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Imperfect Competition and the Firm: Some Equivalence Results

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Imperfect Competition and the Firm: Some Equivalence Results

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Abstract

The paper introduces an abstract economy with imperfect competition; the choice of allocation takes place through an abstract mechanism, where producers choose strategies and the outcome is (a set of) feasible allocations, where the consumers' choices are sustained by the market mechanism at some prices. We show that with a wide range of assumptions on producer preferences, the equilibrium outcomes in this economy are ordinary compensated equilibria, possibly in an economy with production externalities.

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1 Introduction

In the theory of imperfect competition, which has had a considerable development in recent years, the problem of integrating imperfectly competitive behavior into a generel equilibrium framework still has not found a satisfactory solution. Since the seminal contribution of Negishi (1962), there has been a steady growing literature on this field (see, e.g., the survey in Hart, 1985) but many of the fundamental questions in this field remain unanswered. Thus, while the details of strategic behavior with firms has been treated in a still more sophisticated way, our understanding of the way in which this behavior fits into the rest of the economy with its more standard mechanisms is not yet complete.

There are, of course, good reasons for this lagging behind of our understanding of this field, which have its own particular features. Thus, it must be considered whether firms take into consideration the full impact on the economy of their actions – giving rise to the so-called objective demand approach – or whether they act according to a perceived version of this impact, to me modelled as subjective demand. But even then, some fundamental choices in the modelling of firms' behavior remain, connected with the objectives of firms. When competition is incomplete, maximization of profits is no longer an unambiguous way of furthering shareholder interests; as a matter of fact, profit maximization does not even always make sense, as it mas shown already by Gabszewicz and Vial (1972), the rule of normalization of prices may matter for the ensuing equilibrium; thus, the choice of numeraire commodity influences the final allocation obtained in the economy.

In this paper, we consider some alternative formulations of the firms' objective. The first one is a rather intuitive one: Firms will act in such a way that there no alternative action would result in a better final outcome for its shareholders. This clearly, is a rather weak criterion; there is no advantage of being a large or even a dominating shareholder. However, as a first approach it seems uncontroversial, at least as a first condition on firms' behavior.

The second objective to be considered in this paper is that of shareholders' real wealth maximization, proposed by Dierker and Grodal (1996). Here the firm chooses its actions in so that there is no alternative choice with the property that the total wealth of its shareholders resulting from this choice is greater than the value of their consumption at the original outcome (so that at the new prices all shareholders could realize a better final consumption after suitable redistribution of income). This objective may be considered as a refinement of the previous one, and at the same time it appears as more intuitively fitted to the situation of transmitting shareholders' preferences to the firm.

With either of the above two assumptions in the firm's objectives, we consider the set of equilibria under imperfect competition and compare the allocations obtained by those obtainable under perfect competition. In both cases, it turns out that under suitable assumptions of well-behavedness, these allocations are compensated equilibria, that is they could be obtained in the underlying economy by perfect competition after some redistribution of income. This result, which of course should be considered in relation to their assumptions, may be interpreted as pointing to some remaining shortcomings of the formulations of firms' objectives; it does not quite correspond to our intuitive conception of imperfect competition that it should end up with the same result as perfect competition, or differ only in respect to distribution of incomes.

In the construction of our model of an economy with imperfect competition, we have chosen not to be specific on the actual choices of the firm, but rather to formalize the imperfect competition as an abstract game with the firms as players, where the combined strategy choices result in allocations and prices in the underlying economy. Thus, the imperfectly competitive competition is treated as sort of a "black box", without explicit formulation of the way, in which strategies of firms influence the market and its functioning. The advantage of this approach is that we do not have to be specific on whether firms choose quantities or prices or some third strategic variable; the standard models of Cournot and Bertrand competition come out as special cases.

