter into short-term compensation contracts under uncertainty and show that the presence of uncertainty leads the principal to manipulate outputs and thus effort targets of the manager in the first period in order to learn and provide incentives. We show that debt substitutes for compensation

[5] Hirshleifer and Thakor (1992b) study the impact of the managerial tendency to invest in relatively safe projects on the choice of debt. However, they do not incorporate managerial compensation in the model. Hirshleifer and Suh (1992a) study the trade-off between project risk and managerial effort but do not incorporate debt.

in both providing incentives and learning, leading to a rich set of comparative statics with respect to the parameters of the model.

The rest of the paper is organized as follows: in Section 2, we present the model.; in section 3, we present the second period problem; in section 4, we present the first period problem with exogenous debt; in section 5, we present the first-period problem with endogenous debt and finally, in section 6, we conclude. The Appendix contains most proofs and examples.

2 MODEL

In this model, there is a single firm and two types of agents: the shareholders (the principal) and a manager (the agent). There are two time periods. The firm produces output y in each period, using the effort of the manager chosen in each period. The output depends on the productivity of the manager and is subject to a random shock, ε. A key feature of the model is that the manager has private information about his productivity. Further, only the manager observes his effort.

Specifically, output y is given by,

$$\tilde{y} = \tilde{\theta} e + \varepsilon,$$

where $\tilde{\theta}$ is the productivity of the manager and $e \in R_{+}$ is the effort level chosen by him. $\tilde{\theta}$ takes two possible values, $\underline{\theta} < \bar{\theta}$. The corresponding realization of output is $\tilde{y}$. The uninformed agents believe that $\tilde{\theta}$ is $\bar{\theta}$ with probability ρ and $\underline{\theta}$ with probability 1-ρ. We loosely refer to the high-productivity manager as a 'good' manager and the 'low' productivity manager as 'bad'. ε is distributed uniformly over the interval [-η,η], $\eta > 0$, and independently over the two time periods.

The production requires assets in place worth K, to be financed at the beginning of the game, either through equity alone or through equity and debt.[6] The manager's utility function is,

$$\tilde{u} = \tilde{r} - e^2,$$

where $\tilde{r}$ is the expected[7] compensation received from shareholders. Thus the manager's utility is linear in compensation and effort is costly for him. Further the marginal cost of effort increases as effort level increases. Shareholders are risk-neutral and maximize the sum of expected profits over two periods net of expected compensation and debt repayment, by choosing compensation contracts in each period. These contracts satisfy individual rationality and incentive compatibility constraints of the manager in each period.

Timing of Events

1. **Date 0**

   Shareholders choose the debt level to finance assets to maximize the value of the firm. They also choose the compensation plan for the two types of manager to maximize the value of equity. The manager chooses the effort level, given the compensation contract, to maximize his two-period expected utility.

2. **Date 1**

   The output is realized and observed by all agents. If the output is sufficient, the debt is paid off and then the manager's compensation is paid. That is, the manager's compensation is zero in the bankruptcy states.

[6] The analysis is based on the assumption that the entire debt is to be repaid at the end of the first period. Relaxing this assumption adds unnecessary notation without changing the essence of the results because what drives the results is a positive probability of bankruptcy at the end of the first period. As long as this is a feature of the environment, any sequence of repayments is consistent with the results.

[7] The focus of the paper is on the asymmetric information about the manager's type. Therefore, instead of analyzing compensation schedule, linking each realization of output with compensation, we study only the expected compensation for each type. Similarly the performance target is an expected output level for each type. The expectation is taken with respect to the distribution of ε.

If the firm is solvent, the shareholders update their beliefs about the manager's type using the observation of output.

If the firm is solvent, the production process is repeated, without any debt. Thus, the shareholders choose the compensation contract for the second period to maximize the value of equity in the second period and the manager chooses effort, given the compensation contract, to maximize his expected utility in the second period.

3. Date 2

Output of the second period is realized and the manager's compensation is paid.

3 Solving the Model: The Second-Period Problem

We solve the model recursively, starting with the second period. Assuming that the two types of manager choose a distinct effort level, that is, the contract in the first period is separating, the distributions of the output corresponding to the two effort levels either overlap partly or are completely disjoint, due to the uniform density of the shock, ε. That is, the publicly observed realization of output, y, either reveals the type of the manager or does not provide any information about the type. This implies that the posterior belief of the uninformed agents about the type of the manager being high-productivity is either 1 or 0 or ρ. A sufficiently large value of η ensures that there is a non-empty region of output realizations in which no information is revealed about the type of the manager. (See JMS).

Let $\underline{y}$ denote the expected output corresponding to the effort level chosen by the low-productivity manager and let $\bar{y}$ be the expected output corresponding to the effort level chosen by the high-productivity manager. The shareholders believe that the output realization lies in $[\ \underline{y} - \eta, \underline{y} + \eta\ ]$ conditional on the belief that the manager's productivity is low and it lies in $[\bar{y} - \eta, \bar{y} + \eta]$, conditional on the belief that the manager's productivity is high. If η is sufficiently large, we have the following posterior distribution.

$$\rho_2 = \begin{cases} 0, \text{ if } y \in (\underline{y} - \eta, \bar{y} - \eta) \\ \rho, \text{ if } y \in (\bar{y} - \eta, \underline{y} + \eta) \\ 1, \text{ if } y \in (\underline{y} + \eta, \bar{y} + \eta) \end{cases}$$

The probability distribution of ρ_2 then is,

$$\text{Probability that } \rho_2 = \begin{cases} 0, \text{ is } (1-\rho)\frac{\bar{y}-\underline{y}}{2\eta} \\ \rho, \text{ is } \frac{2\eta-\bar{y}+\underline{y}}{2\eta} \\ 1, \text{ is } \rho\frac{\bar{y}-\underline{y}}{2\eta} \end{cases}$$

Let D denote the face value of debt, owed at date1. The realized output can lie in either of the three intervals mentioned above. Since D is chosen at date 0, the manager's choice of output targets determines which interval D falls in. We focus on the parameter values for which D lies in the interval, (Max $(\underline{y} - \eta, 0), \bar{y} - \eta)$, denoting it by I_1.[8] The key implication of this assumption is that there is a positive probability of bankruptcy if and only if the manager's productivity is low.

