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International Parity Relationships between Germany and the United States: A Joint Modelling Approach

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International Parity Relationships Between Germany and the United States: A Joint Modelling Approach

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Abstract

This paper examines the interrelations between the purchasing power parity, uncovered interest parity, the term structure of interest rates and the Fisher real interest rate parity using cointegration analysis. Dynamic adjustment and feed-back effects are estimated jointly in a full system of equations. An important finding is that the very slow, though significant, price adjustment towards sustainable levels of real exchange rates, has been compensated by corresponding changes in the spread of the long-term bond rates. Related to this is the strong empirical support for the weak exogeneity of the long-term bond rates, signifying the importance of the large US trade deficits (i.e. the low levels of US savings) and, hence, their linkage to international finance. Altogether, the results suggest that the transmission mechanisms over the post Bretton Woods period have been significantly different from standard theoretical assumptions.

JEL Classifications: E31, E43, F31, F32.

Keywords: International Parity Conditions.

1 Introduction

Parity conditions are central to international finance and, more specifically, to many open economy macro-models such as the celebrated Dornbusch (1976) overshooting model. Although international parity conditions, such as purchasing power parity (PPP) and uncovered interest rate parity (UIP), have received considerable empirical scrutiny, very little empirical research has focussed on modelling such conditions jointly (exceptions are Johansen and Juselius (1992), Juselius (1991,1995) and MacDonald and Marsh (1997,1999)). This perhaps seems surprising since such parity conditions can be shown to be closely linked through interest rates and expected inflation. By modelling international parity conditions jointly, extra information may brought to bear on each individual parity condition, thereby increasing the likelihood of establishing well-defined results. In this paper we attempt to push this nascent literature further by jointly modelling PPP and UIP with the term spread (TS), or yield gap, for Germany against the United States, over the period 1975 to 1998. In addition to shedding light on the interaction of these parity conditions, we hope to address a number of unresolved issues.

One important issue concerns the persistence in real exchange rates. For example, a number of studies have demonstrated that for the recent floating experience real exchange rates are $I(1)$ processes (see Froot and Rogoff (1995) and MacDonald (1995) for surveys). The modelling approach adopted in this paper shows that although this non-stationarity may be removed using inflation and interest differentials, it, in turn, is an important determinant of interest differentials and inflation. A second issue, which is essentially a corollary of the first, concerns the extent to which German (European) or US variables are the driving variables in the system. For much of the post-war period, particularly during the Bretton Woods period, the US has been seen as the 'locomotive' economy. But with increased integration and convergence in Europe it may be expected that European variables, represented here by Germany, will be as important in international financial linkages as US variables. A third issue we seek to address is the extent to which 'implicit' parity conditions - namely the Fisher conditions and real interest rate parity - hold for our sample period. Thus although the linkage between nominal interest rates, as in UIP, describes capital mobility between financial centres, it is the lock between real interest rates which governs the efficiency with which savings and investment are allocated internationally. To what extent does the joint modelling of UIP, PPP and the TS shed light on this issue?

The outline of the remainder of this paper is as follows. In the next section we provide a motivational discussion of a number of parity conditions used in this paper. In Section 3 a visual interpretation of the parities is presented, while in Section 4 the econometrics of the parities is introduced. Section 5 details both the 'general-to-specific' and 'specific-to-general' testing methods used in this paper. Our tests of the parity conditions begins in Section 6 where we consider a 'small model' which excludes short term interest rates. The representation of the parities in our 'large model', which includes short term rates, is presented in section 7. The estimated long-run impacts of shocks to the system are reported in section 8. The final section of the paper contains summary results and conclusions.

2 International Parity Conditions

Perhaps the best known parity condition in international finance is that of purchasing power parity (PPP). Absolute PPP is usually written as:

$$p_t - p_t^* - s_t = ppp_t, \tag{1}$$

where p_t is the log of the domestic price level, p_t^* is the log of the foreign price level, and s_t denotes the log of the spot exchange rate (home currency price of a unit of foreign currency). For 'strong form' the term ppp_t[1] is assumed to be stationary.

The nature of the empirical support for PPP is very dependent on the sample period chosen in the following sense: if the time it takes for ppp to return to its steady-state value is very long, say ten years, then we need a long sample to get statistically significant mean reversion effects[2]. Over century long historical data spans, there is mounting evidence that a version of the strong-form PPP is valid, but with a very small adjustment coefficient (see, for example, Froot and Rogoff (1995) and MacDonald (1995)). For the recent floating experience the sample period is too short for such a small adjustment coefficient to be statistically significant. Hence, ppp_t should statistically be treated as an $I(1)$ process.

There are in fact a number of potential reasons why the adjustment to strong-form PPP is so slow. For example, the correspondence between the measured prices series -usually the CPI - and the true or theoretical price series might be weak, institutional differences might be important.

[1] Note that the ppp term is also the (logarithm) of the real exchange rate. We prefer to use the label ppp in this paper because we are adopting a parity perspective and also because we do not model the real exchange rate in terms of real fundamentals.

[2] See Juselius (1999) for a discussion of the statistical versus economic interpretation of unit root econometrics.

Another potentially more interesting objection to traditional PPP, is that important real determinants of real exchange rates may introduce a stochastic trend into real exchange rates. The interpretation is that the persistence in deviations from PPP is due to the existence of important real factors working through the current account, such as productivity differences, net foreign asset positions and fiscal imbalances. This hypothesis has received some empirical support by researchers who have explicitly modelled the real determinants of real exchange rates (see the papers contained in MacDonald and Stein (1999)).

However, through the balance of payments constraint we know that any current account imbalance generated by such movements has to be financed through the capital account. The implication of this is that PPP is likely to be strongly related with another parity condition, namely uncovered interest rate parity (UIP) (see Johansen and Juselius (1992), Juselius (1991,1995) and MacDonald and Marsh (1997,1999)). Therefore, by combining the two parity conditions we may pick up the influence of the real factors on PPP indirectly.

The condition of UIP may be stated as:

$$E_t \Delta s_{t+l} - i_t^l + i_t^{l*} = 0, \tag{2}$$

where i_t^l denotes a long term bond yield with maturity $t+l$, E_t denotes the conditional expectations operator, on the basis of time-t information. Assuming that expectations are formed rationally:

$$\Delta s_{t+l} = E_t \Delta s_{t+l} + \varepsilon_{t+l}, \tag{3}$$

where ε_t is a white noise error, we may write a testable version of (2) as:

$$\Delta s_{t+l} - i_t^l + i_t^{l*} = \varepsilon_{t+l}. \tag{4}$$

A number of researchers (see, for example, Cumby and Obstfeld (1981)) have tested this version of UIP and essentially find that ε_t is non-stationary. However, when the UIP condition is modelled jointly with PPP more satisfactory results have been obtained in the sense that deviations from the conditions are stationary and the sign of the coefficients conform with priors. Nevertheless, the empirical evidence strongly suggest that the assumption of market clearing underlying (2) would have to be replaced by an assumption of price adjustment.

There are two further parity conditions, related to PPP and UIP, which are useful in trying to understand some of the puzzles noted in the introduction. The first relates to the term structure of interest rates, or the so-called yield gap. The interest differential in (2) is defined with respect to interest rates with a long-term maturity, although it could equally be written for interest rates with a short-term maturity. However, both short and long bond yields will be linked via a term structure of interest rates relationship, implying that it is unnecessary to define UIP twice for both short and long rates. In the standard expectations model of the term structure 'the' long rate is a weighted average of the current and expected future spot rates. Short rates in this view of the maturity spectrum 'drive' long rates. As Campbell and Shiller (1987) note, the standard expectations model of the term structure implies that the, so-called, term spread (TS) or yield gap should be stationary. The TS is defined as:

$$i_t^l - i_t^s = v_t, \tag{5}$$

where i_t^s denotes the yield on a short maturity bond and v_t generally denotes a random error term. However, the TS model has failed on a variety of empirical tests (see Campbell, *1995)* and v_t has often been found nonstationary. As in the PPP relationship, the average mean reversion time is usually long and stationarity can only be accepted in very long spans of data such as century long data.

It is conventional to think of nominal interest rates being decomposed into real and expected inflation components using the Fisher decomposition. For long bond yields this may be written as:

$$i_t^l = r_t^l + E_t \Delta p_{t+l}, \tag{6}$$

where r denotes the real interest rate. The conventional interpretation of the causality in this relationship would be that excess monetary growth causes inflation and this, combined with a stationary real interest rate (in some models the real rate is assumed to be constant), is reflected in the nominal interest rate. Although a similar decomposition may be written for short rates:

$$i_t^s = r_t^s + E_t \Delta p_{t+s}, \tag{7}$$

the interpretation placed on the latent causality would be different. For example, it is widely accepted that short rates are set by central bank policy, and in the presence of sticky goods prices this shows up in one-to-one movements of the real interest rate. Combining (6) with (7) gives:

$$(r_t^l - r_t^s) = (i_t^l - i_t^s) - E_t(p_l - p_s) \tag{8}$$

which shows that if expected inflation between time s and l is nonstationarity, then the yield gap would also have to be nonstationary for the real rate differential to be stationary. Since inflation is found to be nonstationary in itself this seems very plausible. In this view (6), (7) and (8) are likely to be non-stationary, or $I(1)$.

The final parity condition, the real interest rate parity (RIP) is implicit in the above relationships. In particular, if we take the UIP condition for long bond yields and the relevant Fisher conditions for the home and foreign country, we have (in ex-post terms):

$$r_t^l - r_t^{l*} = v_t. \tag{9}$$

The empirical literature on RIP usually focuses on testing if the restrictions necessary to move from (2) and (6) to (9) actually hold in the data. The majority of such studies find that RIP is strongly rejected for most country pairings (see, for example the overview in Hallwood and MacDonald (1999)).

We now draw out the implications for the modelling of PPP, UIP and TS under assumption that the simple parity conditions are nonstationary and that the very slow adjustment to sustainable real exchange rates is the basic reason for this nonstationarity. We formulate the following hypothetical adjustment relations for the spot exchange rate:

$$\Delta s_t = \omega_1 \Delta(p_t - p_t^*) + \omega_2(i_s - i_s^*) + \omega_3 ppp_{t-1} + v_t \tag{10}$$

where actual depreciation can be related to a change in inflation differential, to the spread in short-term interest rate (a monetary policy intervention effect) and to an (probably very small) adjustment to real exchange rates with adjustment parameters ω_1, ω_2 and ω_3, respectively. If the expected exchange rate in (2) is formed by using (10) we can now derive a relationship combining the PPP and the UIP conditions:

$$i_t^l - i_t^{l*} = \omega_1 E_t(\Delta p - \Delta p^*)_{t+1} + \omega_2 E_t(i_s - i_s^*)_{t+1} + \omega_3 ppp_t + v_t. \tag{11}$$

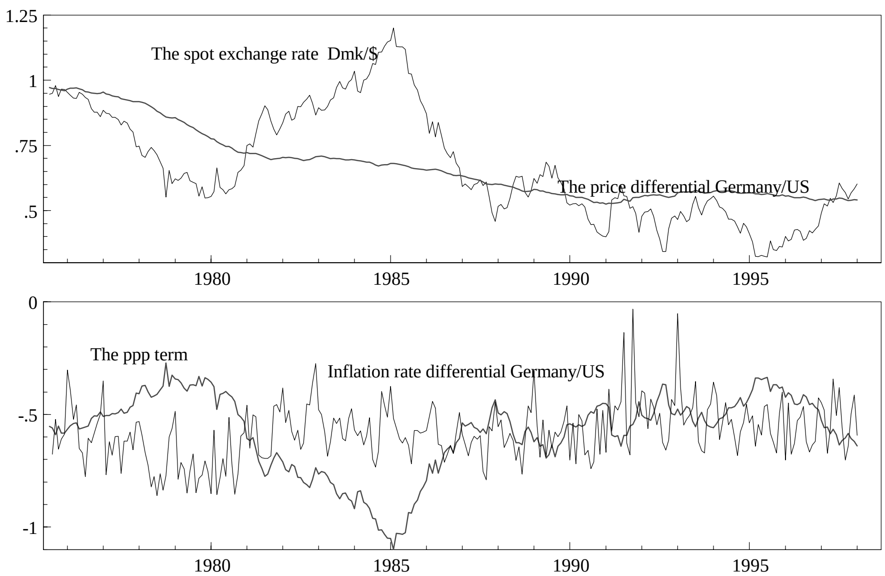
If the difference between $E_t(x_{t+1})$ and x_t is white noise (i.e. agents do not make systematic forecast errors) the cointegration results will be unaffected when replacing expectations with actual values:

$$i_t^l - i_t^{l*} = \omega_1(\Delta p - \Delta p^*)_t + \omega_2(i_s - i_s^*)_t + \omega_3 ppp_t + v_t. \tag{12}$$

Thus, we note that implicit in (12) is all the other parity relationships: the two Fisher conditions, international real interest rate parity condition, the ppp condition, and the term structure condition. For example, (12) becomes the real long-term interest parity relationship for $\omega_1 = 1$ and $(\omega_2 = 0, \omega_3 = 0)$. By modelling these these relationships jointly we can test the stationarity of the simple parity conditions as special cases of (12). If these are rejected we can test whether combinations of the parity relationships become stationary.

3 An ocular analysis of the parities

In this section we offer a first pass at how closely the various parity conditions considered above hold. We also introduce some of the relevant institutional background which will have a bearing on our econometric results.

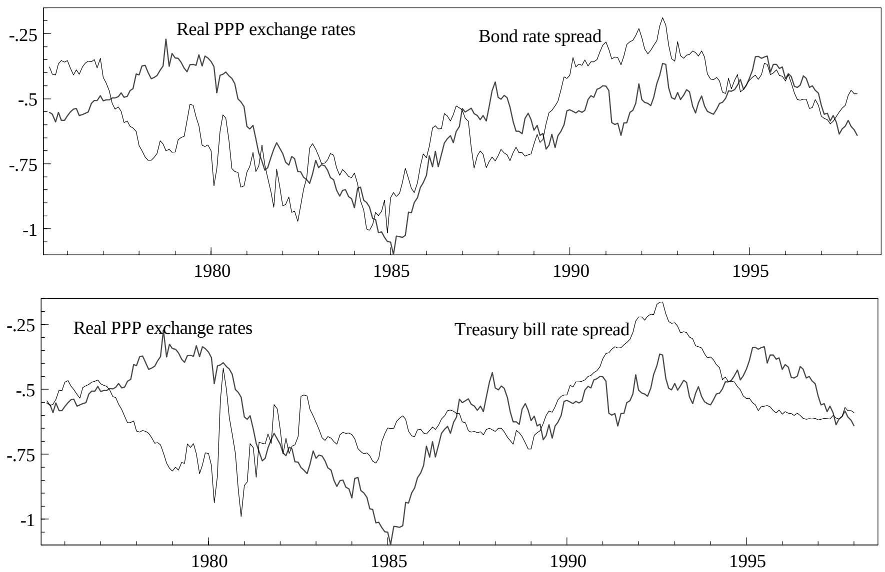


The monthly price differential and the spot exchange rate (upper panel) and the ppp term and the inflation rate differential (lower panel) between Germany and USA .