The paper is structured as follows: In section 2, we introduce the basic model which consists of (1) a basic framework for general equilibrium analysis, (2) a method of determining final allocation as a result of the simultaneous choices of the imperfectly competing firms, in this paper called a mechanism, and (3) an assignment of preferences to firms. In the following sections, we equilibria of the model with particular specifications of the preferences of firms; in section 3 we discuss shareholder welfare maximization, and in section 4 we look at shareholder real wealth maximization by firms. In section 5, we subsume the previous results considering a slightly more general situation allowing for production externalities, and in this framework we derive our main result, which issection 5, we present the general equivalence theorem for imperfect competition (with objective demand). We conclude with some final remarks in section 6.

2 Imperfect competition economies

In this section, we introduce the basic notions to be used in the sequel. Since we discuss imperfectly competitive behavior in general equilibrium, there are three basic ingredients: first of all, we need the basic framework of general equilibrium theory (for considering the outcome of imperfect competition), secondly, we need a notion of strategies for the imperfect competitors (defining the rules of the imperfect competition), and thirdly, we must introduce objectives of the imperfect competitors, since this is no longer trivial.

(1) We start with the first ingredient, defining abstract economies for general equilibrium analysis.

Let $\mathcal{E} = (X_i, \succsim_i, \omega_i)_{i=1}^m, (Y_j)_{j=1}^n, ((\theta_{ij})_{i=1}^m)_{j=1}^n)$ be an abstract economy in the sense of Debreu (1959). Here,

- for $i \in \{1, \dots, m\}$, the triple $(X_i, \succsim_i, \omega_i)$ denotes a consumer with
  - consumption set $X_i \subset \mathbf{R}^l$,
  - preferences $\succsim_i \subset X_i \times X_i$, and
  - initial endowment $\omega_i \in \mathbf{R}^l$, for $j \in \{1, \dots, n\}$,
- for $j \in \{1, \dots, j\}$,
  - $Y_j \subset \mathbf{R}^l$ is the production set of firm j, and
  - for $(i, j) \in \{1, \dots, m\} \times \{1, \dots, n\}$, $\theta_{i,j} \in [0, 1]$ denote the profit share of consumer i in firm j, where $\sum_{i=1}^m \theta_{ij} = 1$ for all j.

The economy is supposed to satisfy the standard assumptions of general equilibrium theory (cf. e.g. Green, MasColell, Whinston (1995) for details):
The economy $\mathcal{E} = ((X_i, \succsim_i, \omega_i)_{i=1}^m, (Y_j)_{j=1}^n, (\theta_{ij})_{i=1}^m{}_{j=1}^n)$ satisfies the following:

(i) For each consumer i, X_i is nonempty, closed, convex, bounded from below, and satisfies $X_i + \mathbf{R}_+^l \subset X_i$, and $\succsim_i$ is complete preorder on X_i which is continuous, monotonic, and convex;

(ii) for each producer j, the set Y_j is closed, convex, and contains $0 \in \mathbf{R}^l$;

(iii) there are $x_i \in X_i$, $i = 1, \dots, m$, $y_j \in Y_j$, $j = 1, \dots, n$, such that $\sum_{i=1}^m x_i = \sum_{i=1} y_j + \sum_{i=1}^m \omega_i$.

An allocation in $\mathcal{E}$ is an array $z = (x_1, \dots, x_m, y_1, \dots, y_n) \in \mathbf{R}^{(m+n)l}$. The allocation z is feasible if

$$x_i \in X_i,\ i = 1, \dots, m,\ y_j \in Y_j,\ j = 1, \dots, n,\ \sum_{i=1}^m x_i \leq \sum_{i=1}^m \omega_i + \sum_{j=1}^n y_j.$$

By Assumption 1.(iii), the set of feasible allocations in $\mathcal{E}$ is nonempty. We let $\mathcal{A}$ denote the set of feasible allocations in $\mathcal{E}$.