We analyze the effect of debt on the compensation contract, which is defined as $((\bar{y}, \bar{r}), (\underline{y}, \underline{r}))$, where $\bar{y}$ and $\underline{y}$ are as defined earlier and $\bar{r}$ and $\underline{r}$ are the corresponding expected compensation to the manager of each type, conditional on solvency. In particular, we analyze the effect of debt on 'pay-performance sensitivity' defined in our setting as the ratio of change in total expected compensation to change in the expected output.

If the realized output at the end of the first period is such that $\rho_2 = 0$ or 1, that is, the type of the manager is revealed, the profit-maximizing contract for the shareholders in the second period must specify the first-best outputs and zero surplus to both types of the manager. On the other hand, if $\rho_2 = \rho$, the shareholders obtain no information from the first-period contract and thus maximize their second-period

[8] The essence of the analysis is the same if D were to lie in the other intervals although the specific results are naturally different.

expected profits subject to the individual rationality and incentive compatibility constraints. Since this is a standard agency problem (see Harris and Townsend (1981)), we skip the details and simply report the maximized expected profits, denoted by A.

From now on, we set $\underline{\theta} = 1$, without loss of generality and replace $\bar{\theta}$ by θ. Thus using the probability distribution of ρ_2, and adjusting it for debt owed, the expected second-period profits of the shareholders are,

$$V(D, \underline{y}, \bar{y}) = (1-\rho)\frac{\bar{y} - \eta - D}{8\eta} + \rho\frac{\bar{y} - \underline{y}}{8\eta}\theta^2 + \frac{2\eta - \bar{y} + \underline{y}}{2\eta}A, \tag{1}$$

where,

$$A = \rho\frac{\theta^2}{4} + \left(\frac{(1-\rho)}{2\left(1 - \frac{\rho}{\theta^2}\right)}\right)^2 \left(1 - \frac{2\rho}{\theta^2}\right). \tag{2}$$

Note that A is independent of the first period outputs as well as η.

In the next section, we examine the impact of given debt on the first period equilibrium outputs, compensation and learning.

4 The First-Period Problem: Exogenous Debt

The first-period maximization problem of the shareholders is to maximize the sum of the two-period profits by choosing the first-period outputs and rewards, given the debt level and given the effect of these choices on the expected second-period profits, subject to the manager's individual rationality (IR) constraint and the incentive compatibility (IC) constraint. Intuitively, in setting the compensation contract variables, the principal faces a trade-off between current and future profits. The closer are the target output levels in the first period, the higher are the current profits due to a smaller ratchet effect payment to the good manager but lower are the future profits due to less learning. As noted above, learning enables the principal to extract all profits above the reservation utility level, in the second period. We examine the effect of debt on this trade-off.

The two-period maximization problem of the shareholders is to choose $\underline{y}, \bar{y}, \underline{r}$ and $\bar{r}$ in the first period to maximize,

$$(1-\rho)\left(\frac{(\underline{y}+\eta-D)^2}{4\eta}-\frac{\underline{y}+\eta-D}{2\eta}\underline{r}\right)+\rho\left(\bar{y}-D-\bar{r}\right)+V(D,\underline{y},\bar{y}),$$

subject to,

$$\begin{aligned}
\frac{\underline{y}+\eta-D}{2\eta}\underline{r} &\geq \underline{y}^2,\ \bar{r}\geq\frac{\bar{y}^2}{\theta^2},\\
\frac{\underline{y}+\eta-D}{2\eta}\underline{r}-\underline{y}^2 &\geq \bar{r}-\bar{y}^2,\\
\bar{r}-\frac{\bar{y}^2}{\theta^2} &\geq \frac{\underline{y}+\eta-D}{2\eta}\underline{r}-\frac{\underline{y}^2}{\theta^2}+\frac{\bar{y}-\eta-D}{8\eta}\left(1-\frac{1}{\theta^2}\right).
\end{aligned}$$

The first two inequalities are the IR constraints and the last two inequalities are the IC constraints. In the event that the manager'productivity is low, the shareholders make no profits due to a positive probability of bankruptcy. In the event of solvency, they get the residual, that is, the realized profit net of the debt owed and the expected compensation, $\underline{r}$. Note that $\underline{r}$ denotes expected compensation conditional on solvency consistent with our assumption that the debtholders have priority in payment. The IR constraint of the low-productivity manager reflects the fact that in the event of bankruptcy, he gets no compensation. Thus for the low-productivity manager to accept the compensation contract, the compensation in the solvency states must be higher than in the absence of debt. That is, the cost of bankruptcy to the low-productivity manager is borne by the shareholders. Recall that by assumption (that D$\in I_1$), the probability of bankruptcy is zero if the manager is good.

The IC constraint of the high-productivity manager reflects the fact that deviating from the prescribed output target leads to a positive probability of bankruptcy and thus a lower gain in the future. This, especially the last term of the IC constraint of the good manager, is a key ingredient in results of this paper. This term measures the ratchet effect (see Freixas, Guesnerie and Tirole (1985))- the up-front payment needed in a dynamic contracting environment to induce the good manager to reveal himself. In the absence of this payment, the good manager has an incentive to mimic the bad manager because doing so implies that there is a positive probability that he is perceived to be the bad manager in the second period. In that event, the good manager gets positive utility in the second period compared to zero utility from revealing himself. Due to the positive probability of bankruptcy brought about by debt, this term is lower than in the absence of debt, other things being equal. That is, given the first period outputs, a lower up-front payment is needed in the presence of debt.

4.1 Analysis

We assume that the IC constraint binds only for the high-productivity manager and the IR constraint binds only for the low-productivity manager and verify these later. The binding IC constraint for the high-productivity manager and the binding IR constraint for the low-productivity manager together imply that in a static model, debt provides no incentives to the high-productivity manager since the shareholders compensate the manager for the bankruptcy cost.