The salient feature of the graphs in Figures 1, 2, and 3[3] is the slow adjustment back to the parities. Figure 1, upper panel, shows clearly that the spot exchange rate does not closely mirror the price differential between Germany and the USA, although there seems to be a tendency to follow the same (very) long-run movements. The much greater variation in the spot exchange rate as compared to the price differential is quite striking[4]. In particular, the period between 1980 and 1985 (showing up here as a depreciation of the mark) is notable. Lothian (1997), for example, has argued that the behavior of the dollar in this period is likely to confound any test of PPP for the recent floating period when the US dollar is used as the numeraire currency. Given the importance of this episode for the kinds of tests conducted in this paper, we believe it merits a brief discussion here.

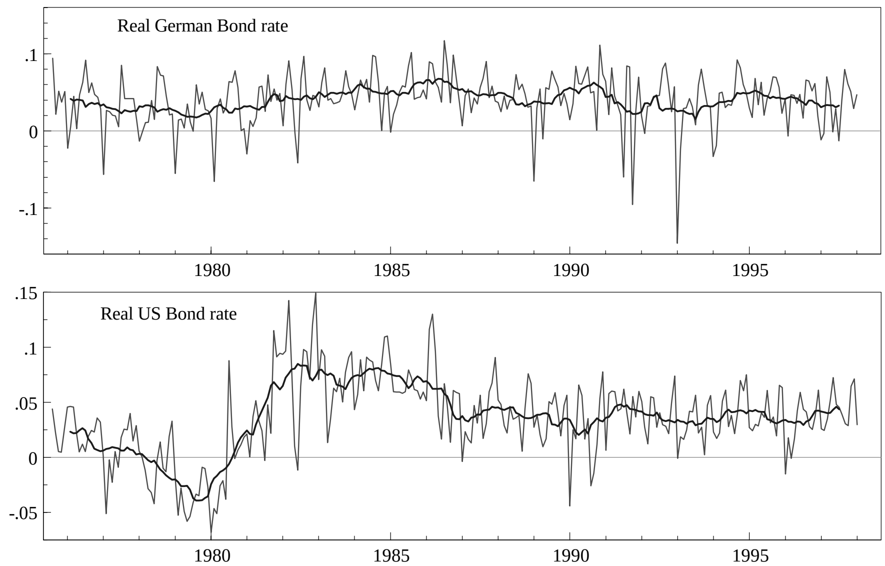
[3] The measurements of the variables discussed in this section are defined in Section 5.

[4] See for example Krugman (1993) for an economic explanation.



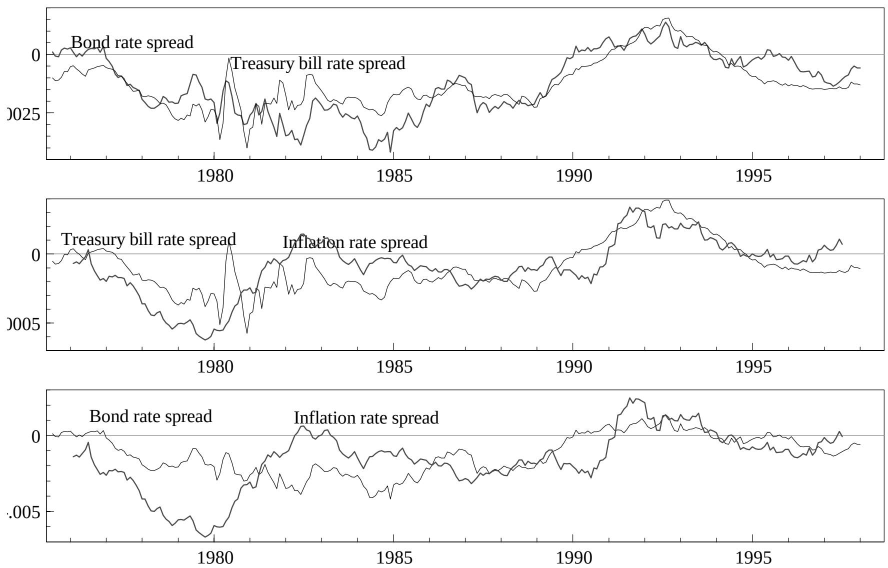
The ppp term relative to the bond rate spread (upper panel) and to the Treasury bill rate spread (lower panel).

The dollar appreciation was kick-started by the effects on interest rates of the so-called 'Reagan Experiment' of increasing the US fiscal deficit. However, the prolonged nature of the appreciation would seem to be unwarranted solely in terms of a real interest differential or, indeed, other fundamentals which were extant at the time, such as portfolio balance or 'safe-haven' effects (see MacDonald (1988)). The consensus view seems to be that in large part it was a speculative bubble, unrelated to economic fundamentals, which drove the currency to such stratospheric levels. However, whatever the actual cause of the dollar's rise we believe that ultimately it could not have behaved as it did if it was not accepted as the key reserve currency in the international monetary system. The role of the dollar as a reserve currency is an important element in how we interpret our results.



Real yearly bond rates (in 0.01%) for Germany (upper panel) and USA (lower panel) together with ±6 months moving averages

The lower panel of Figure 1, shows that the long movements of the *ppp* cannot directly be related to an adjustment of the inflation rates; the inflation spread appears too small to facilitate a long-run adjustment towards a stationary level of real exchange rates. Figure 2 relates the ppp_t term to the bond rate spread in the upper panel and to the Treasury bill rate spread in the lower panel. There is a quite remarkable co-movement in the long-run behavior of the real exchange rate and the long bond differential. However, there is not the same close correspondence with respect to the short-term Treasury bill rates. This in large measure reflects the nature of these two yields. The latter are driven by short term policy considerations, whereas the former are market determined and have a term to maturity which more closely matches the long persistence in the real exchange rate (we discuss the importance of relative interest rates further below). Figure 3 demonstrates the large variation in real bond rates over this period. This is particularly so for the US real bond rate, which has varied between -7% and +15%. These are huge variations considering that theoretically it is usually assumed to be constant!



The monthly bond rate spread and Treasury bill rate spread (upper panel), the inflation rate spread relative to the treasury bill rate spread (middel panel) and to the bond rate spread (lower panel).

Finally, Figure 4 compares the spread of the bond rates and of the Treasury bill rates in the upper panel, and the inflation rate spread with treasury bill rate spread (middle panel) and the bond rate spread (lower panel), respectively. There are clearly periods in which both spreads mirror relative inflation quite closely and periods in which they diverge and the real interest rate spreads open up. The extent to which such real interest rate spreads are consistent with real interest rate parity is something we investigate formally in Sections 6 and 7.

The graphical inspection demonstrated a fair degree of persistence both in the spreads and the parities which is inconsistent with the stationarity assumption of the parities made in theoretical models. Econometrically, we will treat these persistencies as stochastic trends and use cointegration analysis to find out how they are related. This is based on the simple idea that a persistent imbalance in one place should create a corresponding imbalance in another. The purpose is to use the econometric analysis to suggest reasons why these simple parity relationships are inadequate on their own and how they could be modified to describe the variation in the data. In the next section we introduce an analytical framework for the econometric analysis of the international parities both in the ideal situation of efficient market clearing and fast adjustment, and in the more realistic situation of slow adjustment and persistent deviations from steady-states. The methodological approach is similar to Juselius (1999a).

4 The econometrics of the parities

In a world with no market rigidities, no trade barriers, no restrictions on capital movements, stationary transportation costs, fully integrated capital and goods markets, etc. we would expect no more than two nominal trend driving the prices of goods and capital in two countries. For example, the first could describe the cumulated effect of demand and supply shocks on prices and the second the cumulated effect of differences between monetary policy interventions in the two countries. Assuming that price inflation is empirically $I(1)$, as is frequently found in empirical work, the data generating process could then be represented as:

$$\begin{bmatrix} p_t \\ p_t^* \\ s_t \\ i_t^l \\ i_t^{l*} \\ i_t^s \\ i_t^{s*} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_{1i} \end{bmatrix} + \begin{bmatrix} 1\,0 \\ 1\,1 \\ 0\,1 \\ 1\,0 \\ 1\,0 \\ 1\,0 \\ 1\,0 \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + X_0. \tag{13}$$

Hence:

$$\begin{bmatrix} \Delta p_t \\ \Delta p_t^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \end{bmatrix} + \widetilde{X}_0$$

where $\Sigma\Sigma u_{1i}$ represents a second order general trend in price levels, and, consequently, Σu_{1i} the corresponding first order trend in inflation rates (also common to nominal interest rates), and Σu_{2i} is a stochastic trend measuring the relative impact of different monetary policy between the two countries. Finally X_0 is a stationary component. In this case $p_t - p_t^* \sim I\,(1)$, and, hence, $\Delta p_t \sim I(1)$, $\Delta p_t^* \sim I(1)$, $s_t \sim I\,(1)$, and $p_t - p_t^* + s_t \sim I\,(0)$. Because inflation and interest rates share the same stochastic trend it follows that $i_t^l - \Delta p_t \sim I(0)$, $i_t^{l*} - \Delta p_t^* \sim I\,(0)$, $i_t^s - \Delta p_t \sim I(0)$, $i_t^{s*} - \Delta p_t^* \sim I\,(0)$, $i_t^s - i_t^l \sim I(0)$, $i_t^{s*} - i_t^{l*} \sim I(0)$, $i_t^l - i_t^{l*} \sim I(0)$, and $i_t^s - i_t^{s*} \sim I(0)$.

The economy described in (13) is characterized by PPP prevailing as a stationary steady-state relation, there are no persistent price rigidities and nominal exchange rates reflect (mirror) relative prices. Real interest rates and the interest rate spreads are stationary, i.e. deviate from their equilibrium position by a stationary error.

As the ocular analysis of the data demonstrated, (13) is a too simplified representation to be an adequate description of the data and has to be modified accordingly. The trace tests in Section 6 showed that three, instead of two, common stochastic trends were needed to describe the data in the model without the short interest rates and four in the full model. The existence of four common stochastic trends seems *a priori* less likely unless trade barriers and binding regulations had prevented international goods and capital markets from clearing. The trade of goods and capital has been reasonably free between Germany and USA, hence two common trends would a priori have been more likely.

Nevertheless, we will argue that the USA is exceptional in the sense that the US dollar is the main reserve currency in the world and the demand for reserve currency is likely to have permanent (long lasting) effects on exchange rates and US interest rates. This is likely to show up through two channels in our model. First, the role of the dollar as a reserve currency means that agents were prepared to hold dollars for long periods with little or no change in its relative price because of its special status. Hence, there was no great pressure to restore current account balance. Second, and relatedly, although US interest rates are clearly closely integrated with interest rates in other countries, the role of the dollar as a reserve currency and safe haven implies that US interest rates have not had to rise by as much as, say, UK or French rates in order to finance a given current account deficit. We interpret the third trend, therefore, as a 'safe haven' or portfolio balance effect.

The fourth trend trend was found to be specifically related to the short-term interest rates and is, therefore, likely to describe the cumulative impact of monetary intervention shocks.

To facilitate the economic interpretation of the subsequent empirical results we present below a modified version of (13) based on three cointegration relations and four common trends. The restrictions imposed are consistent with the time-series properties of the data in the sense that changing them would violate the test results reported in Sections 6 and 7. Clearly, more restrictions would have to be imposed if (14) would be used as a structural model, rather than as a convenient summary description of the data.

$$\begin{bmatrix} p_t \\ p_t^* \\ s_t \\ i_t^l \\ i_t^{l*} \\ i_t^s \\ i_t^{s*} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ 0 & d_{32} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_{1i} \\ \Sigma\Sigma u_{2i} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & 0 & c_{14} \\ c_{21} & c_{22} & 0 & c_{15} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & 0 & c_{44} \\ c_{51} & c_{52} & c_{53} & c_{54} \\ c_{61} & c_{62} & 0 & c_{64} \\ c_{72} & c_{72} & c_{73} & c_{74} \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \\ \Sigma u_{3i} \\ \Sigma u_{4i} \end{bmatrix} + ... \qquad (14)$$

Hence:

$$\begin{bmatrix} \Delta p_t \\ \Delta p_t^* \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ 0 & d_{32} \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} \\ \Sigma u_{2i} \end{bmatrix} + .. \tag{15}$$

The graphs of the German and US price level and inflation rate in Appendix II clearly suggest that prices are approximately $I(2)$ and, hence, inflation rates $I(1)$. The latter is strongly supported by the tests in Section 6, Table 2. Furthermore, the graph of the price differential in Figure 1 seems to suggest that $p_t - p_t^* \sim I(2)$, implying that $\Delta p_t - \Delta p_t^* \sim I(1)$. (Cf. Figure 1, lower panel.) The latter is strongly confirmed by the statistical tests in Section 6, Table 3. But if the two prices series have exhibited different long-run movements, resulting in a long-run stochastic trend in the price differential, we would expect the nominal exchange rate to exhibit a similar long-run stochastic trend. In this case nominal exchange rates should also be $I(2)$, implying two $I(2)$ trends. It turns out that the spot exchange rate, though behaving very differently from the price differential, seems to move along a similar long-run trend.

Altogether, this suggests that prices are influenced both by a long-run nominal price trend, $\Sigma\Sigma u_{1i}$, but also by another long-run trend, $\Sigma\Sigma u_{2i}$, which is assumed to reflect real imbalances between the two countries. The hypothesis here is that this trend is primarily related to national savings imbalances in the United States (arising from either the private or public sectors) which have been so important during our sample period. Since a current account deficit has to be counteracted by a corresponding change in the capital account, the second stochastic trend is likely to be associated with permanent shocks influencing the capital account. The resulting imbalances will require an appropriate adjustment in long term yields, which will have implications for the exchange rate / relative price configuration. This is the rational for $\{d_{12}, d_{22}, d_{32} \neq 0\}$ in (14).

The large and persistent deviations from the long-run trend in the price differential (cf. Figure 1) suggests the presence of an additional stochastic trend, Σu_{3i}, influencing the spot exchange rate but not the two prices. This is the rationale for $c_{13} = c_{23} = 0$ in (14). Hypothetically we interpret the third trend as a US$ reserve currency trend. Based on the graphs of the real exchange rate and the bond rate spread in Figure 2 it seems likely that the 'reserve currency' stochastic trend, Σu_{3i}, has affected the US bond rate and the spot exchange rate similarly. This is the rationale for $c_{43} = 0, c_{33} = c_{35} = c_{37} \neq 0$ in (14).