A price system is a non-negative l-vector $p \in \mathbf{R}_+^l \backslash \{0\}$; the demand of consumer i with income w_i at the price p is

$$\xi_i(p, w_i) = \{x_i \in X_i \mid p \cdot x_i \leq w_i,\ x_i' \succ_i x_i \Rightarrow p \cdot x_i' > w_i\}.$$

Let $\mathcal{F}$ denote the set of allocation-price pairs (z, p) such that $z \in \mathcal{A}$, and for each i,

$$x_i \in \xi_i(p, p \cdot x_i)$$

(here $\cdot$ denotes the inner product in $\mathbf{R}^l$). Thus, (z, p) belongs to $\mathcal{F}$ if the allocation is feasible and the consumers' bundles are best in their preference orderings among the bundles which cost no more at the price system p.

(2) Now we turn the second category of definitions, those pertaining to the imperfectness of competition between firms in the economy $\mathcal{E}$. Since we do not want to commit ourselves to a particular institutional framework, we take an abstract approach, assuming that firms choose strategies which through the interplay of the market results in allocation-price pairs in $\mathcal{F}$. Of course, even so we are restricting ourselves: The fact that strategies will be chosen with a view to the resulting allocation(s) means that what we use an *objective demand* approach (cf. Hart (1985)).

Thus for each j, let Σ_j be an abstract strategy space, and let $\Sigma = \prod_{j=1}^n \Sigma_j$. We define a mechanism as a non-deterministic game form $\Gamma = ((\Sigma_j)_{j=1}^n, \mathcal{F}, \pi)$ where π is a correspondence $\pi : \Sigma \to \mathcal{F}$ taking strategy arrays $\sigma = (\sigma_1, \dots, \sigma_n)$ to sets of allocation-price pairs $(z', p') \in \mathcal{F}$. The choice of a non-deterministic game form rather than an ordinary game form (for each strategy array we assume that there may be several allocation-price pairs in the outcome set) is made in order not to exclude the most obvious particular cases such as Cournot-competition, cf. below.

One of our reasons for the abstract approach to imperfect competition is that we do not need to commit ourselves to either prices or quantities as strategic variables of the firms. Thus, the prices of the output commodoties of the firm or its actual sales may be determined only as part of the final outcome of the strategy choices of themselves and their competitors. Thus the mechanism reflects that prices and quantities adjust as a response to the strategic choices of the firms until a price-guided allocation in $\mathcal{F}$ prevails.

Example 2.1 (Cournot competition) Suppose that each firm j chooses a net trade $\hat{y}_j \in Y_j$ to be sent to the market, accepting the market prices resulting from this; formally, the strategy space of firm j in the mechanism Γ is then the set of all production plans $\hat{y}_j \in Y_j$, and the outcome of the mechanism is then the set of all Walras equilibria of the economy $\mathcal{E}[\hat{y}_1, \dots, \hat{y}_n] = ((X_i, \succsim_i, \omega_i)_{i=1}^m, (\{\hat{y}_j\} - \mathbf{R}_+^l)_{j=1}^n, (\theta_i)_{i=1}^m{}_{j=1}^m)$. Thus, each firm chooses as strategy a commitment to a certain net production, and the outcomes may either be considered (as above) as the Walras equilibria of the economy with the corresponding simplified production sets, of alternatively, as the Walras equilibria of the exchange economy where each consumer i has endowment $\omega_i + \sum_{j=1}^n \theta_{ij}\hat{y}_j$.

It might be noticed that the definition of Γ as given above is not yet complete, since we need a rule for determination of outcome when $\hat{y}_1, \dots, \hat{y}_n$ are such that $\mathcal{E}[\hat{y}_1, \dots, \hat{y}_n]$ has no Walras equilibria; a precondition for this to happen (given Assumption 1) is that

$$\sum_{i=1}^m \omega_i + \sum_{j=1}^n [\{\hat{y}_j\} - \mathbf{R}_+^l] \cap \sum_{i=1}^n \operatorname{int} X_i = \emptyset$$

(intuitively, the choices of production plans proposed by the firms are such that the demand for inputs exceed the available quantities). To complete the definition of the mechanism, some rationing of firm's demand for inputs might be added; however, the set of equilibria (in the sense to be developed below) will depend on the precise way of defining such a rationing, and we shall not enter into this topic here.

