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# Endogenous Business Cycles and Stabilization Policies

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# Endogenous Business Cycles and Stabilization Policies*

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Abstract

We analyze the e¾ects of simple stylized economic policy rules, or stabilization principles, when À uctuations in economic activity are created endogenously by self-ful¿lling volatile expectations. We study a simple monetary competitive model with intertemporally optimizing agents and a government. We only depart from neoclassical orthodoxy by assuming that a cycle or a sunspot equilibrium, not necessarily a steady state, could be the descriptive dynamic rational expectations equilibrium. The government may then well out of welfare concerns want to conduct systematic stabilization policy through transfers, expenditure, and taxation even though this has distortionary e¾ects. We show that the policy rules that stabilize output in a way that is best for welfare involve countercyclical elements in government activity.

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Keywords: Endogenous business cycles, stabilization policy.

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# 1 Introduction

What are the implications for stabilization policies if economic fluctuations, or business cycles, are to an important extent created endogenously by the economies' equilibrium mechanisms, and are not solely reactions to exogenous shocks? We find that, even under perfectly competitive conditions, the occurrence of endogenous fluctuations due to self-fulfilling volatile expectations may give reasons based upon welfare concerns for systematic government stabilization policy, and the policy rules which best stabilize economic activity with respect to welfare involve a certain kind of countercyclicality in government activity.

We study the most standard and fully competitive dynamic model of expectations-driven endogenous fluctuations. This happens to be the simple overlapping generations model with only labor as input in production, but by the argument of Woodford (1986), the model has an equivalent interpretation with infinitely lived agents and cash-in-advance constraints. The model involves a government which we assume can tax income proportionally and pay transfers to the old. However, (positive) real transfers to the old and government demand for output work in exactly the same way in the considered model as long as government and private demand for goods are assumed to be perfect substitutes in the consumers' utility functions (which is a natural bench mark assumption). Real government spending can therefore be interpreted equally as real transfers or government demand.

We depart from neoclassical orthodoxy by assuming that a cycle or a sunspot equilibrium could be the relevant rational expectations equilibrium describing how the economy evolves over time.[1] As far as steady state is concerned the model is such that neoclassical policy views are strongly supported; active government is unambiguously bad. The essential departure from Keynesian modelling is that we do not assume any price rigidities.

Government spending is linked to the performance of the economy by policy

[1] The sufficient condition normally considered for the existence of endogenous fluctuations under laissez faire in the simple OLG model is that the elasticity of labor supply with respect to the intertemporal real wage measured at steady state is less than minus one half. Some find this problematic. However, the purpose here is not to obtain plausible conditions for the existence of expectations-driven endogenous fluctuations, but rather to demonstrate how certain intertemporal effects of stabilization policies become of (increased) importance, should such fluctuations occur. We may therefore as well start from the simplest possible model of expectations-driven endogenous fluctuations.

rules meant to formalize realistic or frequently suggested stabilization principles. It is ¿nanced either by proportional income taxation or by inÀationary taxation (seigniorage). We consider and axiomatize a simple class of policy rules where in each period real government spending depends homogeneously on the current and the past level of GNP. Two special cases are: śpending proportional to current GNP´, and śpending proportional to past GNP´. These are important because they are simple, manageable and possibly implementable by automatic stabilizers. The ¿rst of these two cases is equivalent to arranging spending such that, in the absence of income taxation, a constant money growth rate results.

Our conclusions are that even under fully competitive conditions, the assumption that expectations driven endogenous Àuctuations could be relevant rational expectations dynamic equilibria may well, on welfare grounds, motivate systematic stabilization policies by the government. Moreover, despite the absence of price rigidities to motivate it, the policy rules which stabilize economic activity in the best way with respect to welfare entail a certain kind of countercyclicality in government activity: government spending should be relatively low in periods up to which output has increased by a relatively large amount.

The intuition for why countercyclical policy rules stabilize output most e¾ectively, and at the lowest welfare costs, is simple and related to certain intertemporal e¾ects of systematic stabilization policies. Assume that GNP increases by a relatively large amount from one period to the next. If this is correctly foreseen from the ¿rst period, and people know and believe in a countercyclical policy rule, then they will expect relatively low transfers during the next period. If leisure and output are normal goods (which is realistic), labor supply and output will increase in the ¿rst period, and thus the increase in output from the ¿rst to the second period will be reduced. If (relatively large) changes in output are reduced, output will become more stable. Interestingly, Benassy (1998) ¿nds that a similar intertemporal e¾ect is important for the stabilization of competitive Àuctuations caused by exogenous shocks, and Benassy also establishes support for countercyclical policy rules.

The present paper is related to contributions such as Grandmont (1986), Goenka (1994), Sims (1994), and Woodford (1994), which also study the e¾ects of ¿xed and realistic policy rules on endogenous Àuctuations, and closest to the ¿rst of these. There are, however, three main di¾erences in assumptions between Grandmont's and our paper, which imply that we are led to radically di¾erent policy conclusions. First, Grandmont makes (implicit) assumptions on fundamentals ensuring that the perfect foresight dynamic he derives has a traditional úni-modal´shape, whereas we consider a set of assumptions also allowing other shapes. Second, Grandmont studies constant money growth rules, whereas we consider a broader class of policy rules containing constant money growth rules as special cases. Third, Grandmont assumes zero substitution between private and public goods (government demand does not enter into utility functions at all), whereas we assume perfect substitution. The basic ¿nding of Grandmont is that constant money growth rules will stabilize the economy at steady state if the money growth rate is large enough. Our results indicate that constant money growth rules are, at best, very poor stabilization instruments. There is no formal contradiction between Grandmont's analysis and ours, but the broader assumptions we consider lead us to results from which the policy implications that could be drawn are very di¾erent from those that could be drawn from Grandmont's analysis.

Our consideration of more di¾erent assumptions on fundamentals reveals that constant money growth rules, although e¾ective in stabilizing output under some assumptions on fundamentals, are incapable of stabilizing output under other and equally plausible assumptions.

Our analysis of a parametrized class of policy rules reveals which aspects of policy rules make them e¾ective with respect to stabilization. It turns out that countercyclicality in the sense explained above is essential. Constant money growth rules are equivalent to śpending proportional to current GNP´-rules, and hence they are procyclical. In fact we ¿nd that constant money growth rules are just at the boundary of the set of policy rules that can be stabilizing at all, and even when they are in this set, they stabilize output in a worst possible way welfarewise.

Our assumption on the degree of substitution between private and government demand gives us, contrary to Grandmont, that constant money growth rules, not only for transfers, but also for government demand, are ine¾ective stabilization instruments. This reveals that the assumed degree of substitution between private and government demand is important for the intertemporal e¾ects of systematic stabilization policies working through government demand.

An important feature shared by Grandmont's and our model is that it is simple enough to give a one-dimensional, ¿rst order di¾erence equation as perfect foresight dynamic. This makes it possible to establish enough global properties to be able to use global determinacy as criterion for stabilization. Other authors study more complicated models yielding two-dimensional dynamic systems, e.g. Schmitt-Grohi and Uribe (1997), and Guo and Lansing (1997), (1998). It is then diÁcult to establish enough global properties of the dynamic system to be able to use global determinacy as a stabilization criterion. These papers then use local determinacy: if government policy can turn the steady state away from being a sink it is considered to stabilize the economy. However, it is only in linear systems that local determinacy is suÁcient for the elimination of endogenous Àuctuations, and linearity only follows from special assumptions on fundamentals. Using local determinacy as stabilization criterion is essentially the same as studying only a linear approximation of the dynamic system around the steady state and is particularly questionable for the two-dimensional case since with non-linearity a local property of the steady state di¾erent from it being a sink can suÁce for the existence of endogenous Àuctuations, see Grandmont, Pintus, and de Vilder (1998). Christiano and Harrison (1996) also stress the importance of one-dimensional dynamics, and for the same reasons as we do, but, like Guo and Lansing (1998), they study the e¾ects of more sophisticated policies involving, e.g., progressive taxation.

In Section 2 we describe the basics of the economic model and the class of policy rules we consider. Section 3 derives the equilibrium dynamics, and Section 4 states the results on the stabilizing and destabilizing e¾ects of di¾erent policy rules. In Section 5 we provide two illustrative examples concerning the particularly interesting rules ģovernment spending proportional to current GNP´and ģovernment spending proportional to past GNP.´ Section 6 summarizes conclusions. Proofs of propositions are given in Appendix A. Appendix B contains a technical result that is of importance for our purposes, and may be of independent interest.