Formulation (14) can now be used to summarize the properties of the data. The order of integration of the variables has also been formally tested in Sections 5 and 6 and the tests support the results in (16):

$$\begin{array}{ll}
\{p_t, p_t^*, s_t, (p_t - p_t^*)\} \sim I\,(2) & \\
\{\,\Delta p_t, \Delta p_t^*, \Delta s_t, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}\} \sim I\,(1) & \\
(\,\Delta p_t - \Delta p_t^*), (i_t^l - i_t^{l*}), (i_t^s - i_t^{s*}) \sim I\,(1) & (16)\\
(\,i_t^l - \Delta p_t), (\,i_t^{l*} - \Delta p_t^*),\,(\,i_t^s - \Delta p_t), (\,i_t^{s*} - \Delta p_t^*) \sim I\,(1) & \\
(i_t^s - i_t^l), (i_t^{s*} - i_t^{l*}), (p_t - p_t^* - s_t) \sim I\,(1) &
\end{array}$$

How can we use this to address questions related to the joint working of PPP and the UIP? From (14) it appears that if $(d_{11} - d_{21}) = 0$ and $(d_{12} - d_{22}) = d_{32}$, then $(p_t - p_t^* - s_t) \sim I\,(1)$. In this case it is a function of all four stochastic $I(1)$ trends. Furthermore, it appears that $(i_t^l - i_t^{l*})$ is also a function of all four stochastic trends and, hence, can (but need not) be cointegrated with the *ppp* term. Hence, (14) is consistent with the finding in Section 6 that the following relation is stationary:

$$\{(i_t^l - i_t^{l*}) - \omega_1(\Delta p_t - \Delta p_t^*) - \omega_5(p_t - p_t^* - s_t)\} \sim I\,(0)\,. \tag{17}$$

Notice that $\omega_1 = 1$ (as we find) corresponds to a relation between real interest rate parity and the real exchange rate. In this case the real interest rate parity condition would hold as a stationary relation only for stationary real exchange rates.

Note, however, that $(\Delta p_t - \Delta p_t^*)$ is only a function of two stochastic trends, Σu_{1i} and Σu_{2i}, and, therefore, cannot be directly cointegrated with the *ppp* term if (14) is correct. This is also what we find in Section 6[5].

Therefore, using cointegration techniques we can test the consistency of (14) with the stationarity of (17) as well as various other hypotheses relating the *ppp* term to the short or long interest rate spread, short or long real interest rates, etc. Furthermore, by imbedding the stationary relations in an equilibrium error-correcting model, we will also be able to find out which variables have taken the burden of adjustment and which have not. All this will be discussed further in Sections 6-7.

5 A 'general to specific' and a 'specific to general' approach

All test and estimation results are based on the VAR model with a constant term, μ, seasonal dummies, S_t, and intervention dummies, D_t, given by:

$$\begin{gathered}\Delta^2 x_t = \Gamma_1 \Delta^2 x_{t-1} + \Gamma \Delta x_{t-1} + \Pi x_{t-2} + \mu + \Phi_1 S_t + \Phi_2 D_t + \varepsilon_t, \\ \varepsilon_t \sim N_p(0, \Sigma\ ),\ t = 1, ..., T\end{gathered} \tag{18}$$

[5] If the *ppp* and $(\Delta p - \Delta p^*)$ were found to be cointegrated, then (12) would have had to be modified accordingly.

where x_t is a vector of monthly variables observed for $t =$ *1975:7-1998:1.* The full set of variables is defined by:

p_t = the German, or 'home', price index,
p_t^*= the US, or 'foreign', price index,
i_t^l = the German long bond yield,
$i_t^{l,*}$= the US long bond yield,
s_t= the spot exchange rate, defined as DM/$,
i_t^s = the German 3 month Treasury bill rate,
$i_t^{s,*}$= the US 3 month Treasury bill rate.

All of the data used in this study have been extracted from the International Monetary Funds CD-Rom disc (December 1998). Both price series are Consumer Prices (line 64), the long interest rates are 10 year bond yields (line 61), the short rates are Treasury bill rates (60c), and the exchange rate is the end of period rate (line ae). All variables, apart from the interest rates (which appear as fractions) are in natural logarithms. The graphs of the variables in levels and in differences are given in the Appendix.

In (18) all parameters $\{\Gamma_1, \Gamma, \Pi, \mu, \Phi, \Sigma\}$ are unrestricted and ML estimates can be obtained by OLS. However, (18) is heavily overparametrized and is a convenient way of describing the covariances of the data rather than a meaningful economic model. By imposing restrictions on the VAR such as reduced rank restrictions, zero parameter restrictions, and other linear or nonlinear parameter restrictions, the idea is to arrive at a more parsimonious model with economically interpretable coefficients. This is essentially the 'general to specific' approach to econometric modelling discussed in Hendry and Mizon (1993) and Juselius (1993).

Because of the generality of the VAR formulation, adding one variable to a p-dimensional VAR system introduces $(p+1) \times k$ new parameters. When the sample is small, typically 50-100 in quarterly macroeconomic models, adding more variables becomes prohibitive. As argued in Juselius (1992) one solution is to model smaller sub-systems and then combine the results of the sub-systems into a larger model. However, even when the sample is large enough for a bigger system to be estimated, it is nevertheless often advantageous to do the VAR analysis first for a smaller set of variables and then gradually increase the system. Because the cointegration property is invariant to extensions of the information set such a procedure is likely to facilitate the identification of cointegration relations without loss of information. If cointegration is found within a smaller set of variables, the same cointegration relations will be found in an extended set. Furthermore, the gradual expansion of the information set facilitates an analysis of the sensitivity of the results associated with the '*ceteris paribus*' assumption, in particular its importance for the empirical analysis of the smaller set of variables. We call this approach 'the specific-to-general in the choice of variables'.

As an illustration of this principle we first focus on the cointegration properties of the DM/US$ spot exchange rates, German and US prices and long-term bond rates and then, in a second stage, include the treasury bill rates into the analysis. The motivation for first including the long-term bond rates rather than the short-term treasury bill rates is partly based on the graphical analysis of Section 2, and partly because the long-term movements in real exchange rates are likely to be more informative about the long-term than the short-term interest rates. Using this approach we are able first to isolate the relations relevant for the longer term movements in the real exchange rates and then to investigate the 'value added' of the short-term interest rates for the foreign transmission mechanisms.

The choice of data vector in the first step then becomes:

$$x_t' = [(s_t, p_t, p_t^*, i_t^l, i_t^{l*}]. \tag{19}$$

Similar systems to (19) have been analysed previously by Johansen and Juselius (1992), Juselius (1991,1995) and MacDonald and Marsh (1997,1999) (although the maturity of interest rates used varies across these studies). As appears from the graphical display in Appendix II, the price variables and, possibly, the spot exchange rates exhibit typical $I(2)$ behavior. The hypothesis that x_t is $I(2)$ can be formulated within the VAR models as two reduced rank hypotheses:

$$\Pi = \alpha\beta' \tag{20}$$

$$\alpha_\perp'\Gamma\beta_\perp = \zeta\eta' \tag{21}$$

where α, β are $p \times r$ and ζ, η are $p - r \times s_1$ matrices. See Johansen (1991) for further details. By solving the first reduced rank problem we get information about the number of stationary cointegrating relations and from the second about the number of second order stochastic trends and how they influence prices and exchange rates, as discussed in Section 4 and exemplified by (14).

The two hypotheses were tested using the likelihood ratio test procedure in Rahbek, Kongsted, and Jorgensen (1998). The results suggested that the reduced rank of Π was two, i.e. $r = 2$, and that of $\alpha_\perp'\Gamma\beta_\perp$ was also two, i.e. $s_1 = 2$. Altogether this is consistent with $p - r = 3$ common stochastic trends of which one is $I(2)$. Based on further estimates of the $I(2)$ model (not shown here) the assumption of one common $I(2)$ trend would have been inconsistent with spot exchange rates and prices being cointegrated, $CI(2,1)$. The problem seemed to arise from relative prices being closer to the $I(2)$ boundary than the spot exchange rates as illustrated by graphs of Figure 1.

A priori one would expect the spot exchange rate to mirror the movements in the price differential, such that $(p_t - p_t^* - s_t)$ is at most an $I(1)$ variable. The results of the $I(2)$ analysis clearly suggested that $\{p_t, p_t^*\} \sim I(2)$, and that $(p_t - p_t^*) \sim I(2)$[6]. A necessary condition for $(p_t - p_t^* - s_t) \sim I(1)$ is that $s_t \sim I(2)$ and cointegrated with $(p_t - p_t^*)$. There was some evidence for this to be the case, but only under the assumption of two common stochastic $I(2)$ trends. Therefore we maintain the hypothesis (14) as our preferred case, though admitting that the econometric evidence of the second $I(2)$ trend was not empirically robust[7].

The empirical analysis of the first data set will be based on the *ppp* transformed vector:

$$[ppp, \Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}] \sim I(1) \tag{22}$$

where $ppp = (p_t - p_t^* - s_t)$. If the *ppp* restriction $(1, -1, -1)$ had been acceptable in all cointegrating relations in the $I(2)$ model of (19) the VAR model based on (22) would have implied no loss of no long-run information (see Juselius and Toro, 1999). However, based on test results not reported here, the joint restrictions were not data consistent and some information (mostly about the mechanisms governing the nominal exchange rate) is lost by moving to (22). The deviations from constant *ppp* are very large in absolute terms compared to the remaining variables in (22). Therefore, the *ppp* term has been divided by 100 to avoid getting very small coefficients in absolute magnitude. Nevertheless, the interpretation of the results are for the original *ppp* term.

Another possibility would have been to perform the analysis based on the differenced data vector:

$$[\Delta s_t, \Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}] \sim I(1). \tag{23}$$

A VAR model based on (23) would be $I(1)$, but all long-run information in the levels of prices and the exchange rate would have been removed by differencing. Using vector (23) instead of (22) would have resulted in a greater informational loss and, therefore, is not acceptable from an econometric point of view.

[6] This is confirmed by the test result in Table 3 that $(\Delta p_t - \Delta p_t^*) \sim I(1)$.

[7] Inference in the I(2) model is based on asymptotic theory and we still need a more complete understanding of the finite sample properties in particular when cointegration is borderline. A more detailed I(2) analysis of (19) is, therefore, left to future research.

6 Testing the parities in the small model

It appears from the graphs of the differenced variables in Appendix II that the normality assumption underlying (18) is not satisfied for many of the marginal processes, particularly not for the short-term treasury bill rates, signifying both the high volatility in the period of money stock targeting by the Fed and many monetary interventions in this period. To secure valid statistical inference we need to control for those intervention effects that fall outside the normality confidence bands. If a residual larger than $|3.5\sigma_{\varepsilon}|$ corresponds to a known intervention, we include it in the information set as a dummy variable. The VAR model based on (23) needed the following dummy variables:

$$D_t' = [Di78.10, D80.07, Di80.02, Di80.03, D82.10, Di84.12, D91.03, Ds91.03]$$

where $Dixx.yy$ is a $..., 0, 1, -1, 0, ...$ dummy measuring a transitory intervention shock in 19xx.yy, $Dxx.yy$ is a $..., 0, 1, 0, ...$ dummy measuring a permanent intervention shock, $Dsxx.yy$ is a $..., 0, 0, 1, 1, 1, ...$ dummy restricted to lie in the cointegration space measuring a shift in the cointegration mean. A more detailed description can be obtained by the authors.

As a first check of the statistical adequacy of model (18) we report some multivariate and univariate misspecification tests in Table 1. A significant test statistic is given in bold face. We also report the estimated eigenvalues of the Π matrix, as well as the five largest roots of the characteristic polynomial.

The multivariate LM test for first order residual autocorrelations is not significant, whereas multivariate normality is clearly violated. Normality can be rejected as a result of skewness (third moment) or excess kurtosis (forth moment). Since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis we report the univariate Jarque-Bera test statistics together with the third and fourth moment around the mean. It turns out that the rejection of normality is essentially due to excess kurtosis, and hence not so serious for the estimation results. The $ARCH(2)$ tests for second order autoregressive heteroscedastisity and is rejected for all equations except the US bond rate. Again cointegration estimates are not very sensitive to $ARCH$ effects. The R^2 measures the improvement in explanatory power relative to the random walk hypothesis, i.e. $\Delta x_t = \varepsilon_t$. They show that with this information set we can explain quite a large proportion of the variation in the inflation rates, but to a much lesser extent the variation in the bond rates and the real exchange rates.

Table 1: Misspecification tests and characteristic roots

Multivariate tests:					
Residual autocorr. LM_1	$\chi^2(25)$	$=$	21.7	p-val.	0.65
Normality: LM	$\chi^2(10)$	$=$	40.6	p-val.	0.00
Univariate tests:	Δp_t	Δp_t^*	Δi_t^l	Δi_t^{l*}	Δppp
ARCH(2)	0.37	6.85	0.64	**14.34**	3.66
Jarq.Bera(2)	**7.58**	**8.49**	**12.77**	**15.17**	0.89
Skewness	0.22	-0.04	0.21	0.22	0.04
Ex. Kurtosis	0.75	0.82	1.08	1.22	0.17
$\hat{\sigma}_\varepsilon \times 0.01$	0.18	0.17	0.01	0.02	2.75
R^2	0.71	0.51	0.37	0.38	0.30
Eigenvalues of the trace test	0.41	0.19	0.06	0.04	0.02
Trace test	231.1 (75.7)	88.7 (53.4)	32.8 (34.8)	15.7 (20.0)	4.5 (9.1)
Modulus of 5 largest roots:					
Unrestricted model	0.98	0.98	0.95	0.53	0.42
$r = 3$	1.00	1.00	0.96	0.52	0.44
$r = 2$	1.00	1.00	1.00	0.48	0.48

The cointegration rank can be seen as an indication of how well markets adjust and, therefore, of market barriers. Both the trace test and the roots of the characteristic polynomial support the choice of $r = 2$, consistent with representation (14) in Section 4. As a sensitivity check, the roots under the choice $r = 3$ are also reported in Table 1. In this case a large root remained in the model supporting the choice of $r = 2$.

Table 2 reports some tests of the individual variables and their role in the system. The test of long-run exclusion investigates whether any of the variables can be excluded from the cointegration space, implying no long-run relationship with the remaining variables. It can be formulated as a zero row in β, i.e.: $H_\beta^i : \ \beta_{ij} = 0, \quad j = 1, ..., r$, where H_β^i is the hypothesis that the variable x_i, $i = 1, ..., p$, does not enter the cointegration space.