Substituting for $\underline{y}$ and $\bar{y}$ from the binding constraints and for the value function from equation 1 into the profit function, we obtain,

$$\begin{aligned}
\Pi_d \equiv\ & \rho\left(\bar{y} - D - \frac{\bar{y}^2}{\theta^2} - \underline{y}^2 + \frac{\underline{y}^2}{\theta^2} - \frac{\bar{y}-\eta-D}{8\eta}\left(1-\frac{1}{\theta^2}\right)\right) + \\
& (1-\rho)\left(\frac{(\underline{y}+\eta-D)^2}{4\eta} - \underline{y}^2\right) + \\
& (1-\rho)\frac{\bar{y}-\eta-D}{8\eta} + \rho\frac{\bar{y}-\underline{y}}{8\eta}\theta^2 + \frac{2\eta-\bar{y}+\underline{y}}{2\eta}A. \qquad (3)
\end{aligned}$$

Note that debt has two types of effects on profits. One is due to bankruptcy and the other is regardless of bankruptcy. The latter leads to a transfer of a fixed payment from shareholders to debtholders, and thus has no effect on firm value. Bankruptcy however changes the compensation contract, including the profit-maximizing output targets. If there were no bankruptcy, debt would have no effect on the compensation contract in this model. In what follows, we refer to the bankruptcy-induced effect of debt, rather than the transfer of D as such. For purposes of comparison, the profit function for the no-debt case is as follows[9].

$$
\begin{aligned}
\Pi \equiv \; & \rho\left(\bar{y} - \frac{\bar{y}^2}{\theta^2} - \underline{y}^2 + \frac{\underline{y}^2}{\theta^2} - \frac{\bar{y} - \underline{y}}{8\eta}\left(1 - \frac{1}{\theta^2}\right)\right) + \\
& (1-\rho)\left(\underline{y} - \underline{y}^2\right) + (1-\rho)\frac{\bar{y} - \underline{y}}{8\eta} + \rho\frac{\bar{y} - \underline{y}}{8\eta}\theta^2 + \\
& \frac{2\eta - \bar{y} + \underline{y}}{2\eta}A. \qquad (4)
\end{aligned}
$$

Comparing equation (3) and (4) reveals first, that the ratchet effect terms are different, as discussed earlier. Second, the expected profits in the second period are different due to the probability of bankruptcy. Bankruptcy not only affects the profits realized at the end of the first period but also the profits realized at the end of the second period. This is because if bankruptcy occurs, the firm is liquidated at the end of the first period. Although this effect is induced by the event of bankruptcy, it has an important implication for shareholders' incentive to learn about the managerial type. In the no-debt case, the shareholders have an incentive to learn, that is, they have the incentive to set first period outputs in such a way that the manager reveals himself with a positive probability. This increases the second period profits of the bank since the first-best outcome can be achieved in the full-revelation states. With debt, some profits from learning are lost. In what follows, we analyze how debt affects compensation and learning by examining the properties of equilibrium of the model.

[9] The derivation is similar to the case when there is debt.

Let $\underline{y}^*, \bar{y}^*$ denote the solutions of the no-debt problem, that is the first-period outputs implied by the optimal contract of the no-debt problem.

Proposition 1

$$\bar{y}_d = \bar{y}^*$$

and

$$\underline{y}_d = \frac{(1-\rho)(\frac{\eta-D}{2\eta}) - \frac{\rho\theta^2}{8\eta} + \frac{A}{2\eta}}{2[1-\frac{\rho}{\theta^2}] - \frac{1-\rho}{2\eta}}. \tag{5}$$

Proof. The first order condition with respect to $\bar{y}$ is,

$$\frac{d\Pi_d}{d\bar{y}} = \rho\left(1 - 2\bar{y}\frac{1}{\theta^2} - \frac{1}{8\eta}\left(1 - \frac{1}{\theta^2}\right)\right) + \frac{1-\rho}{8\eta} + \rho\frac{1}{8\eta}\theta^2 - \frac{A}{2\eta} = 0.$$

This reduces to,

$$\bar{y}_d = \frac{\theta^2}{2}\left(1 - \frac{1}{8\eta}\left(1 - \frac{1}{\theta^2}\right) + \frac{1}{8\eta\rho}\left(1 - \rho + \rho\theta^2 - 4A\right)\right) = \bar{y}^*. \tag{6}$$

Similarly the first order condition with respect to $\underline{y}$ is,

$$(1-\rho)\left(\frac{\underline{y}+\eta-D}{2\eta} - 2\underline{y}\right) + \rho\left(-2\underline{y} + \frac{2\underline{y}}{\theta^2}\right) - \rho\frac{1}{8\eta}\theta^2 + \frac{1}{2\eta}A = 0. \tag{7}$$

Solving for $\underline{y}$ gives us the result. ■

The second order condition of the maximization problem requires that,

$$\eta > \frac{1-\rho}{4(1-\frac{\rho}{\theta^2})}.$$

For later use, the lower output target without debt is,

$$\underline{y}^* = \frac{(8\eta - 1)(1-\rho) - \rho[\frac{1}{\theta^2} - 1] + 4A - \rho\theta^2}{16\eta\left(1 - \frac{\rho}{\theta^2}\right)}. \tag{8}$$

Other conditions are needed to ensure existence of equilibrium in I_1. Specifically, the analysis above assumes that $\max\left(0, \underline{y}_d - \eta\right) < D < \bar{y}_d - \eta < \underline{y}_d + \eta$. This set of inequalities imposes further conditions on the parameters of the model, η, ρ and θ. Example 1 in the Appendix shows that there are parameter values for which equilibrium exists.[10] It can also be verified that the IR constraint of the good manager is satisfied in equilibrium. The IC constraint of the low-productivity manager is satisfied in equilibrium if $\bar{y}_d + \underline{y}_d > 1$.We confine the analysis to parameters for which this condition is met. In particular, Example 1 in the Appendix satisfies this condition. The following corollary of Proposition 1 follows.