Example 2.2 (Bertrand competition in one-commodity producing firms) Assume that each firm j produces a single output commodity $h(j)$; let $L_1 \subset \{1\dots,l\}$ be the set of commodities produced by some firm, and let L_2 be the remainder, and for each $h \in L_1$, let $N_h = \{j \mid h(j) = h\}$.

The following mechanism captures some features of competition a la Bertrand: The strategies of firm j are functions $\sigma_j : \mathbf{R}_+^{l-1} \to \mathbf{R}_+$ determining a price of the output commodity $h(j)$ as a function of the prices of all other commodities. To determine the outcome function, we define for each strategy array $(\sigma_1, \dots, \sigma_n)$ the set of outcomes as the set of allocation-price pairs $(z,p) = (x_1, \dots, x_m, y_1, \dots, y_n, p) \in \mathcal{F}$ for which the following hold: For each $h \in L_1$, there is $j^*(h) \in \{1, \dots, n\}$ with

$$\sigma_{j^*(h)}(p) = p_h = \min_{j \in N_h} \sigma_j(p);$$

then (z,p) is a feasible allocation-price pair in the economy $((X_i, \succsim_i, \omega_i)_{i=1}^m, (\tilde{Y}_j)_{j=1}^n, (\theta_i)_{i=1}^m {}_{j=1}^m)$, where

$$\tilde{Y}_j = \begin{cases} Y_j & \text{if } j = j^*(h) \text{ for some } h \in L_1, \\ -\mathbf{R}_+^l & \text{otherwise;} \end{cases}$$

thus, for each commodity h, a firm quoting the lowest prices gets the whole demand, and the remaining firms producing this commodity get nothing.

It is seen that even in the one-output case considered here, a mechanism which captures some features of Bertrand competition becomes somewhat complicated (and we have not addressed the existence problem involved in its definition). The mechanism presupposes that the firm with the smallest price on its output will get the market in the sense that the remaining competitors can only carry through a trivial production. This is a somewhat drastic assumption in the context of Bertrand competition; in the literature on partial equilibria, there is usually a rationing assumption involved, whereby competitors with prices higher than the lowest in the market do get some demand, at least in the cases where the firm with the lowest price cannot satisfy all of the demand. We have chosen the above simpler – and perhaps less realistic – formulation in order to avoid outcomes which are traded simultaneously at several different price systems, since in that case outcome would no longer belong to $\mathcal{F}$.

(3) To complete our formal framework, having introduced game forms, we need a notion of preferences of producers over outcomes. At present, we shall introduce these preferences formally as correspondences $\Phi_j : \mathcal{F} \to \mathcal{F}$, assigning to each feasible allocation-price pair (z,p) all the allocation-price pairs (z',p') considered by firm j to be superior to (z,p). The preferences of firm j should presumable reflect the shareholders' interest, but at this point we are not yet committed to any interpretation. In the following sections we consider several possibilities.

Now we have completed the introduction of our model; the array $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$ is called an economy with imperfect competition, and the logical next step is to define equilibria for this economy.

Definition 2.3 *Let* $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$ *be an economy with imperfect competition. An equilibrium in* $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$ *is an array* (z, p, σ)*, where*

(1) $(z, p) \in \pi(\sigma)$*,*

(2) σ *is a Nash equilibrium of* $(\Gamma, (\Phi_j)_{j=1}^n)$ *in the sense that there is no* $j \in \{1, \dots, n\}$ *and* $(z', p') \in \Phi_j(z, p)$ *such that*

$$(z', p') \in \pi(\tau_j, \sigma_{-j})$$

for some $\tau_j \in \Sigma_j$ *where* (τ_i, σ_{-j}) *is the strategy array obtained from* σ *by replacing* σ_j *with* τ_j*.*

The definition of an equilibrium is almost self-explanatory, only the standard concept of a Nash equilibrium has had to be slightly extended to be useful in our context of non-deterministic game forms: We demand that for the given choice of allocation-price pair in the outcome set belonging to the equilibrium strategy array, no individual change of strategy can lead to an outcome set containing an allocation-price pair considered better in the preferences of this firm. Thus, we work with what may be called 'optimistic beliefs' with respect to the selection of the particular allocation-price pair from the outcome set.