The test of long-run weak exogeneity investigates the absence of long-run levels feed-back and is formulated as a zero row of α, i.e. $H_\alpha^i : \ \alpha_{ij} = 0, \ \ j = 1, ..., r$, where H_α^i is a hypothesis that the variable x_i, $i = 1, ..., p$, does not adjust to the equilibrium errors $\beta_i' x_t$, $i = 1, ..., r$. If accepted, the variable in question can be considered a driving variable in the system: it 'pushes' the system, but is not being 'pushed' by it.

Finally, the test of stationarity investigates whether any of the variables can be assumed stationary by itself by testing whether the variable in question corresponds to a unit vector in the cointegration space. It

Table 2: Tests of hypothesis about some properties of the system variables

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp	$\chi^2(\nu)$
Long-run exclusion:	124.8	54.1	15.9	8.2	25.8	$\chi^2(2) = 6.0$
Long-run weak exogeneity	117.8	37.6	**0.6**	**2.4**	**2.0**	$\chi^2(2) = 6.0$
Stationarity:	26.9	27.6	44.5	41.6	49.0	$\chi^2(3) = 7.8$

is formulated as $\{\beta\} = \{b, \psi_1\}$, where b is a unit vector and ψ_1 is a $p \times (r-1)$ vector of unrestricted coefficients. Accepting the hypothesis implies that the variable in question can be considered $I(0)$.

It appears that no variable can be excluded from the cointegration space, that the German long-term bond rate, the *ppp* exchange rate, and the US long-term bond rate can be assumed individually weakly exogenous. The test of all three being jointly weakly exogenous was accepted based on $\chi^2(6) = 6.59$, with a p-value of 0.36. Thus, the data strongly support that inflation rates are adjusting to the long-term interest rates and real exchange rates, but not vice versa. We interpret this as evidence against the Fisher real interest rate hypothesis, in which inflation supposedly drives nominal interest rates. Finally, none of the variables can be considered stationary over the sample period.

6.1 Single cointegration hypotheses

The hypotheses reported in Table 3 are of the form $\beta = \{H\phi_1, \psi_1\}$, i.e. they test whether a single restricted relation is in $sp(\beta)$ leaving the other relation unrestricted. If the hypothetical relations exists empirically, then this procedure will maximize the chance of finding them. For a technical derivation of the test procedures, see Johansen and Juselius (1992).

$\mathcal{H}_1$to $\mathcal{H}_4$ are hypotheses tests on pairs of variables, such as relative inflation ($\mathcal{H}_1$), relative interest rates ($\mathcal{H}_2$) and Fisher parity conditions ($\mathcal{H}_3$ and $\mathcal{H}_4$). These tests therefore seek to determine if some of the key parity conditions introduced in Section 2 are empirically verifiable on their own. Since all of the p-values are less than the 5% critical value, the tests reject this. In the remaining hypotheses tests in Table 4, $\mathcal{H}_5$ to $\mathcal{H}_{11}$, we therefore consider combining these parity relationships and introducing the *ppp* term.

$\mathcal{H}_5$ and $\mathcal{H}_6$ are tests of variants of real interest rate parity in which full proportionality has not been imposed. Restricting the two inflation rates to have unitary coefficients and the nominal interest rates to have equal and opposite signs ($\mathcal{H}_5$) is rejected, but relating the *ex post* German real interest rate with *ex post* US real interest rate ($\mathcal{H}_6$) gives a

Table 3: Cointegration properties

	Δp	Δp^*	i_b	i_b^*	$ppp^{1)}$	*Ds91.03*	$\chi^2(v)$	*p.val.*
$\mathcal{H}_1$	1	-1	0	0	0	*	25.9 (3)	0.00
$\mathcal{H}_2$	0	0	1	-1	0	*	45.0(3)	0.00
$\mathcal{H}_3$	1	0	-1	0	0	*	14.6(3)	0.01
$\mathcal{H}_4$	0	1	0	-1	0	*	34.2(3)	0.00
$\mathcal{H}_5$	1	-1	-0.19	0.19	0	*	25.6(2)	0.00
$\mathcal{H}_6$	1	-0.25	-1	0.25	0	*	4.4(2)	**0.11**
$\mathcal{H}_7$	1	-1	0	0	0.66	*	8.8(2)	0.01
$\mathcal{H}_8$	0	0	-1	1	-0.47	*	40.2(2)	0.00
$\mathcal{H}_9$	1	0	-1	0	-0.35	*	2.9(2)	**0.23**
$\mathcal{H}_{10}$	0	1	0	-1	-1.35	*	0.4(2)	**0.82**
$\mathcal{H}_{11}$	1	-1	-1	1	1.01	*	1.8(2)	**0.41**
$\mathcal{H}_{12}$	1	-0.34	-0.66	0	0	*	0.15(2)	**0.93**

[1)] The *ppp* term has been divided by 100

borderline stationary relation with a coefficient 0.25 to US real interest rate. Combining parity conditions, therefore, does seem to be important from the perspective of defining relationships which conform to standard macro theory. This point is underscored in $\mathcal{H}_{11}$ in which *ex post* real interest rates are equalized across countries, if the *ppp* term is included in the vector. Thus, a strict form of real interest rate parity is likely to be found in periods of a stationary *ppp* exchange rate. Note that including the *ppp* term in closed Fisher relationships for Germany and the US also produces stationary relationships ($\mathcal{H}_9$ and $\mathcal{H}_{10}$, respectively), but including the *ppp* term in the relative inflation or relative interest rate terms does not ($\mathcal{H}_7$ and $\mathcal{H}_8$, respectively). Finally, $\mathcal{H}_{12}$ describes a homogeneous relationship (i.e., the coefficients sum to zero) between German inflation, US inflation, and the German bond rate.

Altogether, the results of the tests in Table 3 are consistent with the formulation (14) in Section 4.

6.2 Fully specified cointegrating relations

Table 3 showed that stationarity is found for a real interest rate parity relation between Germany and USA and the *ppp* term ($\mathcal{H}_{11}$), combining the joint information in $\mathcal{H}_9$ and $\mathcal{H}_{10}$, and for a homogeneous relation between German price inflation, US price inflation and German bond rate ($\mathcal{H}_{12}$). We therefore tested the following joint hypothesis on the full cointegration structure:

Table 4: A structural representation of the cointegrating space.

	Eigenvectors β (*appr. t−values in brackets*)		*Weights* α (*t−values in brackets*)		
Var.	$\hat{\beta}_1$	$\hat{\beta}_2$	Eq.	$\hat{\alpha}_1$	$\hat{\alpha}_2$
Δp_t	1.0	-1.0	$\Delta^2 p_t$	$\mathbf{-1.02}$ (9.7)	-0.12 (1.4)
Δp_t^*	-0.31 (7.0)	1.0	$\Delta^2 p_t^*$	$\mathbf{-0.50}$ (5.1)	$\mathbf{-0.59}$ (7.1)
i_t^l	-0.69	1.0	Δi_t^l	0.00 (0.4)	-0.00 (0.1)
i_t^{l*}	0.0	-1.0	Δi_t^{l*}	-0.01 (0.7)	0.00 (0.4)
$ppp_t^{1)}$	0.0	-1.03 (10.0)	Δppp_t	0.03 (2.1)	0.03 (2.2)
$Ds91.03$	-0.001	0.002			
$const$	0.003	-0.003			

1)The ppp term has been divided by 100

$$\mathcal{H}_{13}: \quad \beta = \{H_1\varphi_1, H_2\varphi_2\},$$

where the design matrices H_1 and H_2 correspond to $\mathcal{H}_{11}$ and $\mathcal{H}_{12}$. The four overidentifying restrictions that $\mathcal{H}_{13}$ imposes on the cointegration space were tested based on the LR test procedure in Johansen and Juselius (1994). The test statistic, aymptotically distributed as $\chi^2(4)$, was 2.24 and the restrictions accepted with a p-value of 0.69. The two stationary relations are reported in Table 4. The first vector has been normalized on the German inflation rate and the second on the US inflation rate. The estimates of the freely estimated β_{ij} coefficients and their asymptotic 't-values' indicate that all of them are strongly significant and, hence, that the suggested structure is also empirically identified (Johansen and Juselius, 1994).

The first vector representing a German inflation relation is given by:

$$\Delta p_t = 0.31\Delta p_t^* + 0.69 i_t^l + 0.001 D91.3_t - 0.003 + stat.error. \qquad (24)$$

The interpretation is that German inflation is related both to the US inflation rate (an imported inflation effect) and to the domestic long-term bond rate. The shift dummy is consistent with a small increase in German inflation after the reunification and the constant term shows that German inflation on average is lower than the implied value as given by the determinants. The short-run adjustment to (24) occurs primarily through the changes in Germany inflation rate signifying its importance as a German relationship. However, US inflation has reacted similarly, i.e. negatively, though less strongly so, to positive deviations from this relation.

The second cointegrating relationship, representing international real interest rate parity, is given by:

$$(i_t^{l*} - \Delta p_t^*) = (i_t^l - \Delta p_t) - 0.01ppp_t + 0.002D91.3_t - 0.003 + stat.\,error. \tag{25}$$

The short-run adjustment to (25) occurs primarily through changes in US inflation rate signifying its importances for the US economy. The interpretation is that the US real interest rate increases relative to the German one when the *ppp* term is negative; i.e. when US prices are above German prices measured in the same currency, that US real interest rate is on average lower (0.003) than the German real interest rate given the *ppp* effect (we interpret this as the safe haven effect). *The results suggest that the lack of adjustment in the US inflation rate required to bring the ppp term back to steady-state (cf. the large US trade deficit discussed in Section 3) has been compensated by increasing US bond yield relative to the German yield.* That is, the failure of the exchange rate/ relative price configuration to restore competitiveness has to be compensated by an increased interest rate. This interpretation is supported by $\mathcal{H}_7$ in Table 3, showing that $(\Delta p_t^* - \Delta p_t - 0.007ppp) \sim I(1)$, i.e. the pure price adjustment, $\Delta p_t^* = \Delta p_t - 0.007(p^* - p - s)$, has not been enough to restore price competitiveness, with the financing of the consequent trade deficit requiring an increased capital import. Considering the large variations in real bond rates over this period, illustrated by the graphs of Figure 3, the fact that we have been able to recover a strong-form version of real interest rate parity seems quite remarkable.

Finally, we subject our chosen model to a set of Hansen-Johansen recursive stability tests. These results, reported in the Appendix indicate a remarkable degree of stability for the coefficients in the cointegrating vectors.

7 The extended model

We now apply our knowledge of the small model to an analysis of the full model structure as given by the vector:

$$x_t' = [ppp, \Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}], \tag{26}$$

and it is this vector which we analyze in the remainder of the paper. This vector facilitates an examination of the final parity condition considered in section 2, namely the term structure relationship.

As mentioned above, the short-term treasury bill rates have been subject to many major shocks and interventions in this period, requiring the inclusion of the following dummy variables:

$D_t' = [D78.09,\ Di78.10D79.12, D79.11, Di80.02, Di80.03, D80.05, D80.07,$
$D80.11, D81.01, D81.02, D81.03, D81.05, D81.10, D81.11, Di82.01, D82.08,$
$D82.10, Di84.12, D88.08, D89.02, D91, Ds91.03,].$

A more detailed explanation can be obtained from the authors. By controlling for these extraordinary shocks the residuals of the VAR model became reasonably well-behaved but normality was, nevertheless, rejected (mainly due to excess kurtosis), and both treasury bills exhibited residual ARCH. It appears that most of the observations in the money stock targeting period of the eighties have been classified as outliers and, hence, dummied out. The test results of Hansen and Johansen (1999) show that this period is in fact not representative for the whole sample[8].

In Table 5 we report the estimated eigenvalues and trace statistics associated with this system. Adding the two treasury bill rates to the data set implies three possibilities regarding the effect on the rank:

- $r = 2$, i.e. the rank is unchanged and the stochastic trends have increased to $p - r = 5$, implying that the two short-term rates are not cointegrated with themselves nor with the inflation rates, bond rates, or *ppp* term. Hence, this case is not very plausible, *a priori*.
- $r = 3$, i.e. $p - r = 4$ and including the short-term interest rates have introduced one additional stochastic trend. This means that the short-term interest rates can be jointly cointegrated or cointgrated with the remaining variables of the system.
- $r = 4$, i.e. the number of common stochastic trends remain unchanged, $p - r = 3$. In this case the short-term interest rates would be fully integrated with long-term interest rates, inflation rates, and the real exchange rates consistent with the theoretical foundations of the parities of Section 2.

The trace test suggests four common stochastic trends and, consequently, three cointegration relations. However, the trace statistic for $p - r = 4$ is quite close to the 95% quantile, which might suggest that the theoretically more acceptable case $p - r = 3$ be true. To check the

[8] Since the use of dummies has often been questioned (usually by theorists) the analysis was also done with only a few of the very large dummy variables included. The basic results remained unchanged, but the theoretical parity conditions obtained less support and the stability of the recursive tests was less satisfactory. Hence, the results suggested that the statistically well-behaved model produced economically more interpretable and 'cleaner' results.

Table 5: Eigenvalues, trace tests, and characteristic roots .

The trace test:							
$p-r$	7	6	5	4	3	2	1
λ_i	0.41	0.26	0.13	0.08	0.05	0.02	0.01
Q(r)	309	166	**86**	47	24	10	3
Q_{95}	132	102	76	53	35	20	9
Characteristic roots:							
$r=7$	1.00	1.00	0.94	0.94	0.88	0.57	0.39
$r=4$	1.0	1.0	1.0	0.97	0.87	0.57	0.40
$r=3$	1.0	1.0	1.0	1.0	0.84	0.55	0.47

sensitivity of the model to the choice of r we have also calculated the roots of the characteristic polynomial. There are approximately four 'near unit roots' in the unrestricted system, the choice of $r = 3$ removes all large roots, whereas $r = 4$ leaves a near unit root in the model. We conclude that $r = 3$ is the appropriate choice and, hence, that the treasury bill rates have been subject to permanent shocks (disturbances) which are not shared by the other variables of the system. Therefore, the fourth stochastic trend is likely to describe the cumulative impact of monetary intervention shocks.

Adding variables to the information set can change previous findings of long-run weak exogeneity. Actually, a change of weak exogeneity status is a sign of changing long-run feedback and is, therefore, of economic interest. If, for example, the short-term treasury bill rates are driving the long-term bond rates, then including the former in the analysis should change the previous finding of weakly exogenous bond rates and, instead, we would find that the treasury bill rates are weakly exogenous. In Table 6 the test results of weak exogeneity are reported. The test statistics, asymptotically distributed as $\chi^2(5)$, suggest that the weak exogeneity status is unchanged for the German and US long bond yield but not for the *ppp* exchange rates. The test of the German and US bond rate being jointly weakly exogenous is accepted with a p-value of 0.63.