Corollary 1 *As debt increases within I_1, (i) $\underline{y}_d$ falls, (ii) the first-period expected compensation of the manager falls, (iii) learning increases and (iv) the pay-performance sensitivity declines.*

Proof. Part (i) follows from 5. For part (ii), the expected compensation of the low-productivity manager is given by $\underline{y}^2$, by his binding IR constraint. Since $\underline{y}_d$ falls as D increases, the expected compensation decreases with D. For the high-productivity manager, we combine his binding IC constraint with the binding IR constraint of the low-productivity manager and obtain,

$$\bar{r} = \frac{\bar{y}^2}{\theta^2} + \underline{y}^2\left(1 - \frac{1}{\theta^2}\right) + \frac{\bar{y} - \eta - D}{8\eta}\left(1 - \frac{1}{\theta^2}\right). \tag{9}$$

Since $\underline{y}$ falls as D increases, the second term falls. The third term obviously falls with D. Thus $\bar{r}$ decreases as D increases. Part (iii) follows from the fact that as debt increases, the distance between the two first-period output targets increases.

For part (iv), define pay-performance sensitivity as $\frac{\bar{r} - \frac{\underline{y} + \eta - D}{2\eta}\underline{r}}{\bar{y} - \underline{y}}$, that is, the change in expected compen-

[10] Interestingly, in the absence of debt, the only condition needed for existence of equilibrium is that η be sufficiently large. Thus debt changes the model significantly.

sation as expected performance changes. Thus,

$$\frac{\overline{r} - \frac{\underline{y}+\eta-D}{2\eta}\underline{r}}{\overline{y}-\underline{y}} = \frac{\overline{y}+\underline{y}}{\theta^2} + \frac{\bar{y}-\eta-D}{8\eta(\overline{y}-\underline{y})}\left(1-\frac{1}{\theta^2}\right) \tag{10}$$

Since $\underline{y}$ falls as D increases and $\overline{y}$ remains unchanged, it follows that this ratio falls as debt increases within the first interval. ■

Intuitively, as debt increases within interval I_1, the probability of bankruptcy increases, implying a lower marginal benefit to the shareholders from increasing $\underline{y}$ and thus $\underline{y}$ decreases. An implication of this corollary is that as debt increases, the expected compensation of the good manager decreases on two accounts (see equation (9)): the ratchet effect component (attributable to the dynamic game) goes down due to a higher debt and second, the reduction in the low-productivity manager's target output reduces the rent (attributable to the static game) to the good manager. The expected compensation of the low-productivity manager obviously falls. Finally, a higher debt implies more learning since outputs are set further apart due to a higher probability of bankruptcy. The intuition behind the decline in pay-performance sensitivity is simply that an increase in debt increases the probability of bankruptcy for the low type and thus reduces the future states in which the high type benefits from mimicking. Thus debt substitutes for compensation and reduces the sensitivity of current pay to performance.

4.2 Comparing $\underline{y}_d$ and $\underline{y}^*$

Note that Corollary 1 does not take into account all effects of debt and thus does not imply that the lower performance target with debt is lower than the lower target without debt. That is, it does not imply that $\underline{y}_d < \underline{y}^*$. Intuitively, this comparison must also account for how debt affects the ratchet effect payment and future profits. These effects are one-time effects in the sense that they occur once, as the shareholders move their debt level from zero to a positive amount within interval I_1 and do not occur as D varies within this interval. A comparison of $\underline{y}_d$ and $\underline{y}^*$ is critical to account for all effects of debt, unlike the partial effect captured in the corollary. For this purpose, we analyze the first order conditions that determine the optimum levels of these two lower targets.

$$\frac{d\Pi_d}{d\underline{y}} = (1-\rho)\left(\frac{\underline{y}+\eta-D}{2\eta} - 2\underline{y}\right) + \rho\left(-2\underline{y} + \frac{2\underline{y}}{\theta^2}\right) - \rho\frac{1}{8\eta}\theta^2 + \frac{1}{2\eta}A,$$

$$\frac{d\Pi}{d\underline{y}} = (1-\rho)\left(1-2\underline{y}\right) + \rho\left(-2\underline{y} + \frac{2\underline{y}}{\theta^2} + \frac{1}{8\eta}\left(1-\frac{1}{\theta^2}\right)\right) - \frac{1-\rho}{8\eta} - \rho\frac{1}{8\eta}\theta^2 + \frac{1}{2\eta}A, \tag{11}$$

$$\frac{d\Pi_d}{d\underline{y}} - \frac{d\Pi}{d\underline{y}} = (1-\rho)\left(\frac{\underline{y}+\eta-D}{2\eta} - 1\right) + \frac{-\rho}{8\eta}\left(1-\frac{1}{\theta^2}\right) + \frac{1-\rho}{8\eta}. \tag{12}$$

The first term in Equation (12) represents the differential marginal effect of $\underline{y}$ on current profits when the manager has low productivity. This is the bankruptcy effect of debt. The second term represents the differential marginal effect of $\underline{y}$ on the ratchet effect payment that the high-productivity manager requires in a separating equilibrium. We refer to it as the incentive effect of debt since this term influences the manager's incentive to mimic. Finally, the third term represents the differential marginal effect on future profits of $\underline{y}$. We refer to this as the learning effect of debt.

Now clearly the second term is negative and the third term is positive. The first term is also negative since D is greater than $\underline{y}-\eta$, by assumption. Intuitively, when debt is in I_1, increasing $\underline{y}$ has a smaller positive effect on current profits because some profits are lost in bankruptcy anyway. Interestingly, debt takes away the positive effect of increasing $\underline{y}$ in reducing the ratchet effect payment. In the absence of debt, increasing $\underline{y}$ reduces the up-front payment because it reduces the probability that the good manager is perceived to be the bad manager. However with debt, the set of states in which the good manager is perceived to be the bad manager does not change on the margin, as $\underline{y}$ changes and thus $\underline{y}$ has no marginal effect on the probability of being perceived as the bad manager. Finally, debt also takes away the negative effect of $\underline{y}$ on future profits. In the zero-debt case, increasing $\underline{y}$ lowers the probability of complete learning and thus lowers expected profits. However, with debt, the bankruptcy states are irrelevant to shareholders and thus on the margin, increasing $\underline{y}$ does not reduce future profits as much.