Since deviating firms have optimistic beliefs, the notion of equilibrium (where there are no deviating firms) is correspondingly strong. However, this problem is more apparent than real, since in applications, it seems quite often to be the case that if there is a deviation leading to somewhat preferred according to our definition, then there is also such a deviation for which the outcome set is a singleton; we return to this in the concluding remarks.

In the following two sections, we consider possible specifications of the preference correspondences Φ_j introduced above, and we show that in both cases the equilbria obtained are among some which were known and studied already (with no mentioning of imperfect competition).

3 Equilibria under shareholder welfare maximization

In the present section we show that if preferences of each firm reflect the interests of its shareholders in a weak sense, then the allocation associated with an equilibrium is Pareto optimal; indeed, the price system at which consumers buy their final bundles define a compensated equilibrium, meaning that the imperfectly competitive production sector may effect a redistribution of incomes as compared to the perfectly competitive situation, but otherwise has little impact on resource allocation.

To obtain this result we need a weak smoothness assumption on preferences (namely unique support of sets of strictly preferred bundles). For $i \in \{1, \dots, m\}$ a consumer, $\succsim_i$ a preference on X_i, and $x_i \in X_i$, we use the notation $P_i(x_i) = \{x'_i \in X_i \mid x'_i \succ_i x_i\}$.

For all i and $x_i \in X_i$, the set

$$\{p \in \mathbf{R}^l_+ \mid x'_i \in P(x_i) \Rightarrow p \cdot x'_i > p \cdot x_i\}$$

is a half-line in $\mathbf{R}^l$.

Next, we turn to the firms' preferences on allocation-price pairs. In all that follows, we consider preferences which depend *only* on the allocation component, *not* on the price. Straightforward as such a restriction may seem, it does rule out some obvious objectives of the firms (such as profit maximization) which in certain contexts will induce different preferences on allocations depending on the associated prices (cf. Gabszewicz and Vial (1972)). The preferences to be used in the following are such that e.g. the choice of rule for price normalization will have no impact on the equilibrium selection.

As stated in the previous section, it would seem reasonable that preferences of the firms should reflect shareholder preferences in some way. A precondition for this is that if shareholder unanimity prevails about the ranking of two allocations, then ranking of the firm should be in accordance with this unanimous ranking of its shareholders.

For each firm $j \in \{1, \dots, n\}$, the preference relation Φ^u_j satisfy the following condition: If $(z, p), (z', p') \in \mathcal{F}$ are such that $x'_i \in P_i(x_i)$ for all i with $\theta_{ij} > 0$ then $(z', p') \in \Phi^u_j(z, p)$.

In the following, we shall be interested in economies with imperfect competition $(\mathcal{E}, \Gamma, (\Phi^u_j)^n_{j=1})$ which satisfy Assumptions 1 – 3. We note that the assumption on firms' preferences is not very restrictive, since it amounts only to price-independence and respect of shareholder unanimity. Even so, it turns out that the set of equilibria is rather severely restricted; below it is shown that the set of equilibria correspond to the set of compensated equilibria of the original economy. Recall that a compensated equilibrium of the economy $\mathcal{E}$ is a pair $(z, p) \in \mathcal{F}$ such that the production plans of the allocation $z = (x_1, \dots, x_m, y_1, \dots, y_m)$ are profit maximizers at the prices p, that is

$$p \cdot y_j \geq p \cdot y'_j, \text{ all } y'_j \in Y_j.$$

If (z, p) is a compensated equilibrium then z is Pareto optimal.