# 2 The Economy and the Policy Rules

We consider an overlapping generations model in discrete time. In each period the commodities are labor input, produced output, and money. The money prices of labor and output are $w > 0$ and $p > 0$ respectively, and labor and output markets are perfectly competitive. Subscript $t$ is used for explicit reference to a period.

In each period a representative ¿rm produces output $y \geq 0$ from labor input $l \geq 0$ under constant returns to scale, $y = l$.

There is in each period one young and one old consumer, and a consumer is endowed with one unit of labor time in his youth. The von Neumann-Morgenstern utility function of a consumer is $u(c) + v(e)$, where $c \geq 0$ is output consumption in the consumer's old age, and $e := 1 - n \geq 0$ is leisure consumption in the youth; $n$ is labor supply when young.

The assumption that in the ¿rst period of a consumer's life only leisure enters utility, and in the second only consumption, implies an equivalence to a cash-in-advance constrained economy with an in¿nitely-lived consumer. In the latter type of model the consumer maximizes $\sum_{i=t}^{\infty}(v(e_i) + \tilde{u}(c_i))/(1+\theta)^{i-t}$ in each period $t$ subject to a budget constraint. If, in addition, there are binding cash-in-advance-constraints, he or she can do no better than to maximize $v(e_t) + \tilde{u}(c_{t+1})/(1+\theta)$ independently over each succession of two periods under the constraint that what can be used for consumption in $t+1$ is what was earned from work in $t$, see Woodford (1986). For this alternative model interpretation our $u(c)$ is $\tilde{u}(c)/(1+\theta)$.

We impose standard assumptions on $u$ and $v$: they are continuously di¾erentiable several times, $u'(c)$ and $v'(e)$ are strictly positive and go to in¿nity as $c$ and $e$ respectively go to zero, and $u''(c)$ and $v''(e)$ are strictly negative. We denote the Arrow-Pratt measure of relative risk aversion in $u$ by $R(c) := -u''(c)c/u'(c) > 0$, and also de¿ne $N(n) := -v''(1-n)n/v'(1-n) > 0$. We assume that $R(0) := \lim_{c \to 0} R(c)$, and $N(0)$ both exist (are $< \infty$), and that $R(0) \neq 1$.

Finally, there is a government that in each period decides on a real lump sum transfer $b$ given to the period's old consumer, and on a proportional tax rate $\tau$, where $0 \leq \tau < 1$, by which the income of the period's young consumer is taxed. Both $b$ and $\tau$ are taken as parametric by the consumers. For the interpretation of our model with an in¿nitely-lived agent, the cash-in-advance constraint should be assumed to work such that in the current period the consumer can spend last period's net of tax income plus the transfer received in the current period.

The variable $b$ can, if positive, alternatively be interpreted as government demand for output (or labor). If it is assumed that public and private goods are perfect substitutes, so the utility function of a consumer is $v(1-n) + u(c+b)$, then the resulting dynamic model will be identical to the one in which $b$ is a transfer. This will be demonstrated below.

Policy is conducted according to certain feedback rules linking in a systematic way the value of the real transfer, or government demand, to present and past values of the GNP (in equilibrium $y$ summarizes everything of economic importance in a period). The rules are meant to formalize possible stabilization principles. Government spending is ¿nanced by either proportional taxation or seigniorage or a mix of both. The exact ¿nancing does not matter since direct proportional and inÀationary taxation have the same e¾ects.

The considered policy rules are meant to formalize realistic (or frequently suggested), simple, and manageable stabilization principles. Therefore we con¿ne attention to rules of the form, $b_{t+1} = b(y_{t+1}, y_t)$, and impose the further restrictions:

(i) The variable $b$ should be (weakly) positive in all periods. For the interpretation of $b$ as government demand this is required. For the interpretation as a transfer there is in principle nothing wrong with negative values, but $b < 0$ means lump sum taxation (of the old) together with subsidies (to the young) proportional to income, these subsidies coming either directly or through negative inÀation. Lump sum taxes are seldom observed and variations in lump sum taxes are never seen as part of stabilization policies.

(ii) At a constant GNP, the government behaves as if it taxes GNP by a certain rate and balances the budget in each period. That is, we require $b(y, y) = \beta y$ for some $\beta$ with $0 \leq \beta < 1$. Indeed, to be a formalization of a stabilization principle the rule should dictate ńeutral government behavior´at a steady GNP. We think it is most realistic to let neutral behavior correspond to ¿xed proportional taxation and budget balance (rather than, e.g. to a ¿xed spending that is independent of $y$).

(iii) At a varying GNP the stabilization e¾ort should depend on the relative variation in GNP. If two pairs $(x, y)$ and $(x', y')$ of current and past GNPs represent the same degree of relative up or down swing in economic activity, $x/y = x'/y'$, then the government stabilization e¾ort should be relatively the same in the two situations, i.e. $b(x, y)/x = b(x', y')/x'$.

These requirements are ful¿lled if and only if $b$ is of the form $b(y_{t+1}, y_t) = \beta\phi(y_{t+1}, y_t)$, where $0 \leq \beta < 1$, and $\phi$ is positive and homogeneous of degree one, with $\phi(1, 1) = 1$. We consider such functions because they are axiomatized by the above requirements, that we ¿nd reasonable, but an independent reason is that they suÁce for revealing the intertemporal incentive e¾ects of importance for stabilization of endogenous Àuctuations. It will give simple statements and proofs of our propositions to focus on the case,

$$b(y_{t+1}, y_t) = \beta y_{t+1}^{1-\alpha} y_t^{\alpha}, \tag{1}$$

where there are no a priori restrictions on the parameter $\alpha$.

Each policy rule of the form (1) contains a level (or resting) component given by $\beta$, and a cyclical (or reactive) component given by $\alpha$. This is illustrated by the rewriting $b(y_{t+1}, y_t)/y_{t+1} = \beta(y_{t+1}/y_t)^{-\alpha}$. The level component $\beta$ is the transfers' share in current output when output is constant, and the cyclical component $(y_{t+1}/y_t)^{-\alpha}$ is the responsiveness of this share to changes in output, $-\alpha$ being the elasticity of the transfers' share with respect to the output growth factor. The larger $\alpha$ is, the more negative will be the reaction in the transfers' share to increases in output, that is, the more ćountercyclical´will the rule be. In particular, if output evolves according to a two-period cycle with output levels $h$ and $l$, where $h > l$, and the transfer payed in periods with output $h$ is denoted by $b_h$ etc., then $(b_h - b_l)(h - l) = k(1 - (h/l)^{2\alpha - 1})$, where $k > 0$, so $(b_h - b_l)(h - l)$ is positive if $\alpha < 1/2$, and negative if $\alpha > 1/2$. This means that along two-period cycles policy rules of the considered form give procyclical transfers (in the usual sense) when $\alpha < 1/2$, and countercyclical when $\alpha > 1/2$.

One reason to be interested in the class (1) of policy rules is that it contains some important and frequently considered rules as special cases:

Transfers proportional to current GNP. The case $\alpha = 0$, gives $b(y_{t+1}, y_t) = \beta y_{t+1}$. Although simple, this is a feedback rule. As the economy's activity level varies so will the real value of the transfer in per capita terms (procyclically). The rule is equivalent to setting the income tax rate constantly to $\beta$ and let transfers be determined by budget balance in each period, and, in particular, it is equivalent to arranging the sequence of transfers such that with no income taxation a constant money growth rate results (as shown below). Constant money growth rules are often advocated and were studied by Grandmont (1986) in connection with stabilization of endogenous Àuctuations. Grandmont (1986) found that a large enough $\beta$ will stabilize the economy at steady state.

Transfers proportional to past GNP. The case $\alpha = 1$, gives $b(y_{t+1}, y_t) = \beta y_t$. Note that if taxes come with a delay then this rule may be equivalent to setting the income tax rate constantly at $\beta$ and letting budget balance determine $b$, an implementation most relevant for the model interpretation with a cash-in-advance constraint and an in¿nitely-lived consumer. For the literal overlapping generations interpretation, note that this rule rewards old consumers according to how much they worked and contributed when young. Finally, this rule is closer to standard Keynesian stabilization recommendations since government spending may react countercyclically.