As will be shown below in Table 7, the *ppp* <u>is</u> adjusting to all three cointegrating relations although very slowly so. Therefore, the finding that the *ppp* is close to being weakly exogenous does not imply that future real exchange rates can drift away without any bounds, but only that there is a lot of inertia in the movements back to its fundamental value. Although the predictive value of ppp_t for one-step-ahead predictions may not be very high, when it comes to predictions over longer period it is likely to increase substantially. This interpretation is strongly

Table 6: Tests of long-run weak exogeneity

Single tests of:	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	$i_t^s,$	i_t^{s*}	ppp
$\chi^2(3) = 7.8$	107.8	43.0	**0.3**	**3.5**	11.5	17.1	9.3

supported by the results of the long-run impact analysis in Section 8. Therefore, we consider it of crucial importance to keep ppp_t as part of the joint model.

7.1 Structural hypothesis tests

An advantage of the principle of 'specific-to-general' is that we can keep the two steady-state relations found in the previous section unaltered. Hence, the 'additional' impact of the two new variables on the system will essentially be described by the third cointegrating relation. To obtain information about the 'new' cointegration relation we first estimate the partially restricted long-run structure $\beta = \{H_1\varphi_1, H_2\varphi_2, \psi\}$, where H_1 and H_2 are the design matrices of Section 5 and ψ is an unrestricted cointegration vector. The hypothesis was accepted with a p-value of 0.24 and estimated coefficients of ψ normalized on the US treasury bill rate was:

Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	i_t^{s*}	ppp_t	$cnst$
0.19	−0.92	0.76	−0.36	−0.61	1.00	0.006	0.002

suggesting that the 'new' cointegration relation contains information about real treasury bill rates, the bond spread, and the ppp exchange rate. This led to the following joint hypotheses on the full cointegration structure:

$$\mathcal{H}_1: \quad \beta = \{H_1\varphi_1, H_2\varphi_2, H_3\varphi_3\}, \tag{27}$$

where the design matrices are defined as:

$$H_1 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, H_2 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, H_3 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$

The likelihood ratio statistic, asymptotically $\chi^2(\nu)$, of the nine overidentifying restrictions was 2.92 and the structure is clearly acceptable with

Table 7: A structural representation of the cointegrating space.

Eigenvectors β (*appr. t−values in brackets*)				*Weights* α (*t−values in brackets*)			
Var	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	Eq.	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
Δp_t	**1.0**	**−1.0**	1.0	$\Delta^2 p_t$	**−1.02** (-9.2)	−0.10 (-1.0)	0.04 (0.5)
Δp_t^*	**−0.33** (7.0)	1.0	**-1.0**	$\Delta^2 p_t^*$	**−0.54** (-5.4)	**-0.51** (-5.5)	**0.17** (2.2)
i_t^l	**−0.67**	1.0	1.51 (7.5)	Δi_t	0.01 (0.9)	0.00 (0.4)	−0.01 (−0.7)
i_t^{l*}	0	**−1.0**	−1.51	Δi_t^*	−0.00 (0.1)	−0.00 (0.4)	−0.01 (−1.7)
i_t^s	0	0	−1.0	Δi_t^s	0.01 (1.3)	**0.04** (3.9)	**0.02** (3.1)
i_t^{s*}	0	0	**1.0**	Δi_t^{s*}	−0.02 (−1.1)	**−0.05** (−4.3)	**−0.05** (−4.9)
$ppp_t^{1)}$	0	**-0.96** (10.0)	0	Δppp_t	**0.04** (2.2)	**0.05** (3.4)	**0.02** (2.0)
Ds91.03	-0.001	0.002	-0.002				
const.	0.003	-0.003	0.003				

1) The ppp term has been divided by 100

a p-value of 0.97. In Table 7 the estimates of the identified β_{ij} coefficients and their asymptotic standard errors are reported. All of them are strongly significant, thereby implying that the suggested structure is both formally and empirically identified. The corresponding adjustment coefficients α_{ij} are reported with t-values in brackets. The graphs in Appendix IV show the recursively calculated tests of constant β (i.e. whether the same β vectors would have been accepted if the sample had ended in $1987 + j, j = 1, ..., 144$) and constant α_{ij} for β fixed at the full sample value. The graphs demonstrate that the empirical effects have been remarkably constant over the last 12 years, a total of 144 observations, thereby refuting the Lucas' critique!

The first two vectors are almost identical to the two cointegrating relations in the small model, illustrating the invariance of the cointegration property. The third vector is a function of real short-term interest rates and the bond spread and can be written as:

$$i_t^{s*} - \Delta p_t^* = (i_t^s - \Delta p_t) + 1.51(i_t^l - i_t^{l*}) + 0.002D91.3 + 0.003.$$

This relationship is interesting since it suggests that short-term real interest rate parity would be satisfied as a stationary relation if the long-term bond spread is stationary. However, the nonstationarity of the bond spread is likely to be related to the nonstationary deviations from the steady-state value of the *ppp* rate.

Empirically, this means that only in periods when the ppp rate has returned to its steady-state path and the bond yield differential has become stationary is it possible to find evidence of real interest rate parities as the stationary relations theory would predict.

Thus the analysis suggests that empirical support for the theoretical parities might very well be found in the data, but as long as the economies stay away from their fundamental steady-state positions, direct evidence is unlikely to be found. In that sense the cointegrating relationships which we have established could be said to contain the 'theoretical' parities as a special case. For example, in the hypothetical situation where real exchange rates have returned to their steady-state path, the *ppp* term should be stationary and so the current account should also be balanced. With no need to finance the current account, the spread between bond yields should be stationary and the other parities would also be individually stationary.

Figure 5 shows the graphs of the three equilibrium error correction mechanisms which clearly appear very stationary.

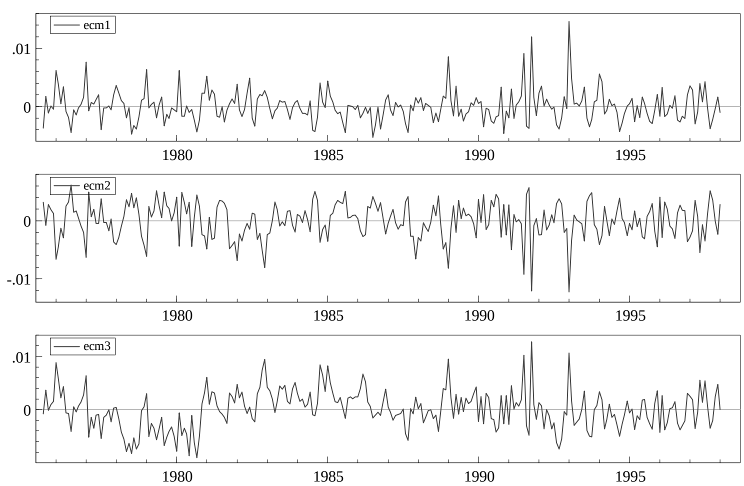


Figure 1: The graphs of the three equilibrium error correction mechanisms

The adjustment coefficients (t-ratios in brackets), reported in Table 7, of the first two relations are similar to those of the small system, with the exception that the US inflation rate is now significantly adjusting to all relations. However, it is noteworthy that the long-term bond rates show no evidence of adjusting to any of the long-run relations, whereas the two treasury bill rates are strongly adjusting to the last two steady-state relations. This seems to be against the expectation's hypothesis

Table 8: The combined long-run effects

The combined effects $\Pi = \alpha\beta'$							
Eq.	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t		
$\Delta^2 p_t$	**-0.88** (-12.8)	**0.19** (2.9)	**0.58** (12.0)	-0.12 (-1.5)	0.15 (1.8)		
$\Delta^2 p_t^*$	0.08 (1.3)	**-0.47** (-7.3)	**-0.13** (-2.6)	**0.44** (7.5)	**0.58** (7.5)		
Δi_t^l	0.01 (0.8)	-0.00 (-0.3)	-0.00 (-0.8)	0.00 (0.0)	0.00 (0.0)		
Δi_t^{l*}	-0.01 (-1.6)	0.01 (0.7)	0.01 (1.5)	-0.02 (-0.2)	-0.00 (-0.2)		
Δppp_t	0.006 (0.5)	0.012 (1.1)	−0.002 (-0.2)	-0.014 (-1.5)	−0.02 (1.5)		
	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	i_t^s	i_t^{s*}
$\Delta^2 p_t$	**-0.88** (-12.5)	**0.18** (2.5)	**0.46** (2.5)	-0.05 (-0.2)	0.10 (1.0)	0.07 (0.9)	0.09 (1.0)
$\Delta^2 p_t^*$	0.13 (2.0)	**-0.53** (-8.2)	-0.18 (-1.1)	0.14 (0.7)	**0.48** (5.5)	0.05 (0.7)	**0.23** (2.8)
Δi_t^l	−0.00 (-0.1)	0.00 (0.6)	−0.00 (-0.1)	0.00 (0.1)	-0.00 (-0.3)	0.00 (0.1)	−0.00 (0.4)
Δi_t^{l*}	-0.01 (-1.7)	0.01 (1.1)	−0.02 (-1.0)	0.03 (1.3)	0.01 (0.6)	0.01 (1.4)	−0.01 (−1.4)
Δppp_t	0.009 (0.9)	0.01 (1.2)	**0.08** (2.7)	**-0.09** (-2.9)	**−0.05** (−3.3)	**−0.03** (−3.0)	**0.03** (2.0)
Δi_t^s	0.00 (0.0)	0.01 (1.4)	**0.06** (4.0)	**−0.07** (−4.0)	**−0.03** (−4.0)	**−0.03** (−4.0)	**0.03** (3.0)
Δi_t^{s*}	−0.01 (−1.4)	0.00 (0.1)	**−0.11** (−5.0)	**0.13** (5.3)	**0.05** (4.4)	**0.05** (5.3)	**−0.05** (−4.7)

which predicts that short-term interest rates should act as exogenous variables and, hence, drive long-term rates. The significant adjustment of the US short rate to the cointegrating vectors reflects its role as a money market determined interest rate, but the lack of adjustment in the long-term bond rates seems to suggest that the transmission of the money market effects to the long-rates is not there, or only weakly so. This will be further investigated in the next section.

7.2 The role of short-term interest rates

To gain a further perspective on the role of the short- relative to the long-term interest rates we report, in Table 8, a comparative analysis of the combined effects, as measured by $\hat{\alpha}_r \hat{\beta}_r' = \hat{\Pi}_r$, where the subscript r stands for the restricted estimates as reported in Tables 4 and 7.

It appears that German inflation is essentially unaffected by the inclusion of the treasury bill rates into the analysis. It is, as before, determined by the long bond rate and US inflation.

The results for US inflation show that the US short-term treasury bill rate has now replaced the long-term bond rates in the small system. However, the results for the US treasury bill rate show significant reaction from the bond yield spread. Thus, it seems likely that the short-run effects go from bond rates influencing treasury bill rates, influencing inflation rates. However, the results in Table 10, next section, show that the long-run impact on US inflation derives from permanent shocks to the short-term treasury bill rates, but the effect is positive and not negative[9]!

Consistent with the weak exogeneity results of Table 6, the equations for the German and US bond rate exhibit hardly any significant effects. The *ppp* term is significantly affected by the bond and the short-term spread, such that the US$ appreciates with an increasing bond spread and depreciates with an increasing treasury bill spread.

The results for the short-term treasury bill equations show strong adjustment to essentially all determinants except for inflation rates! The lack of significant inflationary effects in all four interest rate equations is very pronounced. This is to be contrasted with the significant interest rate effects in the inflation rate equations! These are very strong results and have also been found in Danish, Spanish, and Italian data (Juselius, 1992, Juselius and Toro, 1999, Juselius and Gennari, 1999).

7.3 A short-run adjustment model[10]

Using the identified cointegration relations reported in Table 7 we first estimated a multivariate dynamic equilibrium error correction model for the full system. Because US bond rate was found to be strongly exogenous we re-estimated the system conditional on the marginal model for US bond rate. By first removing insignificant lagged variables from the system based on a F-test and then removing insignificant coefficients from the equations based on a Likelihood Ratio test we arrived at the following parsimonious model:

$$\begin{bmatrix} \Delta^2 p_t \\ \Delta^2 p_t^* \\ \Delta i_t^l \\ \Delta i_t^s \\ \Delta i_t^{s*} \\ \Delta ppp_t \end{bmatrix} = \begin{bmatrix} \underset{(2.7)}{1.11} & 0 & 0 & 0 & 0 \\ \underset{(3.5)}{1.29} & 0 & 0 & \underset{(3.2)}{1.57} & 0 \\ \underset{(8.2)}{0.26} & 0 & \underset{(6.6)}{0.34} & 0 & \underset{(2.0)}{0.03} \\ \underset{(3.8)}{-0.22} & \underset{(7.3)}{1.16} & 0 & \underset{(2.4)}{0.11} & 0 \\ \underset{(10.1)}{0.52} & 0 & 0 & 0 & \underset{(13.0)}{0.34} \\ \underset{(3.0)}{-0.20} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_t^{l*} \\ \Delta i_t^l \\ \Delta i_{t-1}^l \\ \Delta i_{t-1}^s \\ \Delta i_{t-1}^{s*} \end{bmatrix} +$$

[9] This is a frequent empirical finding, the so called "price puzzle".