To explain the nature of the equivalence result to follow, consider for a while a feasible allocation $z = (x_1, \dots, x_m, y_1, \dots, y_n)$ in the economy $\mathcal{E}$. Suppose that there is a firm j, and a bundle $x'_i \in P_i(x_i)$ for each i with $\theta_{ij} > 0$, that is for each shareholder in firm j, such that the aggregate change in consumption $\sum_{i:\theta_{ij}>0}(x'_i - x_i)$ corresponds to a change in the production of firm j, that is

$$\sum_{i:\theta_{ij}>0} (x'_i - x_i) \in Y_j - \{y_j\};$$

if allocation is carried out by direct delivery (rather than through the interplay of the market), and if firms' preferences satisfy Assumption 3 above, so that they respect shareholder unanimity, then (z,p) cannot be an equilibrium choice. In our context, we need to go beyond the notion of direct improvement, since firms do not improve their shareholders' situation by direct delivery of goods, but rather by choosing another strategy in the mechanism which through the interplay of the market will result in other bundles for the consumers.

For our first approach we investigate what will happen if the mechanism Γ is sufficiently flexible to allow for the same type of improvements that could be carried through under direct delivery:

The mechanism Γ has the following property: If $(z,p) \in \pi(\sigma)$ is a feasible allocation-price pair with

$$(Y_j - \{y_j\}) \cap \left(\sum_{\theta_{ij}>0} [P_i(x_i) - \{x_i\}] \right) \neq \emptyset$$

(such that some unanimous improvement for shareholders could be obtained by a change in j's production, everything else being equal) then there exist $\tau_j \in \Sigma_j$ and $(z',p') \in \pi(\tau_j, \sigma_{-j})$ such that $x_i' \in P_i(x_i)$ for all i with $\theta_{ij} > 0$.

Now we may state and prove the following theorem:

THEOREM 3.1 *Let $(\mathcal{E}, \Gamma, (\Phi_j^u)_{j=1}^n)$ be an economy with imperfect competition satisfying Assumptions 1 – 4.*

If (z,p,σ) is an equilibrium then (z,p) is a compensated equilibrium in $\mathcal{E}$.

Proof: Suppose that (z,p,σ) with $z = (x_1, \ldots, x_m, y_1, \ldots, y_n)$ is an equilibrium in in $(\mathcal{E}, \Gamma, (\Phi_j^u)_{j=1}^n)$; then

$$(Y_j - \{y_j\}) \cap \left(\sum_{i:\theta_{ij}>0} [P_i(x_i) - \{x_i\}] \right) = \emptyset$$

for all j. Indeed, suppose that there were $x_i' \in P_i(x_i)$ for each i with $\theta_{ij} > 0$ such that $\sum_{i:\theta_{ij}>0}(x_i' - x_i) = y_j' - y_j$ for some $y_j' \in Y_j$. By Assumption 4 there is $\tau_j \in \Sigma_j$ and $(z'',p'') \in \pi(\tau_j, \sigma_{-j})$ such that $x_i'' \in P_i(x_i)$ for each i with $\theta_{ij} > 0$. By Assumption 3, the latter condition on (z'',p'') implies that $(z'',p'') \in \Phi_j^u(z,p)$, and we have a contradiction since (z,p) is an equilibrium.

Since $Y_j - \{y_j\}$ and $\sum_{i:\theta_{ij}>0} P_i(x_i) - \{x_i\}$ are both convex sets, and 0 belongs to the first of the sets while the second set is open, there is $p' \in \mathbf{R}^l$, $p \neq \emptyset$, such that

$$\begin{aligned} p' \cdot y &\leq 0, \; y \in Y_j - \{y_j\}, \\ p' \cdot x &> 0, \; x \in P_i(x_i) - \{x_i\}. \end{aligned}$$

From the monotonicity properties of $\precsim_i$ (Assumption 1.(i)) we infer that $p' \in \mathbf{R}^l_+$.