When $\alpha$ goes below zero or above one, the dependence of transfers on GNP becomes more complicated. As long as $0 \leq \alpha \leq 1$, the rules have the possible simple implementation that an average (though geometrical) of the last two GNPs is taxed and the revenue used for transfers.

In what follows it is assumed that the policy rule $b(y_{t+1}, y_t)$ used by the government is known by the households who also have rational expectations with respect to next period's output price. Furthermore, the households are assumed to believe in the relevant policy rule.

# 3 Dynamic Equilibrium

In (non-trivial) equilibrium one must have $w = p$ in all periods, and that any level of production and employment is optimal for the ¿rm.

Consider a young consumer whose expectation concerning the next period is that with probability $q_j$ the output price will be $p_j$ and the transfer received will be $b_j$, where $j = 1, ..., r$. A point expectation corresponds to $r = 1$. The consumer chooses labor supply $n$, money holding $m$, and consumption $c_j$ in each of the $r$ future ´states´ to maximize expected utility $v(1-n) + \sum_j q_j u(c_j)$, subject to the budget constraints $m = (1-\tau)wn$, and $c_j = m/p_j + b_j$ for $j = 1, ..., r$, where $w$ and $\tau$ are the nominal wage rate and the tax rate in the consumer's young age respectively. The optimal choices for $n$ and $c_j$ are uniquely given by the ¿rst order condition,

$$\frac{v'(1-n)}{(1-\tau)w} = \sum_{j=1}^{r} q_j \frac{u'(c_j)}{p_j}, \tag{2}$$

and the budget constraints,

$$c_j = (1-\tau)\frac{w}{p_j}n + b_j \text{ for } j = 1, ..., r. \tag{3}$$

In the case of a point expectation (where $p$ and $b$ are expected), the optimality conditions amount to $v'(1-n) = \omega u'(c)$, and $c = \omega n + b$, where $\omega := (1-\tau)w/p$. Solving for $n$ and $c$ gives the labor supply curve $n = n(\omega, b)$, and the future demand for produced goods $c = c(\omega, b)$. It is a consequence of our assumptions that leisure and consumption are both strict normal goods, $n'_b < 0$ and $c'_b > 0$.[2]

## 3.1 Temporary Equilibrium

From (3), $(1-\tau)w/p_j = (c_j - b_j)/n$. Inserting this into (2) gives, $nv'(1-n) = \sum_j q_j(c_j - b_j)u'(c_j)$. Inserting the equilibrium conditions $n = y$ and $y_j = c_j$, then gives $yv'(1-y) = \sum_j q_j(y_j - b_j)u'(y_j)$. Inserting ¿nally the policy rule $b_j = b(y_j, y)$ yields,

$$yv'(1-y) = \sum_{j=1}^{r} q_j \left[y_j - b(y_j, y)\right] u'(y_j). \tag{4}$$

This is the temporary equilibrium equation for the considered economy in terms of production levels. If the young consumer expects output in the next period to be $y_j$ (between zero and one) with probability $q_j$, $j = 1, ..., r$, and knows and believes in the policy rule $b(\cdot, \cdot)$, then a $y$ (between zero and one) is an equilibrium output of the current period if and only if it ful¿ls (4).

All rational expectations dynamic equilibria studied below are de¿ned from the temporary equilibrium equation (4). The tax rates do not enter into this. Hence, for a given policy rule for spending, the rational expectations equilibrium dynamics of the considered economy is independent of how much income taxation vs. seigniorage is used in ¿nancing government spending. Proportional income taxation and inÀationary taxation work and distort in exactly the same way.

[2] Labor supply is given by $v'(1-n) = \omega u'(\omega n + b)$. A larger $b$ implies a lower right hand side, and to recreate equality $n$ must fall since this both decreases the left hand, and increases the right hand, side, so $n'_b < 0$. A similar exercise on $v'(1-(c-b)/\omega) = \omega u'(c)$ shows $c'_b > 0$. For later use, we derive the elasticity of labor supply wrt. $\omega$ by log-di¾erentiation,

$$\varepsilon_\omega := \frac{n'_w \omega}{n} = \frac{1 - R(\omega n + b)\frac{\omega n}{\omega n + b}}{N(n) + R(\omega n + b)\frac{\omega n}{\omega n + b}} > -1.$$

Consider the alternative interpretation of $b$ as government demand. In this case, the consumer would maximize $v(1-n)+\sum_j q_j u(c_j+b_j)$ subject to the budget constraints $c_j = (1-\tau)\frac{w}{p_j}n$, $j = 1, ..., r$. The ¿rst order condition would be,

$$\frac{v'(1-n)}{(1-\tau)w} = \sum_{j=1}^{r} q_j \frac{u'(c_j+b_j)}{p_j}.$$

By use of the budget constraints, $(1-\tau)w/p_j = c_j/n$, one gets $nv'(1-n) = \sum_j q_j c_j u'(c_j+b_j)$. In equilibrium, $n = y$ and $y_j = c_j + b_j$, and hence $yv'(1-y) = \sum_j q_j (y_j - b_j) u'(y_j)$. Inserting a policy rule for government demand, $b_j = b(y_j, y)$, would give exactly (4). The two interpretations of $b$ lead to the same equilibrium condition which veri¿es the equivalence postulated in Section 2.

## 3.2 Perfect Foresight Dynamics and Steady State

The economy's perfect foresight dynamics is obtained from (4) assuming that the next period's output is correctly foreseen from the current period in a deterministic sense. Inserting $y_j = y_{t+1}$ for all $j$, and rewriting current output as $y = y_t$, one arrives at a ¿rst order, one-dimensional di¾erence equation in $y_t$ and $y_{t+1}$,

$$y_t v'(1-y_t) = \left[y_{t+1} - b(y_{t+1}, y_t)\right] u'(y_{t+1})). \tag{5}$$

A dynamic perfect foresight equilibrium is a sequence $(y_t)$ of production levels $0 \leq y_t < 1$, such that (5) is ful¿lled for all $t$. A steady state is a particular case where $y_t = y$ in all periods. For all the policy rules we consider, $b(y, y) = \beta y$, and it follows from (5) that a strictly positive, or monetary, steady state production level $y$ is given by,

$$\frac{v'(1-y)}{u'(y)} = 1 - \beta. \tag{6}$$

Since the MRS on the left hand side goes from zero to in¿nity as $y$ goes from zero to one, there is for any $\beta$ a unique monetary steady state $y(\beta)$, and $y(\beta) < 1$. It follows directly that $y(\beta)$ is strictly decreasing in $\beta$, and that $y(\beta)$ goes to zero as $\beta$ goes to one.

If we de¿ne welfare at the steady state as the common utility of all generations, $W(\beta) := u(y(\beta)) + v(1-y(\beta))$, then $W' = (u' - v')y'_\beta$, and from $y'_\beta < 0$ and (6), $W' < 0$ for all $\beta > 0$, and $W' = 0$ for $\beta = 0$. This proves, Proposition A. For all $\beta$, there is a unique monetary steady state involving production $y(\beta)$, with $0 < y(\beta) < 1$, and $y(\beta)$ is strictly decreasing in $\beta$ and $y(\beta) \to 0$ as $\beta \to 1$. Welfare at steady state $W(\beta)$ is unambiguously decreasing in $\beta$, and optimal policy for steady state is $\beta = 0$.