Thus debt has an important bearing on both incentives and learning implied by the compensation contract. The incentive effect of debt, through the ratchet effect, implies that, ceteris paribus, the first-period output targets are set further apart with debt. By Equation (9), it follows that the expected compensation is lower, ceteris paribus. In this sense, debt acts as a substitute for compensation in providing incentives. However, debt also acts as a substitute for compensation in learning about the manager. That is, how the good manager is compensated in the first period has a smaller marginal effect on the future profits of shareholders. Thus the first period outputs can be set closer together with debt. By Equation (9), it follows that the expected compensation of the good manager is higher with debt, ceteris paribus. Thus, the two effects of debt on compensation go in opposite direction and the net result depends on the fundamentals of the environment, namely, the likelihood of the manager being high-productivity (ρ), the importance of asymmetric information (θ) and the extent of uncertainty (η), as well as the amount of debt (D).

We next examine how these opposite effects of debt on incentives and learning are resolved.

Proposition 2 *$\underline{y}_d < \underline{y}^*$ if and only if*

$$D > \underline{D} \equiv Max[0, \underline{y}^* - \eta - \frac{1}{4}\left(\frac{\rho}{1-\rho}(1 - \frac{1}{\theta^2}) - 1\right)]. \tag{13}$$

Proof. Proof follows by evaluating Equation (12) at $\underline{y} = \underline{y}^*$, setting it to be less than 0 and then solving for D. ■

Corollary 2 *If $D >$ Max $(0, \underline{y}^* - \eta)$, a sufficient condition for $\underline{y}_d < \underline{y}^*$ is*

$$\rho > \frac{1}{2 - \frac{1}{\theta^2}}. \tag{14}$$

Proof is obvious.

Equation (14) implies that when ρ is 'large', debt substitutes for compensation more in providing incentives than in learning and thus outputs of the two types of manager are set further apart with debt than without to maximize learning.[11] Intuitively, the substitution for the incentive-role of compensation is stronger since it is more likely that the manager has high productivity and thus reducing the up-front payment to the good manager is more important than maximizing profits from the low-productivity manager in the future.

We now present some economic implications of Proposition 2, that are similar in essence to Corollary 1.

Corollary 3 *(Effect on Learning) For all $D>\underline{D}$, debt leads to higher learning.*

Proof. The distance between expected outputs increases due to debt and thus learning increases. ■

So far, the discussion has been in terms of output targets rather than effort targets but given that $\underline{y}$ and $\bar{y}$ are simply $\underline{e}$ and $\theta\bar{e}$ respectively, the effect of debt on effort levels is straightforward.

Corollary 4 *(Effect on Effort) For all $D>\underline{D}$, debt leads to lower effort by the low-productivity manager. Debt leads to the same effort for the high-productivity manager for all $D \in I_1$.*

Proof follows from the production function and Proposition 2.

Corollary 5 *(Effect of debt on compensation) For all $D>\underline{D}$, debt leads to lower expected compensation for both types of manager.*

Proof. For the low-type manager, the proof is obvious since by the individual rationality constraint, his expected compensation equals $\underline{y}_d^2$, which has been shown to be lower. The expected compensation of the high type with debt is given by Equation (9). In comparison, the expected compensation without debt

[11] It may seem puzzling at first glance that debt increases learning even though its role in substituting for learning is less important than in providing incentives. It is in fact quite intuitive. Compensation in the absence of debt needs to address incentives as well as learning. Debt substitutes for the incentive role, thus leaving the compensation contract to focus more on learning.

$= \frac{\overline{y}^{*2}}{\theta^2} + \underline{y}^{*2}\left(1-\frac{1}{\theta^2}\right) + \frac{\overline{y}^{*}-\underline{y}^{*}}{8\eta}\left(1-\frac{1}{\theta^2}\right)$.In these two expressions of expected compensation, the first term is identical, the second term is lower with debt and finally the third term is also lower with debt since D $>\underline{D} \Rightarrow D > \underline{y}^{*} - \eta$. ■

Corollary 6 *For all D>$\underline{D}$, debt lowers the pay-performance sensitivity.*

Proof. Since $\underline{y}_d < \underline{y}^{*}$, for all D>$\underline{D}$, the same logic applies as in Corollary 1, part (iv). ■

Thus, these corollaries imply that if debt is above a cutoff level, learning increases, expected compensation of both types of manager falls and the pay-performance sensitivity falls. Thus debt substitutes for compensation in providing incentives. The following Proposition shows that the cut-off level is non-binding if the uncertainty is not too low.

Proposition 3 $\forall\ \eta > 0.75, \underline{D} = 0.$

For Proof, see the Appendix.

Intuitively, $\underline{y}^{*}$ is 'small' relative to η, for all $\eta > 0.75$. Thus, if η is sufficiently high, for all allowable debt levels, the lower target output, $\underline{y}_d$, falls and thus expected compensation of both types of manager falls and learning increases.[12]

Corollary 7 $\forall\ \eta > 0.75$, *debt decreases the first-period expected compensation of both types of manager, increases learning and reduces the first-period pay-performance sensitivity.*

Thus far we have assumed that debt level is exogenously given to be in I_1 and examined its effect on compensation and learning, restricting output targets such that debt indeed lies in I_1. We next determine the optimal level of debt and show that the analysis thus far is non-vacuuous since there exist parameter

[12] We can show that even for small values of η, except for θ close to 1, the cut-off level of debt is zero, implying that debt leads to lower expected compensation and higher learning for all values of parameters, except when θ is close to 1 *and* η is small. However the proof is quite tedious and does not add much to the insights already obtained.

values for which optimal debt level indeed lies in I_1 and further, that debt increases value of the firm. We also conduct comparative statics with respect to ρ, θ and η.