Since $p' \cdot \sum_{i:\theta_{ij}>0} \tilde{x}_i > p' \cdot \sum_{i:\theta_{ij}>0} x_i$ whenever $\tilde{x}_i \in P_i(x_i)$ for each i with $\theta_{ij} > 0$, we conclude that p' is a support of $P_i(x_i)$ at x_i for all i with $\theta_{ij} > 0$. By the smoothness Assumption 2 (and the price independence in Assumption 3(i)), we may assume that $p = p'$. It follows then that $p \cdot y \leq p \cdot y_j$. Since j was chosen arbitrarily, this profit maximization condition holds for every firm, and consequently (z, p) is a compensated equilibrium. □

REMARK 3.2 Note that in the proof of Theorem 1 only Assumption 2 on preferences of consumers is needed, so there is no need to assume that $\succsim_i$ is a monotone, continuous, total preorder for all i.

If preferences of firms not only respect shareholder unanimity but are actually defined by this condition, that is if

$$(z', p') \in \Phi_j^u(z, p) \Leftrightarrow [x_i' \in P_i(x_i) \text{ all } i \text{ with } \theta_{ij} > 0], \tag{1}$$

then the converse of Theorem 1 holds, at least under the additional assumption of universal shareholding (every consumer has shares in every firm). In this case, every compensated equilibrium is obtainable as an equilibrium of the economy with imcomplete competition $(\mathcal{E}, \Gamma, (\Phi_j^u)_{j=1}^n)$.

THEOREM 3.3 *Let $(\mathcal{E}, \Gamma, (\Phi_j^u)_{j=1}^n)$ be an economy with imperfect competition satisfying Assumptions 1 and 4, such that $\theta_{ij} > 0$ for all i and j, and assume that for each j, the correspondence Φ_j^u satisfies (1) above.*

If $(z, p) \in \mathcal{F}$ is a compensated equilibrium of $\mathcal{E}$ with $(z, p) \in \pi(\sigma)$ for some $\sigma \in \Sigma$, then (z, p, σ) is an equilibrium of $(\mathcal{E}, \Gamma, (\Phi_j^u)_{j=1}^n)$.

Proof: Suppose that $(z, p) \in \pi(\sigma)$ is a compensated equilibrium. If (z, p, σ) is not an equilibrium of $(\mathcal{E}, \Gamma, (\Phi_j^u)_{j=1}^n)$, then there must be some j and $\tau_j \in \Sigma_j$ such that $\pi(\tau_j, \sigma_{-j})$ contains an allocation price pair $(z', p') \in \Phi_j^u$. Applying (1) and the fact that $\theta_{ij} > 0$ for all j, we get that $x_i' \in P_i(x_i)$ for all i. Since $z' \in \mathcal{A}$ we have that z' is a Pareto improvement of z, contradicting that z is Pareto optimal. We conclude that (z, p, σ) is an equilibrium. □

The results of this section show that equilibria under conditions of incomplete competition are to be found among the allocations which are also studied in the context of perfect competition. In a certain sense, then, imperfect competition does not add anything new as far as the set of allocations obtainable in equilibrium is concerned. Also, Theorem 2 indicates that imperfect competition may give a rather large set of allocations as equilibrium outcomes.

Clearly, these results depend on the assumptions, and some of them are rather strong. We shall relax most of them in due course, getting somewhat less sharp results as a consequence. The next section represents a small step in this direction.

4 Equilibria under real wealth maximization

In this section, we show that results similar to those of Theorem 1 can be obtained also with a somewhat weaker assumption on the mechanism than that stated as

Assumption 4 in the previous section. This assumption states that there is a close connection between choice of production plans in the firms and choices of strategies. In the version of the assumption stated below this connection is still present but in a weaker sense: only the feasible directions of change in production are sustained by strategy choices.