Proposition A is a simple version of a familiar neoclassical proposition. In the absence of distributional reasons for transfers, one is left, at steady state, with the pure distortional e¾ect of the taxation, direct or inÀationary, implied by giving the transfers. Proposition A implies that government activity has to be motivated by the steady state not being the appropriate descriptive equilibrium. Furthermore, should endogenous Àuctuations prevail (under laissez faire) and should one, by use of a policy rule belonging to the considered class, manage to stabilize the economy at steady state, then it is unambiguously to be preferred that this is done for as low a value of $\beta$ as possible, since $\beta$ measures the degree of distortion at steady state.[3]

The left hand side of (5) increases from zero to in¿nity as $y_t$ goes from zero to one. If $\alpha \geq 0$, or $\beta = 0$, then $b$ is (weakly) increasing in $y_t$, so the right hand side will, for any given $y_{t+1} > 0$, decrease weakly from a strictly positive value as $y_t$ increases from zero. This means that for every positive $y_{t+1}$, there is a unique $y_t$ between zero and one that solves (5), which thus everywhere implicitly de¿nes $y_t$ as a function $f$ of $y_{t+1}$. From the Implicit Function Theorem, $f$ is continuously di¾erentiable. So, for $\alpha \geq 0$, or $\beta = 0$, the backward perfect foresight dynamic $y_t = f(y_{t+1})$ is well-de¿ned globally. For $\alpha < 0$ and $\beta > 0$ it is not. In that case there are for $y_{t+1}$ small enough several solutions in $y_t$ to (5), and for $y_{t+1}$ large enough there are none. As just shown there is, however, a unique monetary steady state $y(\beta)$, and locally around $y(\beta)$ the backward perfect foresight dynamic $f$ is again well-de¿ned and continuously di¾erentiable.[4]

[3] It could be argued that the right welfare measure at steady state is rather $V(\beta) = u(y(\beta))/(1+\theta) + v(1 - y(\beta))$, where $\theta > 0$ is a time preference rate. In a free optimization one will then ¿nd that optimal policy for steady state is some $\beta < 0$, which, in the absence of direct taxation, is equivalent to a constant negative money growth rate, a so-called Friedman rule. If one only allows $\beta \geq 0$, then also in this case $\beta = 0$ is optimal for steady state.

[4] From the Implicit Function Theorem, $f$ is locally well-de¿ned by (5) around steady state if the derivative of $y_t v'(1 - y_t) - [y_{t+1} - b(y_{t+1}, y_t)]\, u'(y_{t+1}))$ wrt. $y_t$ measured at steady state is not zero. This derivative is $v'(1 - y(\beta))(1 + N(y(\beta)) + \alpha\beta u'(y(\beta))$, which, for any given $\beta$, is zero only for one particular (non-generic) negative value of $\alpha$.

## 3.3 Rational Expectations Fluctuations

A deterministic $r$-cycle is a collection of $r$ di¾erent production levels $0 < y_1, ..., y_r < 1$ in the range where $f$ is well-de¿ned such that $y_1 = f(y_2), ..., y_r = f(y_1)$. An $r$-state stationary (Markov) sunspot equilibrium, SSE, consists of $r$ production levels $0 < y_1 \leq \cdots \leq y_r < 1$, where $y_1 < y_r$, and $r^2$ transition probabilities $q_{ij}$, $\sum_{j=1}^{r} q_{ij} = 1$ for $i = 1, ..., r$, where the matrix $(q_{ij})$ is irreducible, such that, whenever the young consumer expects that the output level $y_j$ will occur with probability $q_{ij}$ next period, $j = 1, ..., r$, then the current temporary equilibrium output level according to (4) is exactly $y_i$, that is,

$$y_i v'(1 - y_i) = \sum_{j=1}^{r} q_{ij} \left[y_j - b(y_j, y_i)\right] u'(y_j) \text{ for } i = 1, ..., r. \tag{7}$$

The well-known idea is that one can imagine that an irreducible Markov chain (a sunspot) on states $1, ..., r$, sending state $i$ into state $j$ with transition probability $q_{ij}$, though exogenous to the economic system, may govern its performance. If the agents know the transition probabilities and believe that in any period output must be $y_i$ if the state is $i$, then output will indeed be governed by the sunspot and À uctuate accordingly, and the agents will have no reason to revise their beliefs since their expectations are probabilistically correct, i.e. rational. An $r$-cycle is a particular, non-stochastic $r$-state SSE.

Deterministic cycles and SSE are our candidates for rational expectations dynamic equilibria exhibiting endogenous À uctuations.

Our results concerning stabilization of endogenous business cycles will rely on some relationships between the perfect foresight dynamic $f$ and the existence of cycles and sunspot equilibria. It is well-known that if $f$ is such that an $r$-cycle exists then there is also a truly stochastic $r$-state SSE close to the cycle, see Guesnerie and Woodford (1992). It is not generally true that the existence of a SSE implies the existence of deterministic cycles, or, equivalently, that non-existence of cycles implies non-existence of SSE. For our purposes it is, however, important to establish such a connection. In Appendix B we prove a proposition stating some general conditions under which the existence of a SSE implies the existence of a 2-period cycle. The conditions are such that we will be able to conclude that the policy rules which eliminate all cycles through establishing global stability according to $f$

of the monetary steady state, also eliminate all SSE.[5] By virtue of these and some other well-known results it will suffice in what follows to study the perfect foresight dynamic $f$. To be precise we will make use of the following standard "dynamic properties":

Indeterminacy. If $f$ is locally well-defined around steady state and the slope of $f$ at the steady state is below minus one or above one, then the steady state is locally stable in the forward direction under perfect foresight, and the steady state is said to be indeterminate. It is well known that indeterminacy implies the existence of SSE arbitrarily close to the steady state, see Guesnerie and Woodford (1992), and for the dynamics we consider, if $f'(y(\beta)) < -1$, there are also deterministic cycles. It is an "opening assumption" of this paper that indeterminacy is a sufficient condition for a cycle or a sunspot equilibrium to be the relevant dynamic equilibrium (if it were the steady state there would not be a stabilization problem). In favor of this assumption is the fact that for plausible backward looking learning rules, a steady state (or cycle) is locally unstable according to learning dynamics exactly when it is stable according to forward perfect foresight dynamics, see Grandmont (1985), Marcet and Sargent (1989), Evans and Honkapojha (1995).[6]

Determinacy. Assume that by appropriate use of one of the policy rules considered it can be obtained that the steady state $y(\beta)$ becomes globally stable according to $f$, implying that $f$ is globally well-defined. Then there can be no deterministic cycles and, from Theorem B shown in Appendix B, for the policy rules that we find indeed can make $y(\beta)$ globally stable according to $f$, no SSE either. The steady state is then the only reasonable bounded and continuously well-defined rational expectations equilibrium, and one says that the steady state is (globally) determinate. Determinacy will be considered a sufficient condition for stabilization at steady state.

[5] The method used in Appendix B to establish that existence of a SSE implies existence of a 2-period cycle is similar to the one used by Grandmont (1986). However, the dynamics arising from our policy rules are not covered by the generality of Grandmont's result. Therefore the theorem in Appendix B generalizes Grandmont's result and it may therefore be of independent interest.

[6] The robustness of this "reversion of stability properties" result has more recently been disputed by Grandmont (1998).

# 4 Stabilization

Inserting the considered speci¿c functional form of policy rules into (5) gives,

$$y_t v'(1-y_t) = (y_{t+1} - \beta y_{t+1}^{1-\alpha} y_t^{\alpha}) u'(y_{t+1}), \tag{8}$$

which de¿nes $y_t = f(y_{t+1})$ at least locally around steady state $y(\beta)$. For any $x > 0$, at which $f(x)$ is well-de¿ned, the slope of $f$ is obtained by implicit di¾erentiation of (8) written as $f(x)v'(1-f(x)) = [x - \beta x^{1-\alpha} f(x)^{\alpha}]\, u'(x)$. This gives,

$$f'(x) = \frac{f(x)}{x} \frac{1 - \beta(1-\alpha)(\frac{f(x)}{x})^{\alpha} - \left(1 - \beta(\frac{f(x)}{x})^{\alpha}\right) R(x)}{1 - \beta(1-\alpha)(\frac{f(x)}{x})^{\alpha} + \left(1 - \beta(\frac{f(x)}{x})^{\alpha}\right) N(f(x))}. \tag{9}$$

Measuring $f'$ at steady state where $x = f(x) = y(\beta)$ gives,

$$f'(y(\beta)) = \frac{1 - \beta(1-\alpha) - (1-\beta)R(y(\beta))}{1 - \beta(1-\alpha) + (1-\beta)N(y(\beta))}. \tag{10}$$

Assume $\beta > 0$. If also $\alpha > 0$, then $f$ is globally well-de¿ned, and for any $y_{t+1} > 0$, the $y_t$ that solves (8) is below $y_{t+1}/\beta^{1/\alpha}$. Hence, as $y_{t+1}$ goes to zero, so must this $y_t$, implying $f(0) := \lim_{x \to 0} f(x) = 0$. If $\alpha = 0$, then $f$ is still globally well-de¿ned, and (8) reads $y_t v'(1-y_t) = (1-\beta) y_{t+1} u'(y_{t+1})$. As $y_{t+1}$ goes to zero, so will the right hand side if and only if $R(0) < 1$.[7] Hence, if $R(0) < 1$, one still has $f(0) = 0$, whereas if $R(0) > 1$, one has $f(0) = 1$.