5 The First-Period Problem: Endogenous Debt

We assume that the decision on financing the assets in place is made by the shareholders to maximize the value of the firm as a whole rather than only equity.[13] Thus the maximization problem of the shareholders is to choose D at date 0 to maximize, (We omit the subscript 'd' on the lower output for convenience.).

$$
\begin{aligned}
&\rho\left(\bar{y} - D - \frac{\bar{y}^2}{\theta^2} - \underline{y}^2 + \frac{\underline{y}^2}{\theta^2} - \frac{\bar{y} - \eta - D}{8\eta}\left(1 - \frac{1}{\theta^2}\right)\right) \\
&+(1-\rho)\left(\frac{(\underline{y} + \eta - D)^2}{4\eta} - \underline{y}^2\right) \\
&+(1-\rho)\frac{\bar{y} - \eta - D}{8\eta} + \rho\frac{\bar{y} - \underline{y}}{8\eta}\theta^2 + \frac{2\eta - \bar{y} + \underline{y}}{2\eta}A \\
&+\frac{1-\rho}{2\eta}\left[\int_{\underline{y}-\eta}^{D} y dy + \int_{D}^{\underline{y}+\eta} D dy\right] + \rho D.
\end{aligned}
$$

The last row represents the value of debt[14] and the remaining terms represent the value of equity. Due to the envelope theorem, only the direct effect of D on the value of equity needs to be considered. However, since the compensation scheme does not take into account the value of debt, we must consider the direct as well as the indirect (through $\underline{y}$) effect of D on the value of debt.

The first order condition with respect to D is,

[13] Note that if D were chosen to maximize the value of equity, a positive level of D would not arise because a positive D implies a transfer from equityholders to debtholders. This transfer outweighs any positive effects of D.

[14] Note that the debtholders own the firm if output is less than D. If the lowest realization of output is negative, the expression for value of debt suggests that they bear the loss. While this may seem unrealistic, it is simply a normalization. In particular, one can shift the support of output to the right without affecting the results.

$$\frac{1-\rho}{2\eta}(D-(\underline{y}-\eta))\frac{d\underline{y}}{dD}+\rho\frac{1}{8\eta}(1-\frac{1}{\theta^2})-(1-\rho)\frac{1}{8\eta}=0. \tag{15}$$

By Corollary 1, $\frac{d\underline{y}}{dD}<0$, and thus the second order condition for maximization is satisfied. The first term in the above derivative captures the indirect effect of debt on the value of debt, the second term captures the direct effect of debt on managerial incentives through the ratchet effect and the last term captures the direct effect of debt on the second period profits. The first term of equation (15) is negative for all D in I_1 - debt lowers the lower output target which reduces the value of debt. The second term of the derivative (reflecting the direct incentive effect of debt) is positive. Thus debt alleviates the managerial incentive problems. Finally, the last term of the derivative, reflecting the direct effect of debt on future profits, is negative. That is, debt, due to bankruptcy, reduces future profits from learning. Thus a necessary condition for debt to emerge in our model is that the incentive effect dominates the learning effect.[15]

Proposition 4 *A necessary condition for debt to emerge in the dynamic model is that the positive direct effect of debt on incentives dominate the negative direct effect of debt on learning and future profits. That is,*

$$\rho>\frac{1}{2-\frac{1}{\theta^2}}. \tag{16}$$

This condition is also sufficient for optimal debt to be in I_1, if $\underline{y}-\eta>0$.

Proof. The necessity part follows from the fact that the first term is negative. The sufficiency follows from the fact that if to the contrary, $D^*\leq\underline{y}-\eta$, the derivative with respect to D is positive. ■

Thus optimal debt is positive only if the probability of high-productivity manager is sufficiently high. This is intuitive since bankruptcy is costly - the firm is liquidated even when on average, it is worth continuing. Thus debt is a costly mechanism for providing incentives to the manager. However, if the manager is much

[15] Note that the equilibrium level of debt must be zero in the static model since only the first term of the first order condition remains and this term is negative. Thus debt is a deadweight loss in the static model.

more likely to have high productivity, the future loss from shutting the firm down is outweighed by the reduced compensation in the first period.[16] This condition is also sufficient to lead to a positive amount of debt if the firm never makes a loss. In this case, there is a benefit from increasing debt until the negative effect of debt on current value of the firm offsets the net positive incentive effect of debt. In contrast, if the lowest realized output is negative, zero debt may be optimal, due to relatively stronger negative effect of distorted lower target on the value of debt.

For derivation of the optimal debt level, denoted by D^*, it is convenient to let (see Equation (5)) $\underline{y}_d = \alpha - \beta D$ so that $\frac{dy}{dD} = -\beta, \alpha > 0, \beta > 0$. If an interior solution for debt exists, it is obtained by simplifying (15):

$$D^i = \frac{1}{1+\beta}\left(\alpha - \eta + \frac{\frac{\rho}{1-\rho}(1-\frac{1}{\theta^2})-1}{4\beta}\right). \tag{17}$$

To be consistent with our assumption that D$\in I_1$, D^* must satisfy $\bar{y}-\eta \geq D^*$ >Max(0, $\underline{y}_d - \eta) = Max(0, \alpha - \beta D - \eta)$. That is,

$$0 < D^* = Min\left(D^i, \bar{y}-\eta\right). \tag{18}$$

The equality is tedious to characterize. However, an example is provided in the appendix where this set of inequalities is indeed satisfied and the optimal level of debt increases the value of the firm.

Next, we analyze how optimal debt level varies with the underlying parameters.

Proposition 5 *(i)* $\frac{dD^*}{d\rho} > 0$ *iff* $D = D^i$. *(ii)* $\frac{dD^*}{d\theta} > 0$.*(iii)* $\frac{dD^*}{d\eta} > 0$ *iff* $D = D^i$.

For proof, see the Appendix.

Intuitively, as ρ increases, the positive effect of debt in alleviating incentives becomes stronger. Thus if the debt level weren't already the maximum consistent with positive probability of bankruptcy only for the

[16] Clearly, allowing for a liquidation value in the event of bankruptcy only strengthens our results.

low-type, that is, if there is room to increase debt, it is increased. However, the maximum level goes down as ρ increases because the marginal future benefit from increasing $\bar{y}$ falls for all ρ satisfying the necessary condition (given by equation (16)) while the marginal cost (current compensation) increases. Example 3a in the Appendix shows that the upper bound on optimal debt binds as ρ increases. Thus optimal debt level increases for intermediate values of ρ and decreases for high values of ρ. An empirical implication of this result is that firms or industries in which the probability of the manager being high-ability is moderately (significantly) higher, borrow more (less) and thus have lower (higher) pay-performance sensitivity and higher (lower) learning.