[10] All calculations have been performed in PcFiml (see Doornick and Hendry (1998)).

$$
+\begin{bmatrix}
-\underset{(11.2)}{1.06} & \underset{(2.5)}{-0.20} & 0 \\
\underset{(6.0)}{-0.51} & \underset{(9.4)}{-0.67} & 0 \\
\underset{(3.0)}{0.01} & 0 & 0 \\
0 & \underset{(4.1)}{0.02} & \underset{(2.1)}{0.01} \\
0 & \underset{(2.7)}{-0.02} & \underset{(2.9)}{-0.02} \\
0 & \underset{(2.8)}{0.03} & \underset{(2.0)}{0.02}
\end{bmatrix}
\begin{bmatrix} ecm1_{t-1} \\ ecm2_{t-1} \\ ecm3_{t-1} \end{bmatrix} + \Phi D_t + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}
$$

$$
\hat{\Sigma}(r_{ij}) = \begin{bmatrix}
(0.00193) & & & & & \\
0.25 & (0.00169) & & & & \\
0.04 & 0.12 & (0.00014) & & & \\
-0.12 & -0.05 & -0.51 & (0.00018) & & \\
-0.10 & -0.08 & 0.01 & -0.01 & (0.00020) & \\
-0.01 & -0.02 & 0.03 & -0.02 & -0.09 & (0.00030)
\end{bmatrix}
$$

where

$$
\begin{aligned}
ecm1 &= \Delta p - 0.33\Delta p^* - 0.67 i^l - 0.001 D91.3 + 0.003 \\
ecm2 &= (i^{*l} - \Delta p^*) - (i^l - \Delta p) + 0.0096 ppp - 0.002 D91.3 + 0.003 \\
ecm3 &= (i^{*s} - \Delta p^*) - (i^s - \Delta p) - 1.54(i^{*l} - i^l) - 0.002 D91.3 + 0.003
\end{aligned}
$$

and the off-diagonal terms of the $\hat{\Sigma}(r_{ij})$ matrix are given as residual correlations, whereas the diagonal terms correspond to the residual standard errors[11]. Except for a negative correlation between the shocks to the German interest rates (-0.55) the residual cross correlations are essentially zero. The estimated coefficients of the included dummy variables are presented in Appendix I. The LR test of overidentifying restrictions, distributed as $\chi^2(136)$, was 156.7 and the restrictions were accepted with a p-value of 0.11. Of the 136 exclusion restrictions only 12 are related to the system variables. The latter were accepted with a p-value of 0.71. The remaining restrictions are associated with the many intervention dummies needed to account for the turbulent movements in US treasury bill rate during the period of monetary targeting in the beginning of the eighties. In addition the monthly seasonal dummy variables are only included in the US and German inflation rate equations.

In terms of the contemporaneous effects, we note that the weakly exogenous US bond rate has a pervasive effect, appearing in all equations,

[11] Note that the standard deviation of the residuals from the monthly changes in CPI inflation rates is approximately 0.2%.

whereas a change in the German bond rate only has an immediate effect on the German treasury bill rate. The effect of lagged changes to the system variables are altogether very modest.

The significant adjustment effects of the error correction terms are notable. Both inflation rates are strongly adjusting to $ecm1$, the German inflation relation, and $ecm2$, the long-term real interest parity relation, but German inflation much stronger to $ecm1$ and US inflation much stronger to $ecm2$. The adjustment coefficients to the two ecm terms are negative both in the German and US inflation equations which might seem surprising. To be able to interpret this result we have calculated the underlying steady-state relation, being a combination of the significant ecm's weighted by the adjustment coefficients, for each of the two inflation rates.

For the German inflation rate the combined effects became:

$$\Delta p = 0.18\Delta p^* + 0.59 i^l + 0.23 i^{l*} + 0.003 ppp.$$

and for the US inflation:

$$\Delta p^* = 0.32\Delta p + 0.68 i^{l*} + 0.65(i^{l*} - i^l) + 0.010 ppp.$$

It appears that German inflation has adjusted homogeneously to German and US bond rates and to US inflation. US inflation has similarly adjusted homogeneously to the German inflation rate and the US bond rate, and additionally also to the bond rate rate spread. Altogether, the results seem to indicate that the long-term interest rates play a very fundamental role for determination of inflation rates *implying that the cost of long-term financing has an important effect on prices*. Furthermore, US inflation is equilibrium error correcting to the ppp term (though not sufficiently fast to restore fundamental equilibrium exchange rates) whereas the ppp effect on German inflation is neglible.

The German bond rate is only very weakly reacting to $ecm1$, i.e. to 'excess' German inflation consistent with the previous finding that it is essentially weakly exogenous. The two treasury bill rates and the ppp term adjust similarly to $ecm2$ and $ecm3$, i.e. to deviations from the long-term and short-term real interest rate parity conditions. To facilitate interpretation we also derive the combined steady state relations for these variables. The combined steady-state relation for German treasury bill rate became:

$$i^s - i^{s*} = 3.5(i^l - i^{l*}) + (\Delta p^* - \Delta p) - 0.02 ppp,$$

for US treasury bill rate:

$$i^{s*} - i^{s} = 2.5(i^{l*} - i^{l}) + 0.01ppp.$$

and finally for the *ppp* term:

$$ppp = 0.35(\Delta p^{*} - \Delta p) - 2(i^{l*} - i^{l}) + 0.65(i^{s*} - i^{s}).$$

Thus, the treasury bill rates adjust strongly to the long-term bond spread, but also to the deviation from the *ppp* and the inflation rate differential.

The *ppp* term adjusts homogeneously to the inflation spread and the short-term interest spread and shows a strong negative effect from the US-German long-term bond spread. The results confirm the crucial role of the long-term, but also the short-term interest rates for the development of the real exchange rates in this period. It is quite interesting that an increase in the spread between US and German bond rates is associated with an appreciation of the dollar, whereas the opposite is the case with an increase in the short spread and the inflation rate differential.

Altogether the results seem to suggest that the reserve currency (safe haven) effect of the dollar has indeed prevented the adjustment towards equilibrium exchange rates and resulted in the overvalued dollar. The need to finance the low US saving drives up the US bond rate relative to the German rate and the increase in the bond yield results in the US$ appreciating, making the adjustment towards stationary real exchange rates very slow.

8 Weak exogeneity and the long-run impact of shocks

We noted above that the German and US long bond yields are weakly exogenous for the long-run parameters β implying that they act as driving variables (a common stochastic trend) in the system. By inverting the VAR subject to the reduced rank restriction $\Pi = \alpha\beta'$ we get the so called moving average representation:

$$x_t = C\sum_{1}^{t}\varepsilon_i + C\Phi_1\sum_{1}^{t}D_i + C\Phi_2\sum_{1}^{t}S_i + C^{*}(L)(\varepsilon_t + \mu + \Phi_1 S_t + \Phi_2 D_t) + Z_0 \tag{28}$$

where $C = \beta_{\perp}(\alpha'_{\perp}\Gamma\beta_{\perp})^{-1}\alpha'_{\perp}$, $C^{*}(L)$ is an infinite polynomial in the lag operator L, and Z_0 is a function of the initial values. Based on (28) it is possible to calculate the impulse responses of a shock to one variable and how it is transmitted over time within the system. Instead of reporting

Table 9: The estimates of the long-run impact matrix C

	$\Sigma\varepsilon_{\Delta p}$	$\Sigma\varepsilon_{\Delta p*}$	$\Sigma\varepsilon_{i^b}$	$\Sigma\varepsilon_{i^{b*}}$	$\Sigma_{\varepsilon i^s}$	$\Sigma\varepsilon_{i^{s*}}$	$\Sigma\varepsilon_{ppp}$
Δp_t	-0.00 (0.1)	0.02 (1.4)	**0.57** (2.8)	0.24 (1.4)	0.14 (1.4)	**0.24** (2.1)	**0.35** (5.3)
Δp_t^*	0.00 (0.1)	0.03 (1.4)	-0.34 (-1.0)	0.31 (1.0)	0.11 (0.7)	**0.65** (3.3)	**0.98** (8.5)
i_t^l	-0.00 (-0.3)	0.01 (0.9)	**1.21** (4.7)	0.38 (1.7)	0.03 (0.3)	-0.07 (-0.5)	0.05 (0.6)
i_t^{l*}	-0.02 (-1.3)	0.03 (1.2)	-0.28 (-0.8)	**1.64** (5.3)	0.15 (0.9)	-0.25 (1.3)	0.10 (0.9)
i_t^s	-0.00 (-0.2)	0.01 (0.5)	**1.18** (3.5)	-0.29 (-1.0)	**0.97** (6.0)	**0.38** (2.0)	**-0.26** (-2.4)
i_t^{s*}	-0.02 (-1.1)	0.05 (1.4)	**-1.70** (-3.3)	**2.00** (4.5)	**0.78** (3.1)	0.34 (1.1)	**0.57** (3.4)
ppp_t	0.01 (1.3)	0.00 (0.2)	**0.70** (2.1)	**-0.97** (3.5)	$\mathbf{-0.41}$ (2.7)	**0.50** (2.8)	**0.74** (7.1)

the impulse response functions for a unitary change of $\hat{\varepsilon}_{it}$,[12] we report only the final impact matrix, C in Table 9.

The estimates of the columns of the C matrix in Table 9 measures the total impact of permanent shocks to each of the variables on all other variables of the system. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behavior of the variable in that specific row. The t-ratios in parenthesis are based on the asymptotic standard errors suggested by Paruolo (1997).

These results reinforce our previous findings from the analysis of the long-run relations. We note that cumulative shocks to the inflation rates have no significant long-run impact on any of the variables, accentuating our previous findings that inflation rates are solely adjusting in this system, but not pushing. We also note that the two long term bond yields have significant cumulative impacts on short term interest rate yields, the *ppp* term and to some extent also on inflation rates, whereas shocks to the short-term interest rates have no long-run impact on the bond rates. The latter result is again in conflict with the basic premise of the expectations hypothesis of the term structure. Furthermore, permanent shocks to the short-term US treasury bill rate do have a permanent positive impact on inflation rates. Thus, increases in the US short-term interest rate tend to increase inflation and not the other way around. Permanent shocks to the *ppp* term are also important as they have a significant long-run impact on inflation rates and short term bill yields.

Therefore, the results strongly suggest that the developments in 'world' financial markets, as measured by the dominant rate yields - the US and

[12]Note that the $\hat{\varepsilon}_{it}$ are almost orthogonal as demonstrated in Section 7.3.

German long rates and the treasury bill rates - are driving this system and inflation rates are essentially adjusting. This latter finding reinforces the point made earlier that the Fisher conditions do not seem to work in the predicted manner.

9 Summary and conclusions

This paper has empirically examined the joint determination of a number of key parity conditions for Germany and the US using monthly data from the recent experience with floating exchange rates. The full vector of variables considered in this paper, consisted of the German Mark-US dollar exchange rate, prices, short term interest rates and long term interest rates. We used the cointegrated VAR model to define long-run stationary relationships as well as common stochastic trends, and a general-to-specific approach to produce parsimonious dynamic short-run equations. In constructing our long-run relationships we advocated a specific-to-general approach, in which we initially excluded the short term interest rates from our cointegration analysis. We now summarize our main findings.

The results strongly rejected the stationarity hypothesis of the 'pure' parity conditions. However, by allowing them to be interdependent stationarity was recovered. The important finding was that the nonstationarity of the 'simple' parity relationships was primarily related to the nonstationarity of the *ppp* exchange rate and the long-term bond rate differential. An obvious interpretation of the results was that the lack of empirical support for the simple parity conditions was due to the lack of (or very, very slow) adjustment to a stationary *ppp* steady state and increasing long-term bond spreads as a plausible consequence of the latter. Thus, the theoretical assumption of stationary parity conditions appeared to be a special case of a more general formulation allowing for persistent deviations from steady-state and, hence, market failure in a simple model framework.

Therefore, the theoretical assumption of two common driving trends had to be replaced by the empirical finding of four common trends, hypothetized as: (1) a nominal price trend driving the goods market, (2) a trend describing relative national savings behavior, (3) a 'safe haven' trend capturing the role of the dollar as a world reserve currency, and (4) a short-term capital market trend describing central bank policy behavior.

Not surprisingly, the empirical modification of the original parity conditions as a result of the above 'market failure' trends, produced a number of new results related to the dynamics of the international transmission mechanism. Some of the major (empirically strong) findings were the following:

1. In the big system of inflation rates, *ppp* exchange rates, 10 year bond rates and 3 months treasury bill rates, it was the US and German long-term bond rates which were the main driving forces and not the short-term interest rates.

2. US and German inflation rates were strongly adjusting to the other variables of the system, primarily to the bond rates and the real PPP exchange rates, but they were not affecting the other variables, in particular, they did not push nominal interest rates.

3. The nonstationary movements in the bond and inflation rate differential were closely related to the nonstationary movements in the *ppp* exchange rate.

4. The short-term interest rates (the 3 months treasury bill rates) were important for the determination of the *ppp* exchange rate both in the short and the long run. They had essentially no impact on the bond rates and the inflation rates, with the caveat that US short rate had a positive (cost push) effect on US inflation.

5. Permanent shocks to long-term as well as short-term interest rates had a positive long-run impact on inflation, signifying the cost effect of interest rates on capital stock.

The above findings were shown to be remarkable robust (empirically as well as econometrically) over a period of fundamental changes and hence, could not be discarded as sample dependent results. They seemed to suggest that:

1. The role of the dollar as a reserve currency (the 'safe haven' effect) has facilitated relatively cheap financing of the large US current account deficits in this period. This might explain one of the 'market failure' puzzles: why an adequate adjustment toward purchasing power parity between USA and Germany has not taken place.

2. The large differences between national savings rates seemed to be an important reason why the long-term bond rates were found to be so crucial in this system.

3. Though the role of central bank policy for stabilizing the short-term capital market has evidently been crucial as the turbulent years of monetary targeting in the eighties demonstrated, its role for controlling inflation seemed much more modest than is usually believed.

However, although the non-stationary of the parities will disappear with the disappearance of other disequilibria in the economy, in the presence of free capital movements we do not believe that the parity reversals in the term structure and Fisher relationships, will disappear. The latter finding would appear to have important policy implications.

Finally, by joint modelling of the parities we have managed not only to recover stationary parity conditions, but also to describe the variation of the data with a remarkable degree of precision as evidenced by the very small residual standard errors. Hence, the results should be used as a benchmark against which the results of other models, possibly with more theory content, could be evaluated.