Taken together, if $\alpha, \beta > 0$ or if $\alpha = 0$ and $R(0) < 1$, the globally well-de¿ned backward dynamic $f$ starts at zero, $f(0) = 0$, and stays everywhere below one, $f(x) < 1$. If $f$ ends at zero, $f(\infty) = 0$, it must have a number of critical points $(x^c, f(x^c))$ at which $f'(x^c) = 0$. If it ends elsewhere it may or may not have critical points. In any case, $f$ has a shape such that if all critical points are below the 45°-line, i.e. ful¿ll $f(x^c)/x^c < 1$, then $y(\beta)$ is globally stable according to $f$. This excludes deterministic cycles, and since the general conditions of Proposition B in Appendix B are satis¿ed when $\alpha \geq 0$, there is no SSE either, and the steady state is determinate. This argument is used to establish Proposition 1.[8]

[7] Note that $R(0)$ is the elasticity measure of how fast $u'(y_{t+1})$ goes to in¿nity as $y_{t+1}$ goes to zero. Hence, if $R(0) < 1$, the product $y_{t+1} u'(y_{t+1})$ goes to zero as $y_{t+1}$ goes to zero etc.

[8] Since Proposition 1 is on stabilization, one could naturally expect an underlying assumption of indeterminacy of steady state under laissez faire, $f'(y(0)) < -1$ ($> 1$ is not possible). However, $f'(y(0)) < -1$ is not strictly necessary for the existence of rational expectations endogenous Àuctuations, and Proposition 1 does not assume it.

Proposition 1. (Stabilization: suÁ cient conditions for policy rules to establish determinacy).

(i) For any $\alpha > 0$, there is a $\beta^*(\alpha) < 1$, such that if the policy rule involves $\alpha$ and $\beta$ with $\beta > \beta^*(\alpha)$, then the steady state $y(\beta)$ is determinate and there are no cycles or stationary sunspot equilibria.

(ii) If $\alpha = 0$ and $R(0) < 1$, there also exists a $\beta^* < 1$, such that $\beta > \beta^*$ implies determinacy of the steady state and non-existence of cycles and stationary sunspot equilibria..

(ii) For any $\beta > 0$, there is an $\alpha^*(\beta) > 0$, such that if the policy rule involves $\alpha$ and $\beta$ with $\alpha > \alpha^*(\beta)$, then the steady state $y(\beta)$ is determinate and there are no cycles or stationary sunspot equilibria.

Proposition 1 is our main result. For any strictly positive choice of the cyclical component $\alpha$ of the policy rule, a suÁ ciently large level component $\beta$ will stabilize the economy at steady state, and for some assumptions on fundamentals the same is true for $\alpha = 0$. It also says that for any strictly positive choice of the level component $\beta$, in particular for (arbitrarily) small values, a large enough cyclical component $\alpha$ will stabilize the economy at steady state.

These are suÁ cient conditions for stabilization. It is of particular interest to know if a low value of $\beta$ necessitates a high value of $\alpha$ for stabilization. It may not, of course, if the economy does not have any business cycle problem at all. The issue should therefore be addressed under an explicit assumption of the presence of a stabilization problem. Therefore Proposition 2 assumes $f'(y(0)) < -1$. In this case large values of $\alpha$ are indeed necessary for stabilization, given small values of $\beta$.

Proposition 2. (Stabilization: a necessary condition for policy rules to establish determinacy). Assume $f'(y(0)) < -1$. For all small enough $\beta > 0$, it is necessary and suÁ cient for a policy rule to imply $-1 \leq f'(y(\beta)) \leq 1$, and therefore necessary for determinacy of the steady state $y(\beta)$, that $\alpha$ is greater than or equal to a certain $\alpha^{**}(\beta)$, where $\alpha^{**}(\beta)$ goes to in¿nity as $\beta$ goes to zero.

The two propositions above do not exclude that large enough values of $\beta$ could also stabilize the economy for negative values of $\alpha$, or generally for $\alpha = 0$. Neither do they exclude that for some values of $\beta$, negative and small enough values of $\alpha$ could stabilize the economy. Proposition 3, however, rules out these possibilities.

Proposition 3. (Destabilization: suÁ cient conditions for policy rules to imply indeterminacy).

(i) If $\alpha < 0$, then $f'(y(\beta)) > 1$ for all suÁ ciently large $\beta$; hence, the steady state $y(\beta)$ is indeterminate and stationary sunspot equilibria exist.

(ii) If $\alpha = 0$ and $R(0) > 2 + N(0)$, then $f'(y(\beta)) < -1$ for all suÁ ciently large $\beta$; hence, the steady state $y(\beta)$ is indeterminate and both deterministic cycles and stationary sunspot equilibria exist.

(iii) If $\beta > 0$, then $f'(y(\beta)) > 1$ for all negative and suÁ ciently small $\alpha$; hence, the steady state $y(\beta)$ is indeterminate and stationary sunspot equilibria exist.

We proceed by raising a number of important remarks:

Elasticity and indeterminacy. It is well-known that for the suÁ cient condition for indeterminacy under laissez faire, $f'(y(0)) < -1$, to be ful¿ lled it is required that the elasticity of labor supply with respect to the real wage at steady state is less than minus one half. Inserting into the $\varepsilon_\omega$ of footnote 2, that at steady state $\omega n + b = y$, $\omega n = (1-\beta)y$, and $b = \beta y$, one gets for the elasticity at steady state,

$$\varepsilon_\omega(\beta) = \frac{1-(1-\beta)R(y(\beta))}{N(y(\beta)) + (1-\beta)R(y(\beta))}.$$

From (10), $f'(y(0)) < -1 \iff R(0) > 2 + N(0)$, and it is easy to see that this implies $\varepsilon_\omega(0) < -1/2$. Although not necessary, $f'(y(0)) < -1$ is kind of a śine qua non´condition for endogenous À uctuations under laissez faire, and it has often been held against the theory of such À uctuations that $f'(y(0)) < -1$ can only be ful¿ lled for unrealistic values of the elasticity of labor supply. Proposition 3 says that if $\alpha < 0$, or if $\alpha = 0$ and $R(0) > 2 + N(0)$, a suÁ cient condition for indeterminacy, $f(y(\beta)) < -1$ or $f(y(\beta)) > 1$, is ful¿ lled for all suÁ ciently large $\beta$. As $\beta$ goes to one, $\varepsilon_\omega(\beta)$ goes to $1/N(0) > 0$, so for all large enough $\beta$, one has both indeterminacy, and $\varepsilon_\omega(\beta) > 0$. All that it is needed to overcome the unrealistic requirement on the elasticity of labor supply is an inappropriate government policy, and this does not have to be more peculiar than a constant money growth rate rule.[9]

Welfare. The above propositions are about output stabilization which should only be an aim for economic policy if output stabilization has good welfare implications. For the literal overlapping generations interpretation of our model, output

[9]This way of overcoming the requirement of a (very) negatively sloped labor demand curve is closely related to the ímperfect competition and positive pro¿ ts´way found in Jacobsen (2000).

stabilization can never be generally Pareto improving. Along a two-period cycle the generations who are young when output is low are fortunate, since they work little while young and consume much when old. Stabilization can only give these generations lower utility. However, for that model interpretation it seems reasonable to let also a concern of equity across generations enter into welfare considerations. For the model interpretation with an in¿nitely-lived consumer and cash-in-advance constraints, output stabilization is good because of the concavity of utility functions, but at the same time it is bad because of the distortion of the steady state it implies. For both model interpretations an economic policy that stabilizes output at steady state can be considered to have good welfare implications if the steady state is not too distorted by the policy. Therefore the above propositions, in combination with Proposition A, have strong welfare implications. For any given $\alpha > 0$, the economy can be stabilized at the steady state $y(\beta)$, if $\beta$ is large enough. This may require a high value of $\beta$, and therefore imply a large distortion of the steady state. However, for any strictly positive value of $\beta$, no matter how small, the economy will be stabilized at the steady state $y(\beta)$, if $\alpha$ is set suÁciently high. That is, one can stabilize the economy at steady state for an arbitrarily small distortion $\beta$, by choosing a large enough $\alpha$. Our propositions point to output stabilization by policy rules with low values of $\beta$ and correspondingly high values of $\alpha$: more elements of countercyclicality help to give stabilization with lower levels of distortion.