As θ increases, the optimal level of debt increases unambiguously since providing incentives becomes more important. The upper bound on debt increases as well since unlike ρ, an increase in θ lowers current compensation and this effect outweighs the decrease in the future marginal benefit. The empirical prediction of this result is that firms or industries in which the managerial productivity differential is large, borrow more in equilibrium and thus have a lower pay-performance sensitivity and higher learning.

Finally, an increase in η increases optimal debt if there is room to do so, that is, if the optimal debt remains in the interior of the feasible range. Intuitively, this is because as η increases, the marginal effect of debt on the lower target, measured by β, decreases, implying that the negative marginal effect of debt on the value of debt is weaker. However, the upper bound on the debt level falls as η increases. Example 3c (in the Appendix) shows that optimal debt decreases as η increases for all η consistent with existence of equilibrium. This is consistent with empirical evidence since a higher η in our setting implies a higher variance of output, keeping the mean constant. The empirical literature shows that high-risk firms are associated with lower debt. Combining it with Proposition 3 and Corollaries of Proposition 2 imply that riskier firms have higher pay-performance sensitivity and lower learning, ceteris paribus.

6 Conclusion

In this paper, we have developed a model of debt, managerial incentives and learning in a dynamic context. We have shown that when the manager has private information about his productivity and his effort, due to the threat of bankruptcy, debt serves as a substitute of compensation in both providing incentives as well as learning about the manager's type. We have determined conditions under which debt reduces the pay-performance sensitivity and increases learning. In particular, these results hold if uncertainty is not too low. Further, the more likely it is that the manager is good and/or the higher the productivity of the good manager, the greater is the role of debt in providing incentives and weaker is the role of debt in facilitating learning. We have also determined conditions under which optimal debt increases value of the firm through its effect on managerial compensation even abstracting from all other costs and benefits of debt. Finally, illustrations of comparative statics show that optimal debt decreases with uncertainty, increases in the manager's productivity, increases for intermediate values of the probability of a good manager and decreases for higher values of the probability.

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Appendix

Example 1 (Existence of Equilibrium) *Suppose* $\theta = 4,\ \rho = 0.7 \Rightarrow A = 2.8225.$

The existence condition for no-debt case are $\bar{y}^* > \underline{y}^*$ and $\bar{y}^* - \eta < \underline{y}^* + \eta$. Substituting for the parameters into the expressions for $\underline{y}^*$ (Equation (8)) and $\bar{y}^*$(Equation (6)), existence of equilibrium requires $\eta >$Max $(0.085, 3.8346)$.

For D$\in I_1$, the following conditions are required for existence of equilibrium in the presence of debt:

1. $\bar{y}_d - \eta < \underline{y}_d + \eta \Rightarrow 8 - \frac{0.6375}{\eta} - \eta < \frac{0.45 - 0.3\frac{D}{\eta} + \frac{0.0225}{\eta}}{1.9125 - \frac{0.3}{\eta}} + \eta$, after substituting in the parameter values assumed for θ and ρ, into (5) and (6).

2. $D > Max(0, \underline{y}_d - \eta)$.

3. $D < \bar{y}_d - \eta \Rightarrow D < 8 - \frac{0.6375}{\eta} - \eta$

Note that condition (3) requires that η be bounded above by $\bar{y}_d$. Further $\bar{y}_d \uparrow 8$ as η increases. Thus, combining the existence condition for the no-debt case with condition 3, $3.8346 < \eta < 8$. We illustrate our results by setting $\eta = 4$. Since, $\underline{y}_d - \eta < 0 \Rightarrow Max(0, \underline{y}_d - \eta) = 0 \Rightarrow D > 0$. Thus the non-empty interval for D is $(0, 3.84)$.

Proof. (Proposition 3) This result follows by noting first that the last term in equation (13) is less than $\frac{1}{4}$. Second, $\underline{y}^*$ is bounded above by $\frac{1}{2}$, the first-best output for the low-productivity manager. This is because increasing $\underline{y}^*$ above $\frac{1}{2}$ decreases profits from the low-type manager as well as the high-type manager. At the same time, increasing $\underline{y}^*$ decreases future profits because it lowers learning. Thus $\eta - \underline{y}^* > \frac{1}{4}$ for all $\eta > 0.75 \Rightarrow \underline{D} = 0.$ ■

Example 2 *(***Optimal Debt)**: *Suppose* $\theta = 4,\ \rho = 0.7, A = 2.8225,\ \eta = 4,$ *as in Example 1 above. Note that the values of* ρ *and* θ *satisfy the necessary condition for existence of debt, given by equation (16).*

At given parameter values, by (5), $\underline{y}_d = 8.1499 \times 10^{-2} - 0.02D$. It follows from (17) that

$D^i = 10.711 \notin (0, 3.84)$, required for existence of equilibrium (see (18)). Thus $D^* = 3.84$.

$\underline{y}_d = 0.0047, \underline{y}^* = 0.16415$ and $\bar{y}^* = 7.84$.

Total profits with debt $= 5.6495$, Total profits without debt $=5.5378$.Thus debt increases the value of the firm.

Proof. (Proposition 5) First, we examine how D^i varies with the three parameters. The first order condition with respect to D (see Equation (15)) makes it clear that as ρ increases, the last two terms increase. We find that if D^i is held fixed, the first term also increases as ρ increases (details are tedious and thus omitted.). Thus $\frac{dD^i}{d\rho} > 0$. Next, as θ increases, the incentive effect of debt becomes stronger (the second term in (15) increases) while the learning effect remains unchanged. Similar to ρ, as θ increases, the first term increases as well. Thus $\frac{dD^i}{d\theta} > 0$. Finally, as η increases, the net incentive effect becomes smaller. However, the first term increases, meaning that the negative effect of bankruptcy is smaller too. The overall effect of an increase in η on D^i is positive. Next, we examine the properties of the upper bound on D, namely, $\bar{y} - \eta$.