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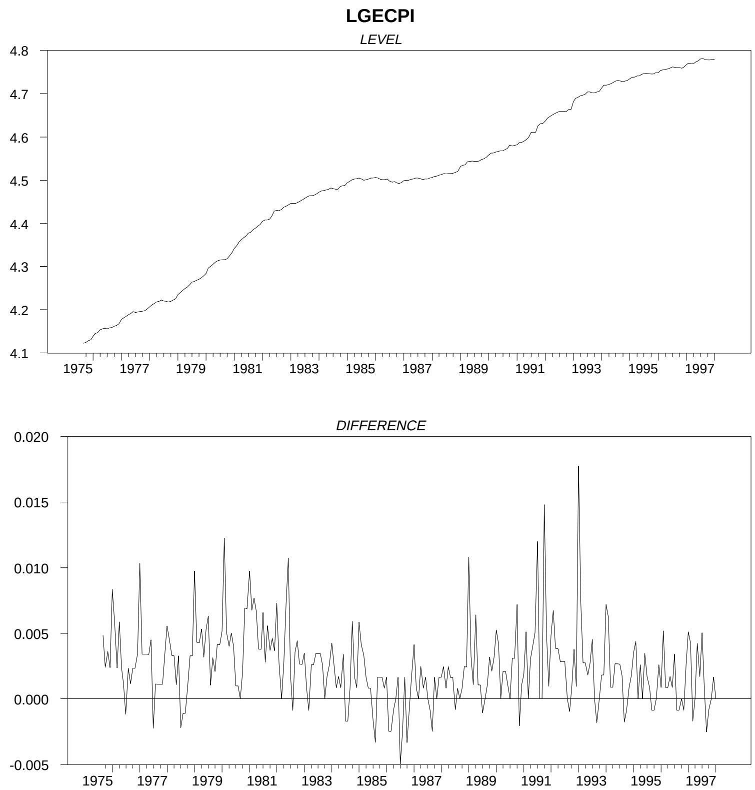
Table 10: The estimated intervention effects in the short-run adjustment model

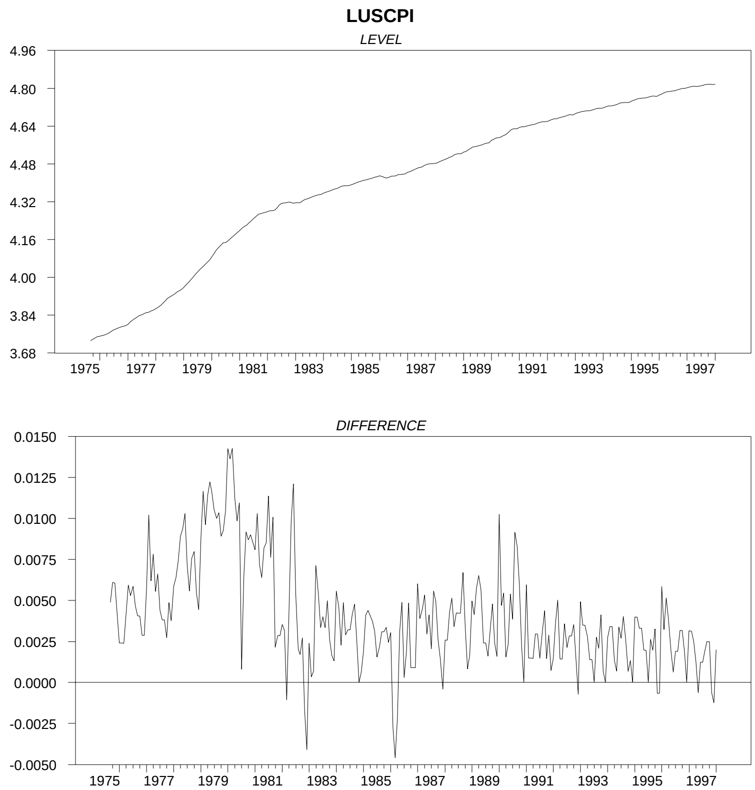
	Δp_t	Δp_t^*	Δi_t^l	Δi_t^s	Δi_t^{s*}	Δppp_t
Di78.10	0	0	0	0	0	0.0010 (4.6)
D79.11.12	0	0	0	0.0007 (6.4)	0	0
D80.03	0	0	0	0	0	0
Di80.03	0	0	0	0	0.0017 (11.3)	0
D80.05	0	0	0	0	-0.0031 (14.9)	0
D80.07	0	-0.008 (4.8)	0	0	0.0012 (5.8)	0
D80.11	0	0	0	0	0.0012 (5.7)	0
D81.01	0	0	0	0.0008 (4.9)	-0.0012 (5.9)	-0.0008 (2.5)
D81.03	0	0	0	0.0005 (3.4)	-0.0012 (5.9)	0
Di81.05	0	0	0.0002 (2.3)	0	-0.0020 (13.4)	0
D81.10	0	-0.007 (4.0)	-0.0004 (3.0)	-0.0008 (4.3)	-0.0007 (3.4)	0
D81.11	0	0	0	0.0003 (2.2)	-0.0012 (5.7)	0
Di82.01	0	0	0	0	0	0
D82.08	0	0	0	0	-0.0021 (10.4)	0
Di84.12	0	0	-0.0002 (2.1)	0	-0.0004 (2.7)	0
D88.08	0	0	0	0.0009 (6.0)	0	0
D89.02	0	0	0	0.0007 (4.5)	0	0
D91	0.01 (10.9)	0	0	0	0	0

11 Appendix I: The intervention dummies

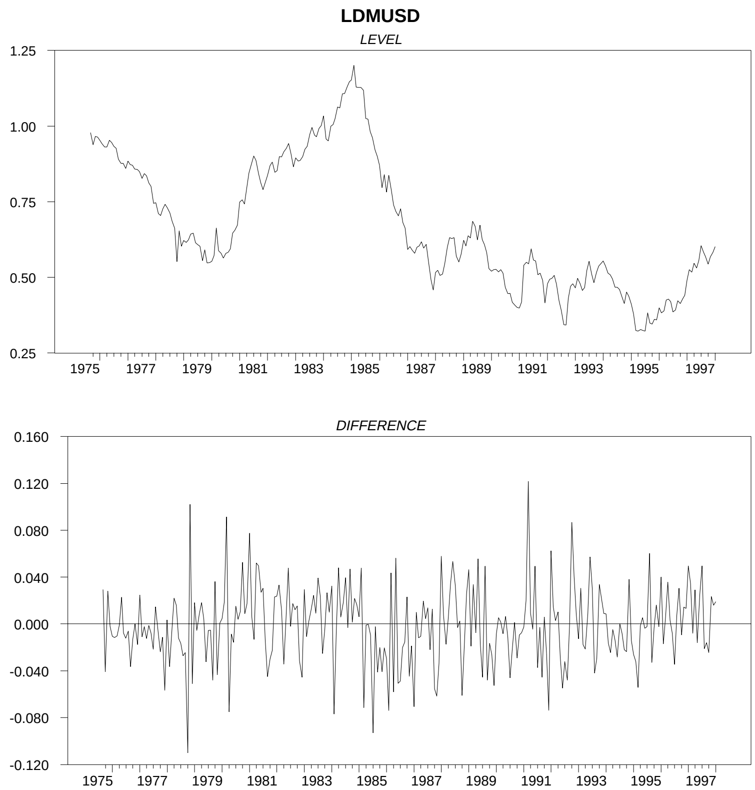
12 Appendix II: The data



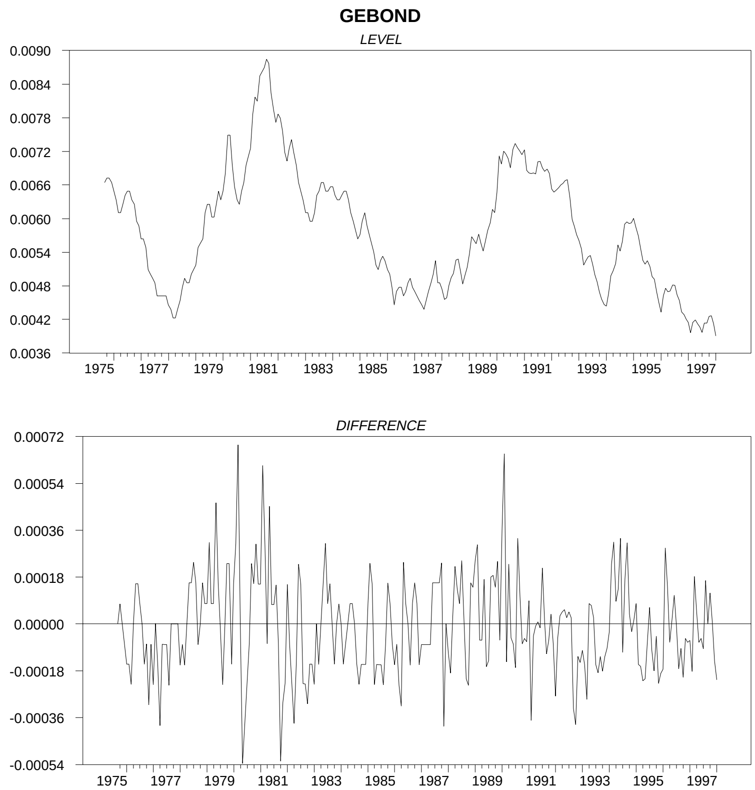
The German CPI in levels and differences



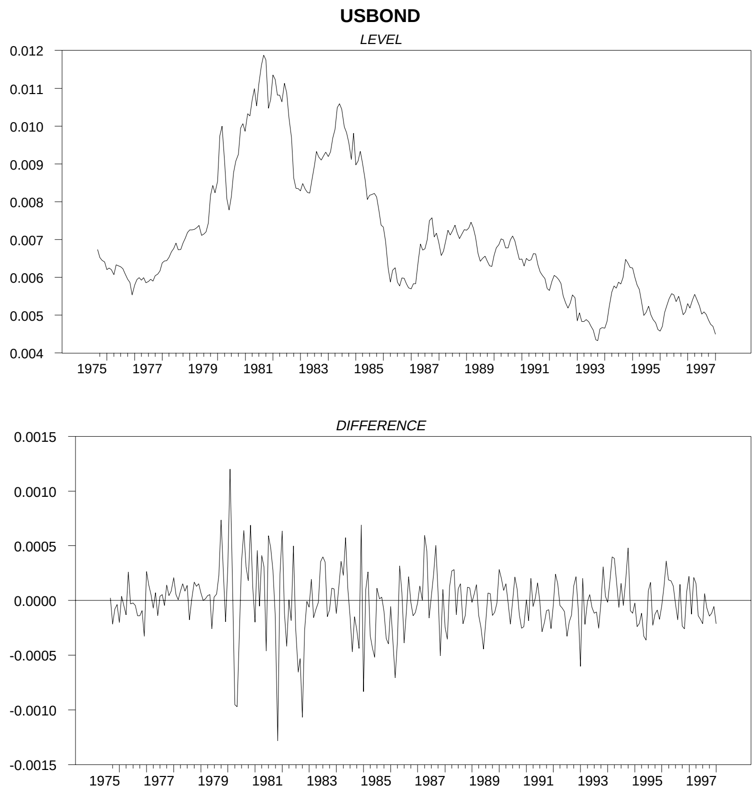
The log of US CPI in levels and differences



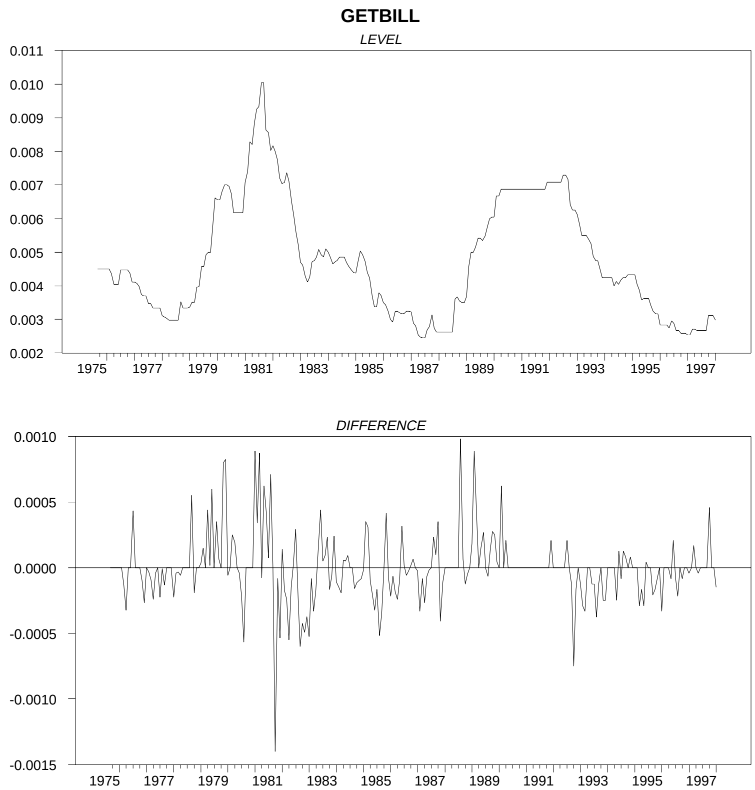
The log of spot exchange rate of German mk in dollars.



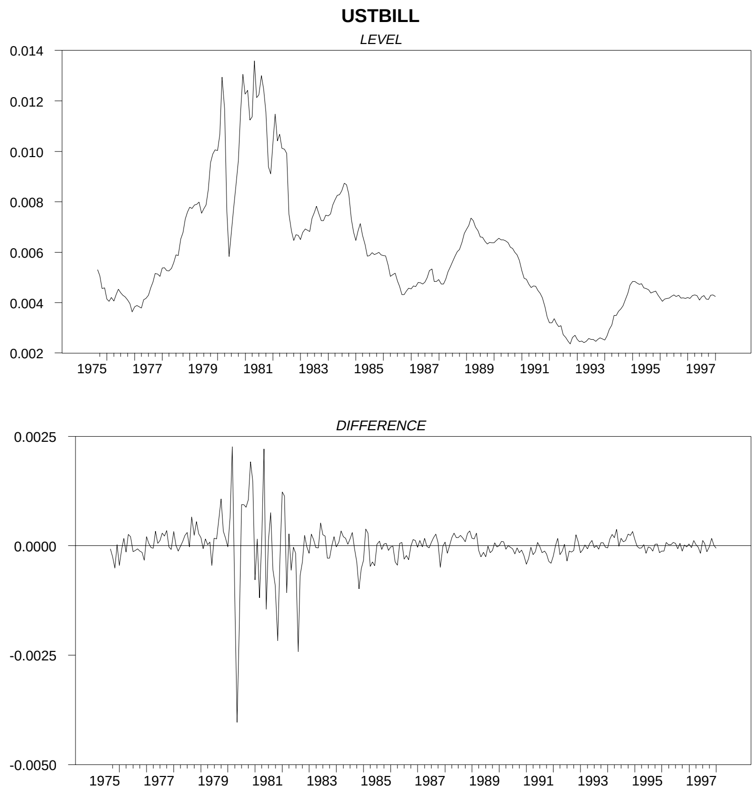
The German monthly bond rate in levels and differences.