Countercycality. It is in the sense described earlier that the rules pointed to are countercyclical. They will require $\alpha > 1/2$ and, easily, values of $\alpha$ above one, for low enough values of $\beta$ (Proposition 2). So, the policy rules which are best in terms of welfare are such that government activity is relatively low in periods up to which output has increased by a relatively large amount. This is not exactly countercyclicality in the usual sense of relatively low government activity when output is relatively high, but such rules will, nevertheless, often appear countercyclical in the usual sense (e.g. over two-period cycles), and they certainly do have a Keynesian Àavor - but not for Keynesian reasons.

Intuition. No nominal or real rigidities have been assumed. So, why is it that the policy rules we have called countercyclical are the most stabilizing? If a policy can eliminate changes in GNP, it will have stabilized the economy. Assume that the economy evolves according to some cycle, and that output increases from the current to the next period. The more countercyclical the policy rule is, the lower a real transfer it will pay the next period, because the higher $\alpha$ is, the more negative dependence on GNP increases the rule contains. Because goods are normal, a lower value of the real transfer in the next period is exactly what it takes to make the consumer work more in the current one, thus increasing output here and diminishing the change in GNP from the current to the next period.

Related literature. Grandmont (1986) has assumptions with the same e¾ect as $R(0) < 1$ here and considers constant money growth rate rules. One of his results is similar to Proposition 1(ii). In view of Proposition 2, policy rules with $\alpha = 0$ are just at the boundary of the set of rules that can be stabilizing for large enough values of $\beta$, and even when they are in this set, they may well be the ones giving output stabilization in the worst possible way welfarewise, requiring the largest $\beta$.

Simplicity and Credibility. Policy rules with values of $\alpha$ way above one and with very low values of $\beta$ stabilize in the best way. As already argued such rules are not very simple. Furthermore, they may involve a credibility problem. At the steady state $y(\beta)$, at which the economy is stabilized, one will not see much government activity, only the constant and low $\beta y(\beta)$. The government may have problems convincing the public that this is only because À uctuations do not presently occur, and that should À uctuations occur the government would react strongly in accordance with its high $\alpha$. Simplicity and credibility considerations point to rules with non-extreme values of $\alpha$, say $0 \leq \alpha \leq 1$. We will therefore, for a speci¿cation of $u$ and $v$, consider the two particular cases $\alpha = 0$ and $\alpha = 1$. The two resulting rules are situated symmetrically around the ácyclicallity point´ $\alpha = 1/2$, with one end ($\alpha = 0$) being the often suggested constant money growth rate rule, and both are of equal structural simplicity. The examples will nicely illustrate the importance of the cyclicallity of policy rules.

# 5 Transfers Proportional to Current or Past GNP

Consider the speci¿cations,

$$u(c) = \delta\frac{(c + c_0)^{1-R}}{1 - R},\ v(e) = \frac{e^{1-R}}{1 - R} \text{ where } c_0 \geq 0,\ \delta > 0,\ R > 0, \tag{11}$$

for which $R(c) = Rc/(c + c_0)$, and $N(n) = Rn/(1 - n)$. Thus, if $c_0 > 0$, one has $R(0) = 0 < 1$, and when $c_0 = 0$, one has $R(c) = R$, and in particular $R(0) > 2 + N(0)$, whenever $R > 2$.[10]

For this example we consider the policy rules “transfers proportional to current GNP”, $b(y_{t+1}, y_t) = \beta y_{t+1}$,[11] and “transfers proportional to past GNP”, $b(y_{t+1}, y_t) = \beta y_t$, corresponding to $\alpha = 0$ and $\alpha = 1$ respectively.

First we let $R = 4$, $c_0 = 0.06$, and $\delta = 0.006$. This is a case where, even for $\alpha = 0$, a large enough $\beta$ will be stabilizing. It is, in addition, a case that gives a traditional “hump-shaped”, or uni-modal, dynamic $f$ with one critical point. For each case of $\alpha = 0$ and $\alpha = 1$, we iterate according to the relevant $f$ starting at the critical point. One is then led to the (deterministic) dynamic equilibrium which is stable according to $f$ (at most one is), and hence plausibly learning stable. This is a two-period cycle under laissez faire, $\beta = 0$, and for both $\alpha = 0$ and $\alpha = 1$, it remains as such for $\beta$ up to the level that stabilizes the economy.

In Figure 1, where (i) is for $\alpha = 0$, and (ii) is for $\alpha = 1$, the solidly drawn curves show the common utility of all generations at steady state as a function of $\beta$; this curve is the same for $\alpha = 0$ and $\alpha = 1$. In the model interpretation with an in¿nitely lived agent the curve shows the steady state utility of this agent. The $*$-dotted curves show, for $\alpha = 0$ in Figure 1(i) and for $\alpha = 1$ in Figure 1(ii), the utilities of the fortunate and the unfortunate generations respectively, at the stable two-period cycle as a function of $\beta$. The fortunate generations are those who are young when output is low and therefore work little, and old when output is high and therefore consume much. In the alternative interpretation the utility of the representative consumer along the two-period cycles would be more or less the

[10] This example does not ful¿ll all above requirements since $u'$ does not go to in¿nity as $c$ goes to zero when $c_0 > 0$. However, when we consider the example we simply ¿nd equilibria by computation and we do not need nice “boundary behavior”. The lack of an in¿nite marginal utility at zero is no problem for the computations as long as $\beta$ is below an appropriate upper limit, which is satis¿ed in all numerical simulations below.

[11] It was said earlier in the paper that this is equivalent to a constant money growth rate rule. To see this note that without income taxation the money stock must evolve as $M_{t+1} - M_t = p_{t+1} b_{t+1}$. The growth rate $d_{t+1}$ of the money stock from end of period $t$ to end of period $t+1$ is thus $d_{t+1} = p_{t+1} b_{t+1}/M_t \iff M_t = p_{t+1} b_{t+1}/d_{t+1}$. The second period budget constraint for the consumer reads $M_t = p_{t+1}(c_{t+1} - b_{t+1})$, where it is used that in equilibrium the amount of money held by the consumer at the end of $t$ must be the economy's entire money stock at the end of $t$. By equalizing the two expressions for $M_t$ we get $b_{t+1}/d_{t+1} = c_{t+1} - b_{t+1}$, or $b_{t+1} = (d_{t+1}/(1 + d_{t+1}))y_{t+1}$, where it was used that in equilibrium $c_{t+1} = y_{t+1}$. Hence a rule of no income taxation and constant money growth rate $d$ is equivalent to our rule with $\beta = d/(1 + d)$.

average of the two $*$-dotted curves.

It follows from Figure 1 that for this example, which has been deviced such that the policy rule with $\alpha = 0$ is indeed capable of stabilizing the economy, the stabilization obtained by increasing $\beta$ from zero has much better welfare implications for $\alpha = 1$, than for $\alpha = 0$.

$<$ Figure 1 here $>$

Figure 2 reports on the example (11), with $R = 4$ and $c_0 = 0$. Now $R(0) > 2 + N(0)$, implying that for $\alpha = 0$, increasing $\beta$ will not be stabilizing. Hence, for $\alpha = 0$, we assume $\delta = 0.1$, which implies that $f'(y(0)) > -1$, and the steady state is stable according to $f$ under laissez faire. For $\alpha = 1$, we consider $\delta = 0.05$, which implies $f'(y(0)) < -1$, and under laissez faire it is a two-period cycle that is stable according to $f$. Otherwise Figure 2 is like Figure 1, and shows, for various values of $\beta$, the dynamic equilibrium that is stable according to $f$, for $\alpha = 0$ in (i), and for $\alpha = 1$ in (ii). For $\alpha = 0$, increasing $\beta$ ¿rst changes the stable equilibrium from the steady state to a cycle, and from then on it implies increasing volatility of utility. For $\alpha = 1$, one obtains stabilization by increasing $\beta$, and the implications for welfare are good.