(i) First, we show that $\frac{d\bar{y}^*}{d\rho} < 0$, thus implying $\frac{dD^*}{d\rho} > 0$ iff $D^* = D^i$.

$$\frac{d\bar{y}^*}{d\rho} = -\frac{\theta^2}{16\eta\rho}[4\frac{dA}{d\rho} - \frac{4A-1}{\rho}] < 0 iff 4\frac{dA}{d\rho} - \frac{4A-1}{\rho} > 0.$$

Substituting for A from equation (2) yields $\frac{d\bar{y}^*}{d\rho} < 0$ iff $\theta^2 > \frac{1-\rho^2}{2(1-\rho)}$. This inequality holds for all $\rho \in [0,1]$ and $\theta > 1$.Hence, $\frac{dD^*}{d\rho} > 0$ if $D^* = D^i$, $\frac{dD^*}{d\rho} < 0$ if $D^* = \bar{y} - \eta$.

(ii) We now show that $\frac{d\bar{y}^*}{d\theta} > 0$, thus implying $\frac{dD^*}{d\theta} > 0$.

$$\frac{d\bar{y}^*}{d\theta} = \theta + \frac{\theta}{8\eta}(\frac{1}{\rho} - 1) + \frac{\theta}{8\eta\rho}\left(\rho(2\theta^2 - 1) + 4(\theta\frac{dA}{d\theta} - A)\right).$$

Now

$$\frac{dA}{d\theta} = \frac{\rho\theta}{2} + \frac{(1-\rho)^2\rho}{\theta^3\left(1-\frac{\rho}{\theta^2}\right)^3}\left(\rho(1-\frac{\rho}{\theta^2})(2-\rho) + \frac{\rho}{\theta^2}(1-\rho)^2\right).$$

Substituting for $\frac{dA}{d\theta}$ and A into $\frac{d\bar{y}^*}{d\theta}$ yields,

$$\frac{d\bar{y}^*}{d\theta} = \theta + \frac{\theta}{8\eta}(\frac{1}{\rho} - 1) +$$
$$\frac{\theta}{8\eta}\left(3\theta^2 - 1 + \frac{(1-\rho)^2}{\left(1-\frac{\rho}{\theta^2}\right)^3}\left(\frac{1}{\theta^2}\left(\left(6-(1-\rho)^2\right)-1\right)(1-\frac{\rho}{\theta^2}) + \frac{\rho}{\theta^4}(1-\rho)^2\right)\right)$$
$$> 0, \text{ since } \theta > 1.$$

(iii) Finally, we show that $\frac{d\bar{y}}{d\eta} - \eta < 0$, for all ρ satisfying (16) and for all η such that

$\underline{y}^* + \eta > \bar{y} - \eta > 0$. Thus $\frac{dD^*}{d\eta} > 0$ iff $D^* = D^i$.

$$\frac{d\bar{y}}{d\eta} - \eta = \frac{\theta^2}{16\eta^2}\left(1 - \frac{1}{\theta^2} + \frac{1}{\rho}\left(-1 + \rho - \rho\theta^2 + 4A\right)\right) - 1 < 0$$

$$iff\ 1 - \frac{1}{\theta^2} + \frac{1}{\rho}\left(-1 + \rho - \rho\theta^2 + 4A\right) < \frac{16\eta^2}{\theta^2}.$$

$$\Rightarrow 1 - \frac{1}{\theta^2} + (-\frac{1}{\rho} + 1 + \frac{(1-\rho)^2}{\left(1-\frac{\rho}{\theta^2}\right)^2}\left(\frac{1}{\rho} - \frac{2}{\theta^2}\right)) < \frac{16\eta^2}{\theta^2},$$

$$\Rightarrow 2 - \frac{1}{\theta^2} - \frac{(1-\rho)^2}{\left(1-\frac{\rho}{\theta^2}\right)^2}\frac{2}{\theta^2} - \frac{1}{\rho}\left(1 - \frac{(1-\rho)^2}{\left(1-\frac{\rho}{\theta^2}\right)^2}\right) < \frac{16\eta^2}{\theta^2}.$$

This inequality holds for all ρ satisfying (16) and for all η such that $\bar{y} - \eta > 0$. ■

Example 3 *Comparative Statics Illustrations*

a. Changes in ρ : Recall that $0 < D^{*} = Min\left(D^{i}, \bar{y} - \eta\right)$.

Let $\eta = 5$ and $\theta = 4$. Substituting in A yields, $A = 4\rho + \frac{(1-\rho)^2}{\left(2-\frac{1}{8}\rho\right)^2}\left(1 - \frac{1}{8}\rho\right)$. Substituting these values in D^{i} yields the solid curve in the diagram. And substituting the same values in $\bar{y}_d - \eta$, yields the dashed curve. Thus for $\rho \in (0.6013, 0.6323)$, D*=D^{i} and increases as ρ increases. For $\rho > 0.6323$, D* = $\bar{y}_d - \eta$ and falls as ρ increases. The fall in $\bar{y}_d - \eta$, is clearer in the second panel, with the scale adjusted.

The value of D^{i} increases significantly as ρ approaches 1. For the sake of comparison with the upper bound on debt, given by $\bar{y} - \eta$, we do not show the higher values of ρ in this diagram.

Effect of Changes in ρ on D^{*} Effect of Changes in ρ on $\bar{y} - \eta$

b. Changes in θ : Let $\rho = 0.7$ and $\eta = 5$. As in part (1), D^{i} is given by the solid curve and $\bar{y} - \eta$ is given by the dashed curve.

Effect on D^* of changes in θ

Since $D^* > 0$, $\theta > 3.387$, and D $= \bar{y} - \eta$, for $\theta < 6.5$ and D $= D^i$ for $\theta > 6.5$. In either case, D^* increases as θ increases.

c. Changes in η : Let $\rho = 0.7$ and $\theta = 4$. Substituting these values in A yields $A = 2.8225$. Once again, the solid curve represents D^i and the dashed curve represents $\bar{y} - \eta$. We have also drawn the curve representing $\underline{y}^* + \eta$, represented by the dotted curve. Recall that the existence of equilibrium requires that $\underline{y}^* + \eta > \bar{y} - \eta \Rightarrow \eta > 3.8346$.

Thus, D^* decreases as η increases.