The US monthly bond rate in levels and differences

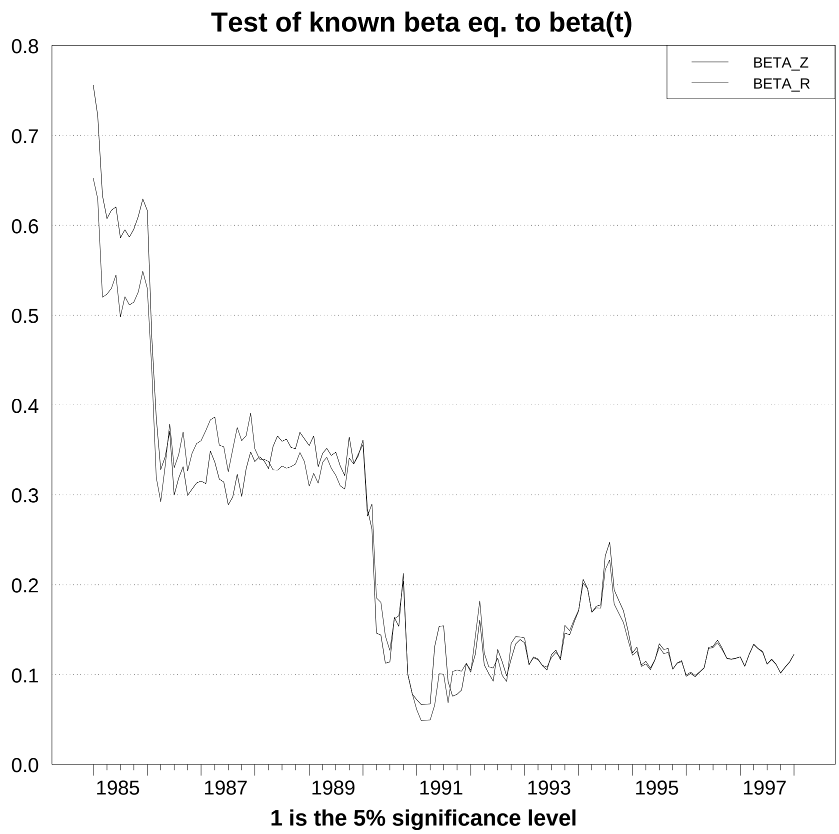


The German 3 months treasury bill rates in levels and differences

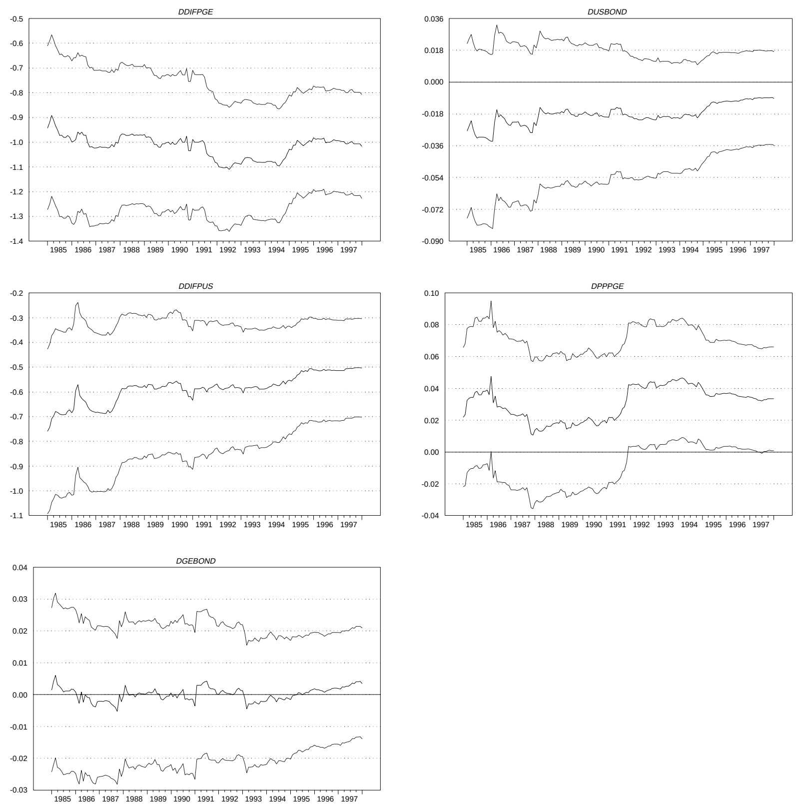


The US three months treasury bill rates in levels and differences.

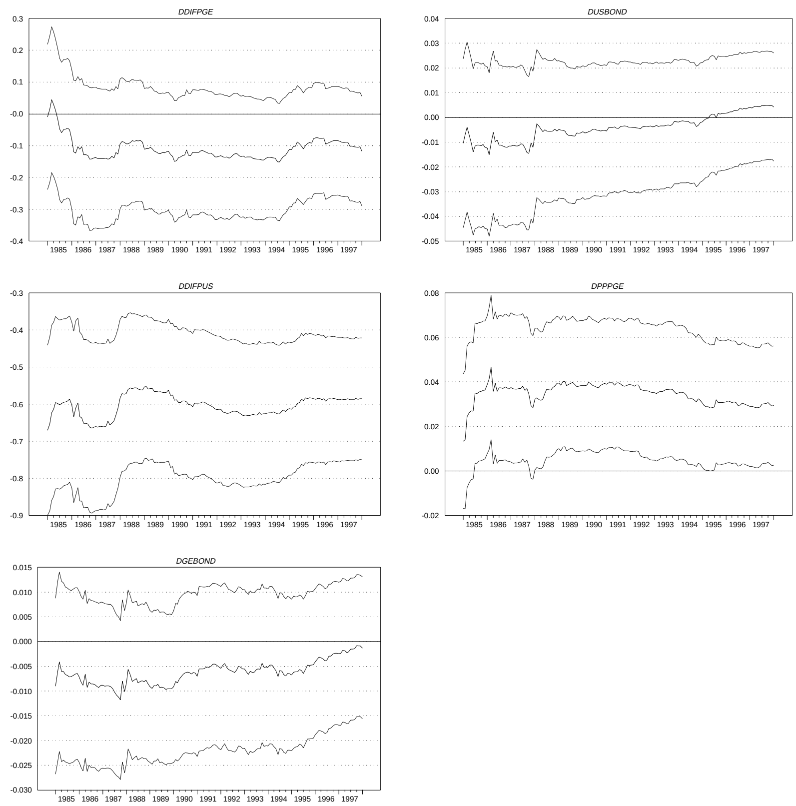
12.1 Appendix III: Recursive graphs of the small model



Recursively calculated test statistics (1.0 corresponds to the 5% significance level) for the constancy of the cointegration space.

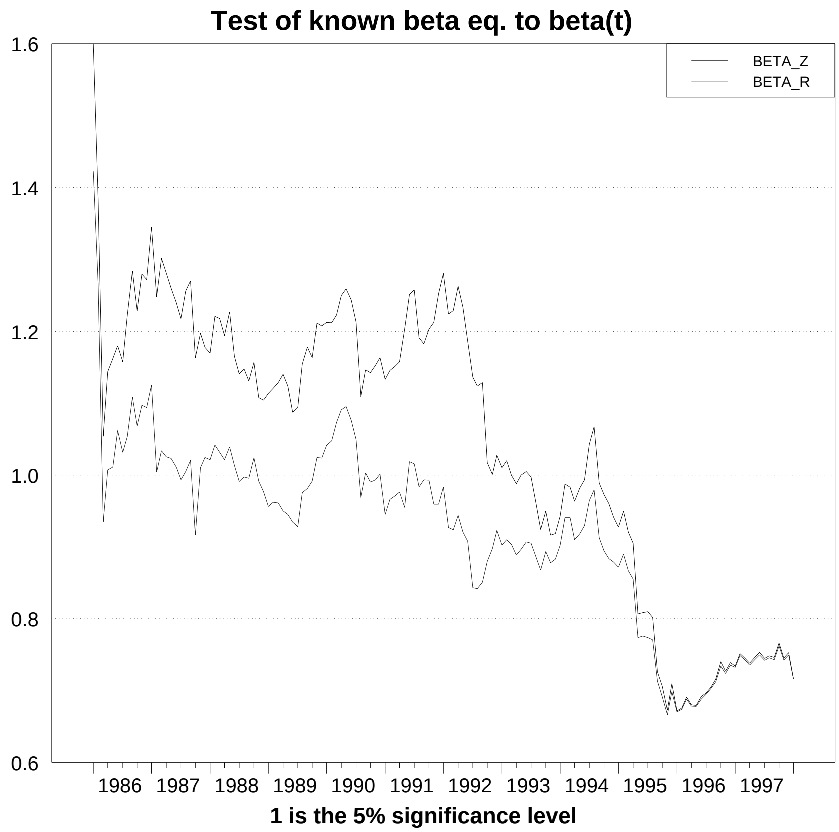


Recursively estimated alpha coefficients to the first cointegrating relation of Table 4.

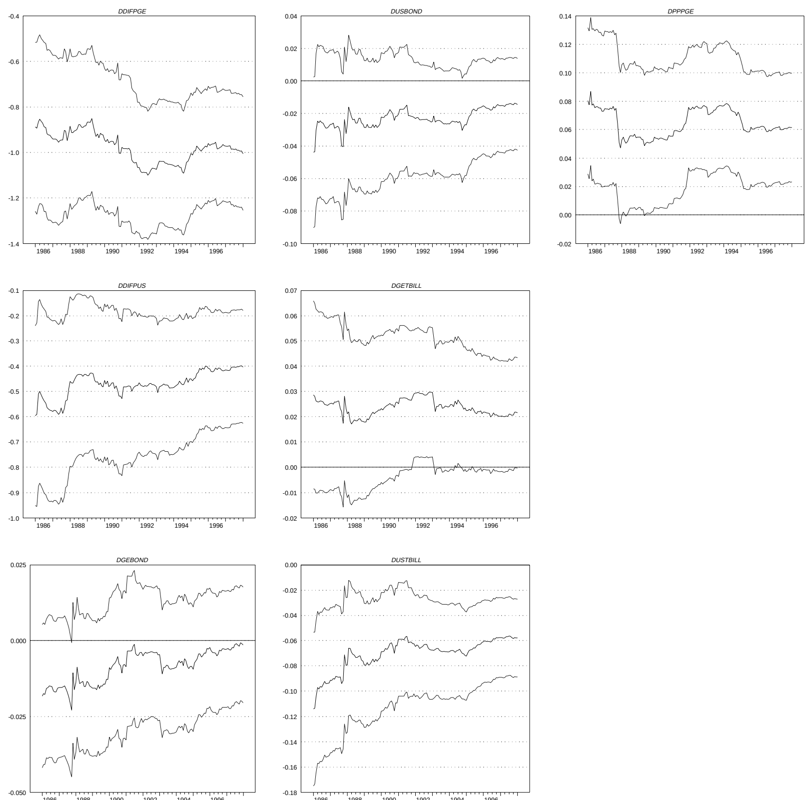


Recursively estimated alpha coefficients to the second cointegrating relation of Table 4.

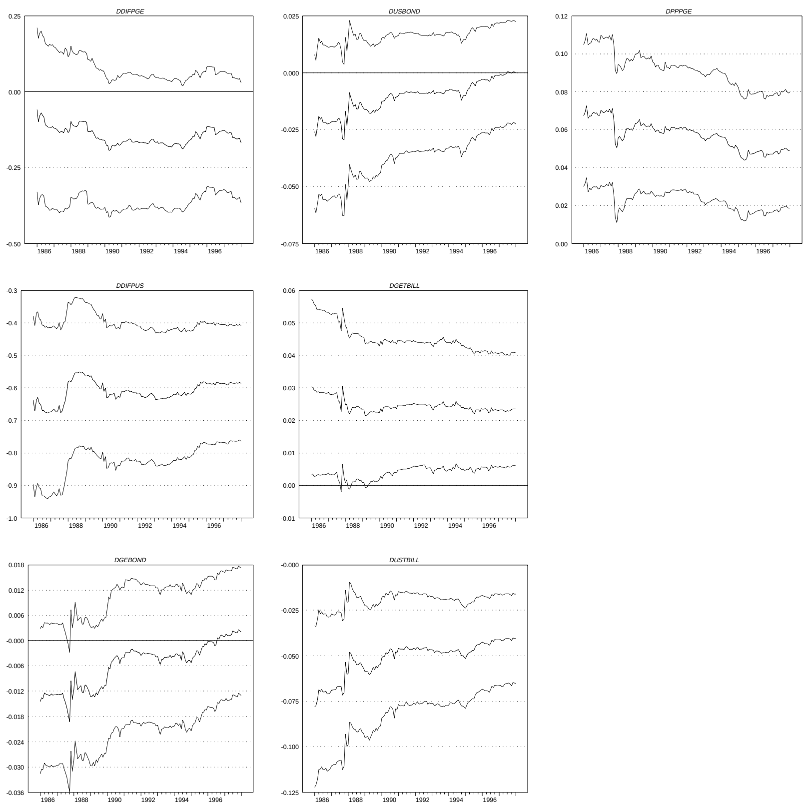
12.2 Appendix IV: Recursive graphs of the big model



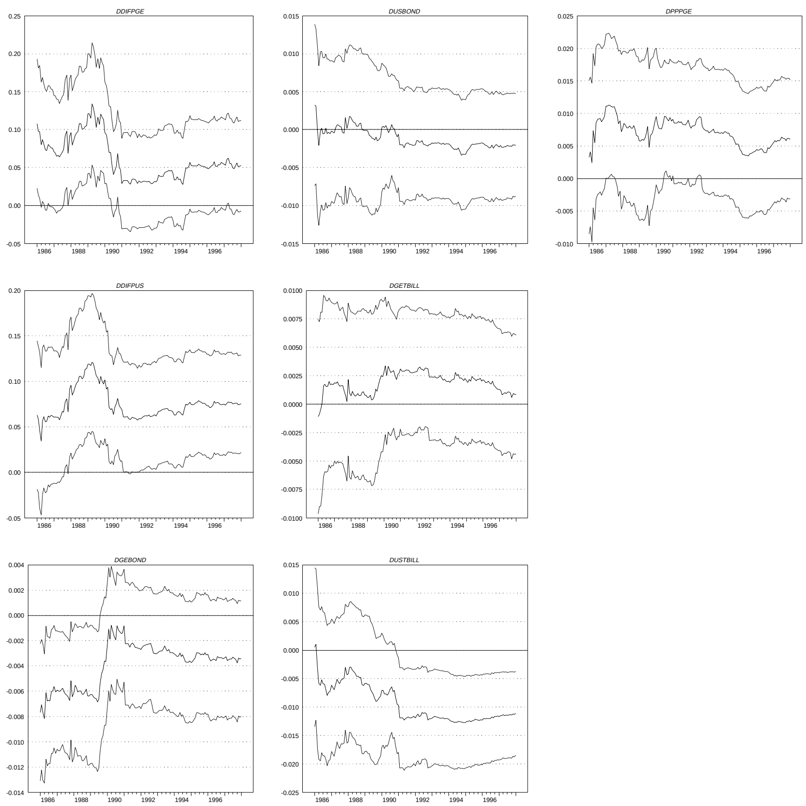
Recursively calculated test statistics (1.0 corresponds to the 5% significance level) for the test of a constant cointegration space.



Recursively calculated alpha coefficients to the first cointegration vector of Table 7.



Recursively calculated alpha coefficients to the second cointegration vector of Table 7.



Recursively calculated alpha coefficients to the third cointegration vector of Table 7.