These examples illustrate how constant money growth rate rules, or rules where government activity is linked to current GNP ($\alpha = 0$), are outperformed with respect to stabilization, and the related welfare implications, by a class of rules which are structurally as simple; namely rules where government activity is linked to GNP with a certain delay ($\alpha = 1$). The latter contains an element of countercyclicality which is important for stabilizing endogenous competitive À uctuations.

$<$ Figure 2 here $>$

# 6 Conclusions

We have studied a simple monetary competitive model with intertemporally optimizing agents. The model can be interpreted either as an overlapping generations model or as a model with infinitely lived agents and cash-in-advance constraints. In any case there is a unique monetary steady state according to the model.

If this steady state could always be assumed to be the relevant rational expectations dynamic equilibrium, government activity would be, in the considered model, unambiguously bad on welfare grounds. However, under some circumstances there also exist under laissez faire other bounded and continuously well-defined rational expectations equilibrium trajectories, i.e. deterministic cycles or sunspot equilibria.

If a cycle or sunspot equilibrium is the relevant rational expectations dynamic equilibrium under laissez faire we are led, within the considered model, to two (interdependent) main conclusions with respect to government stabilization policy.

The first conclusion is that government intervention may be well motivated since it may stabilize economic activity in a way that has positive consequences for welfare. It is the departure from the steady state to some kind of endogenous fluctuation as the relevant equilibrium that leads to this conclusion. Even in the presence of exogenous shocks, if it could be safely assumed that the economy was always at - or close to and approaching - a competitive steady state (as in RBC models), then it would be hard to justify government intervention for stabilization reasons.

The second conclusion derived from the model is that the best stabilization principles - i.e. the policy rules that stabilize economic activity in a way that is best for welfare - entail a certain kind of countercyclicality in government activity; government should provide relatively small transfers and/or small amounts of public goods in periods up to which GNP has increased by a relatively large amount.

We take a modest view concerning the significance for actual stabilization policies of these model results. Insofar as fluctuations or cyclical movements in economic activity can be viewed as (at least partly) created endogenously by volatile and self-fulfilling expectations, some intertemporal effects of stabilization policies, which do not usually gain so much attention, become important. It is a logical possibility that these intertemporal effects work in such a way that good stabilization principles involve a kind of countercyclicality in government activity that is reminiscent of what is advocated by Keynesians.

# A Proofs

Proof of Proposition 1. First note that for all the stated conditions in Theorem 1 under which the steady state is determinate we have $\alpha \geq 0$. It then follows from Proposition B of Appendix B, that global stability of $y(\beta)$ according to $f$ (global determinacy), which obviously must eliminate all cycles, also eliminates all SSE.

We are going to show that for the $\beta^*(\alpha)$ of (i), one can use $\max_{x\in[0,1]} \frac{R(x)-1}{R(x)-1+\alpha} < 1$ if this is non-negative and zero otherwise. For the $\alpha^*(\beta)$ of (iii), one can use $\frac{1-\beta}{\beta}\max_{x\in[0,1]}(R(x)-1))$ if this is strictly positive and an arbitrarily small strictly positive value otherwise.

From (9), a critical point is given by $1-\beta(1-\alpha)(\frac{f(x)}{x})^\alpha - \left(1-\beta(\frac{f(x)}{x})^\alpha\right) R(x) = 0$. This implies that at a critical point one must have $R(x) > 1$, whenever $\alpha > 0$, and,

$$(\frac{f(x)}{x})^\alpha = \frac{1}{\beta}\frac{R(x)-1}{R(x)-1+\alpha}.$$

A critical point $(x^c,\ f(x^c))$ is below the 45°-degree line if $f(x^c)/x^c < 1$, which has to be ful¿lled if $x^c \geq 1$, since $f(x) < 1$ for all $x$. The denominator above is strictly positive at a critical point when $\alpha > 0$, so for $\alpha > 0$, $f(x^c)/x^c < 1$ is equivalent to,

$$\beta > \frac{R(x^c)-1}{R(x^c)-1+\alpha}, \tag{12}$$

and to,

$$\alpha > \frac{1-\beta}{\beta}(R(x^c)-1). \tag{13}$$

Now, if $\beta > \beta^*(\alpha)$, then in particular (12) is ful¿lled for any critical point $x^c < 1$, implying that $f(x^c)/x^c < 1$. This proves (i). If $\alpha > \alpha^*(\beta)$, then in particular (13) is ful¿lled for any critical point $x^c < 1$, implying that $f(x^c)/x^c < 1$. This proves (iii).

For (ii) note that the perfect foresight dynamic (8) for $\alpha = 0$ becomes $y_t v'(1-y_t) = (1-\beta)y_{t+1}u'(y_{t+1})$, so for $\beta$ going to one the $y_t$ that solves it must go to zero for any value of $y_{t+1}$. This means that $f(x)$ is pulled down arbitrarily close to the $x$-axis. Further, from (9) a critical point is given by $R(x) = 1$ independently of $\beta$. So, as $\beta$ is increased all critical points $(x^c, f(x^c))$ move downwards along the same value of $x^c$ with $f(x^c)$ getting arbitrarily close to the $x$-axis, so eventually they all go below the 45°-line. ■

Proof of Proposition 2. From (10) one sees that if the denominator of $f'(y(\beta))$ is negative (which it can be for $\alpha < 0$), then $f'(y(\beta)) > 1$. So, to exclude $f'(y(\beta)) > 1$, one must set $\alpha$ such that the denominator is positive, for which $\alpha \geq 0$ suffices. On the other hand, for such an $\alpha$, the necessary condition for avoiding indeterminacy, $f'(y(\beta)) \geq -1$, is equivalent to,

$$\alpha \geq \alpha^{**}(\beta) := \frac{1}{2}\frac{1-\beta}{\beta}(R(y(\beta)) - N(y(\beta)) - 2).$$

From (10), $f'(y(0)) < -1$ implies $R(0) - N(0) - 2 > 0$, which means that for a small enough $\beta$, the parenthesis on the right hand side is positive, so an $\alpha$ fulfilling the inequality also fulfils $\alpha \geq 0$. Finally, as $\beta$ goes to zero, the required $\alpha^{**}(\beta)$ goes to infinity because the parenthesis goes to $R(0) - N(0) - 2 > 0$, and $(1-\beta)/\beta$ goes to infinity. ■

Proof of Proposition 3. (i) When $\alpha < 0$, one sees from (10), that as $\beta$ goes to one, $f'(y(\beta))$ goes to $\alpha/\alpha = 1$, so both numerator and denominator become negative for a large enough $\beta$, but the numerator is numerically the largest, so $f'(y(\beta))$ goes to one from above. Hence, for all sufficiently large $\beta$, one has $f'(y(\beta)) > 1$, meaning that the steady state is indeterminate and an SSE exists.

(ii) Again from (10), if $\alpha = 0$, the slope of $f$ at steady state is $f'(y(\beta)) = \frac{1-R(y(\beta))}{1+N(y(\beta))}$. As $\beta$ goes to one, $y(\beta)$ goes to zero (Proposition A), and hence $f'(y(\beta))$ goes to $\frac{1-R(0)}{1+N(0)}$, which is less than -1 exactly because $R(0) > 2+N(0)$. If $\lim_{\beta\to 1} f'(y(\beta)) < -1$, then from continuity also $f'(y(\beta)) < -1$ for all large enough $\beta$. Hence $y(\beta)$ is indeterminate, which suffices for the existence of SSE close to it. When $f$ is globally well-defined and known to stay below a "ceiling," $f(x) < 1$ for all $x$, then $f'(y(\beta)) < -1$ also suffices for the existence of deterministic cycles.

(iii) For $\beta > 0$, when $\alpha$ becomes negative and sufficiently large numerically, both the numerator and the denominator in (10) become negative with the numerator numerically the largest, so $f'(y(\beta)) > 1$. ■

# B Conditions for the Existence of SSE to Imply the Existence of Deterministic Cycles

Inserting the speci¿c form (1) of policy rules into the equations (7) that a SSE must ful¿ll gives the equations,

$$y_i v'(1-y_i) = \sum_{j=1}^{r} q_{ij} \left[ y_j - \beta y_j^{1-\alpha} y_i^{\alpha} ) \right] u'(y_j) \text{ for } i = 1, ..., r.$$

The left hand side can be called $v_1(y_i)$, and if one on the right hand side uses $v_2(y_i, y_j) := \left[ y_j - \beta y_j^{1-\alpha} y_i^{\alpha} ) \right] u'(y_j)$, the equations become,

$$v_1(y_i) = \sum_{j=1}^{r} q_{ij} v_2(y_i, y_j) \text{ for } i = 1, ..., r. \tag{14}$$

The backward perfect foresight dynamic $f(x)$ is then given implicitly (as the solution in $z$) by $v_1(z) = v_2(z, x)$. Under the assumptions made in this paper, $v_1(y_i)$ is strictly increasing, and when $\alpha \geq 0$, $v_2(y_i, y_j)$ is either independent of $y_i$ (for $\alpha = 0$), or strictly decreasing in $y_i$ (for $\alpha > 0$). Furthermore, still for $\alpha \geq 0$, the perfect foresight dynamic $f$ is globally well-de¿ned and continuous (and di¾erentiable), stays below one, and with exactly one monetary steady state. This motivates,

Assumption 1. $v_1(y_i)$ is strictly increasing in $y_i$, and $v_2(y_i, y_j)$ is (weakly) decreasing in $y_i$.

Assumption 2. For every $x > 0$, there is a unique solution in $z$ to $v_1(z) = v_2(z, x)$, and the backward perfect foresight dynamic $f(x) = z$ thus de¿ned is continuous, $f(x) < 1$ for all $x$, and there is exactly one $y > 0$, that solves $f(y) = y$.

So, for all policy rules with $\alpha \geq 0$, these two assumptions are ful¿lled for the model considered in the main text of this paper. They are also the assumptions underlying Proposition B below. This is why we have been able to conclude that for policy rules with $\alpha \geq 0$, if there are no deterministic cycles (as there cannot be if the steady state $y > 0$ is globally stable according to $f$), then there are no SSE either.

Theorem B. Let $v_1$ and $v_2$ be such that Assumptions 1 and 2 are ful¿lled. If there are $y_1 \leq \cdots \leq y_r$, with $y_1 < y_r$, and an irreducible matrix $(q_{ij})$ of transition probabilities, such that (14) is ful¿lled, then there are also $y', y''$ with $0 < y' < y'' < 1$, such that $y' = f(y'')$ and $y'' = f(y')$. That is, if there is a stationary Markov sunspot equilibrium SSE, then there is also a two-period cycle, or, if there is no two-period cycle, then there is no SSE either.

Proof.[12] One can safely assume that all transition probabilities ful¿ll $q_{ij} > 0$.[13] For each $i = 1, ..., r$ de¿ne,

$$y_i^{\min} := \arg\min_{j\in\{1,\ldots,r\}} v_2(y_i, y_j),$$

$$y_i^{\max} := \arg\max_{j\in\{1,\ldots,r\}} v_2(y_i, y_j).$$

Since from (14), each $v_1(y_i)$ is an average of the $r$ values of $v_2(y_i, y_j)$, $j = 1, ..., r$, one must have $v_2(y_i, y_i^{\min}) \le v_1(y_i) \le v_2(y_i, y_i^{\max})$ for $i = 1, ..., r$. In particular for $i = 1$ and $r$,

$$v_2(y_1, y_1^{\min}) \le v_1(y_1) \le v_2(y_1, y_1^{\max}),$$

$$v_2(y_r, y_r^{\min}) \le v_1(y_r) \le v_2(y_r, y_r^{\max}).$$

Since $v_2$ is decreasing in its ¿rst argument we have: $v_2(y_r, y_r^{\max}) \le v_2(y_1, y_r^{\max}) \le v_2(y_1, y_1^{\max})$, and $v_2(y_1, y_1^{\min}) \ge v_2(y_r, y_1^{\min}) \ge v_2(y_r, y_r^{\min})$. So, now using that $v_1(y_i)$ is strictly increasing in $y_i$, we get,

$$v_2(y_r, y_r^{\min}) \le v_2(y_1, y_1^{\min}) \le v_1(y_1) < v_1(y_r) \le v_2(y_r, y_r^{\max}) \le v_2(y_1, y_1^{\max}).$$

Part of this is $v_2(y_1, y_1^{\min}) < v_2(y_1, y_1^{\max})$, and since all transition probabilities $q_{1j}$ are strictly positive, one gets $v_1(y_1) > v_2(y_1, y_1^{\min})$. Similarly, $v_1(y_r) < v_2(y_r, y_r^{\max})$. We have thus established,

$$v_2(y_1, y_1^{\min}) < v_1(y_1) \le v_1(y_2) \le \cdots \le v_1(y_r) < v_2(y_r, y_r^{\max}). \tag{15}$$

For one $i$, one has $y_i = y_1^{\min}$, and hence $v_1(y_1^{\min}) > v_2(y_1, y_1^{\min}) \ge v_2(y_1^{\min}, y_1^{\min})$, where the latter follows since $v_2$ is decreasing in its ¿rst argument. Hence, $v_1(y_1^{\min}) >$

[12] This proof extends the result of Grandmont (1986) from the case where $v_2$ is independent of $y_i$, to the case where $v_2$ is weakly decreasing in $y_i$.

[13] We appeal here to standard results. For dynamic systems as considered here, if there is a deterministic cycle, that is, a completely non-stochastic SSE where for each $i$, only one $q_{ij}$ is greater than zero (equal to one), then there is also a fully stochastic SSE where all $q_{ij}$ are strictly positive. By the same reasoning, if there is an SSE where for each $i$, some, but not all, $q_{ij}$ are strictly positive, then there is also a fully stochastic SSE, cf. Guesnerie and Woodford (1992).

$v_2(y_1^{\min}, y_1^{\min})$, but this implies that $f(y_1^{\min}) < y_1^{\min}$. (Remember that $f(y_1^{\min})$ is the solution in $z$ to $v_1(z) = v_2(z, y_1^{\min})$. For $z = y_1^{\min}$, one gets ŕstrictly larger than´. The solution is then to be found strictly below $y_1^{\min}$, since $v_1$ is strictly increasing, and $v_2$ is decreasing, in $z$). Similarly, for one $i$, one must have $y_i = y_r^{\max}$, so $v_1(y_r^{\max}) < v_2(y_r, y_r^{\max}) \leq v_2(y_r^{\max}, y_r^{\max})$, implying $f(y_r^{\max}) > y_r^{\max}$. So, we have both $f(y_1^{\min}) < y_1^{\min}$ and $f(y_r^{\max}) > y_r^{\max}$. This implies, of course, that $y_1^{\min} \neq y_r^{\max}$, but also that,

$$y_r^{\max} < y_1^{\min}.$$

Otherwise one would have $f(y_1^{\min}) < y_1^{\min} < y_r^{\max} < f(y_r^{\max})$, which from the continuity and $f < 1$ parts of Assumption 2 would imply the existence of a monetary steady state strictly between $y_1^{\min}$ and $y_r^{\max}$, and one strictly above $y_r^{\max}$, contradicting the uniqueness of monetary steady state part of Assumption 2.

Also from (15), one has directly that $v_1(y_1) > v_2(y_1, y_1^{\min})$, which implies $f(y_1^{\min}) < y_1$ (by the same reasoning as above), and similarly $v_1(y_r) < v_2(y_r, y_r^{\max})$, implying $f(y_r^{\max}) > y_r$. Since also $y_1 \leq y_r^{\max}$, and $y_1^{\min} \leq y_r$, one has,

$$f(y_1^{\min}) < y_r^{\max} \text{ and } y_1^{\min} < f(y_r^{\max}).$$

Combining the two last displayed inequalities gives,

$$f(y_1^{\min}) < y_r^{\max} < \ y_1^{\min} < f(y_r^{\max}).$$

Given that $f$ is continuous and stays below the ćeiling´one, this suÁces for the existence of a two period cycle: Note that the obtained inequality states that $f$ has a negative slope below minus one over an interval around the steady state, not necessarily in¿nitesimally close to it. However, the kind of non-local negative slope below minus one obtained suÁces from a standard argument. If one constructs the mirror image of $f$ around the 45°-line then this has, under the obtained condition and Assumption 2, to intersect $f$ itself at two points $y'$ and $y''$ di¾erent from the steady state. These $y'$ and $y''$ de¿ne a two-period cycle. ■

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FIGURE 1, $R = 4$, $c_0 = .06$, $\delta = .006$

FIGURE 2, $R = 4$, $c_0 = 0$, and in (i) $\delta = .1$, and in (ii) $\delta = .05$