Before stating the assumption, we need some notions of non-smooth analysis, in particular that of a tangent cone, cf. Clarke (1983): A vector $v \in \mathbf{R}^l$ is a tangent of $C \subset \mathbf{R}^l$ at $x \in C$ if for every sequence $(x_\nu)_{\nu=1}^\infty$ in C converging to x and for every sequence $(t_\nu)_{\nu=1}^\infty$ of nonnegative reals decreasing to 0, there is a sequence $(v_\nu)_{\nu=1}^\infty$ in $\mathbf{R}^l$ converging to v such that $x_\nu + t_\nu v_\nu \in C$. The set of tangents of C at x is denoted by $T_C(x)$ and is called the tangent cone of C at x.

Let $(z, p) \in \mathcal{F}$ be an allocation-price pair, and let $\sigma \in \Sigma$ be such that $(z, p) \in \pi(\sigma)$. If $C_j(z, p, \sigma) \subset (\mathbf{R}^l)^n$ is the set of productions obtained in allocation-price pairs $(z', \sigma') \in \pi(\tau_j^t, \sigma_{-j})$ for some $\tau_j \in \Sigma_j$, then

$$(0, \dots, 0, y_j', 0, \dots, 0) \in T_{C_j(z,p,\sigma)}(y_1, \dots, y_n),$$

for each $y_j' \in Y_j$, where $(0, \dots, 0, y_j', 0, \dots, 0)$ is the vector in $(\mathbf{R}^l)^n$ with 0 everywhere and y_j at the places corresponding to firm j.

The assumption states that locally, that is at any allocation-price pair with corresponding strategy choices, a firm can change its final production in any feasible direction, which here means in any direction which is in the tangent cone of $Y - \{y_j\}$ at y_j, by a unilateral change of strategy; moreover, this can be effectuated in such a way that the final production plans of the other firms are affected only slightly. Thus, the assumption gives for each producer j a wide variety of alternative production plans, at least in a neighbourhood of the actual choice. In the case of Cournot competition with quantity-choosing firms (Example 1 above), this is a rather natural assumption, but if the firm chooses prices rather than quantities, it is less obvious that every (or at least something nearby) production plan is an obtainable choice no matter what the other firms have chosen, given that the production is assumed actually to satisfy the consumer demand resulting from the strategy choice. The restrictiveness of this assumption in general models should of course be taken into consideration when assessing the results to be presented.

Instead of repeating the analysis in the previous section with this assumption instead of assumption 4, we consider a situation where firms' preferences are not derived from shareholder unanimity but depend on shareholder real wealth. An intuitive reason for this change of formulation is that with shareholder unanimity, a decision among shareholders based on their final bundles of consumption is called for, whereas decisions involving only the financial position of shareholders seem somewhat simpler. Formally, we demand that firms should choose strategy in such a way that no other strategy could give a Pareto improvement for shareholders, when these are allowed to redistribute the profits of the firm in any way desired. Formally, this amounts to the following:

Firm preferences Φ_j^r, $j = 1, \dots, n$, are shareholder compatible in the following sense: Let $(z, p) \in \mathcal{F}$, and suppose that $(z', p') = (x'_1, \ldots, x'_m, y'_1, \ldots, y'_n, p')$ is such that

$$\sum_{i:\theta_{ij}>0} p' \cdot (x_i - \omega_i) < \sum_{i:\theta_{ij}>0} \sum_{j=1}^{n} \theta_{ij} p' \cdot y'_j.$$

Then $(z', p') \in \Phi_j^r(z, p)$.

The assumption states that if at the allocation-price pair (z', p'), the aggregate wealth of firm j's shareholders is large enough to allow these shareholders to buy the bundles which they obtained at allocation-price pair (z, p), then firm j consider (z', p') as better than (z, p). Note that in our interpretation of the assumption, the firm has the opportunity of redistributing the shareholder incomes, or, alternatively, we assume that shareholders may agree on income transfers conditioned on a change of the strategy of the firm.

REMARK 4.1 If $(z', p') \in \Psi_j(z, p)$, that is if (z', p') is better than (z, p) from the point of view of real wealth maximization, then each shareholder i of firm j can buy something better than x_i at the prices p' and after a suitable redistribution of shareholder incomes; this does not mean however that the bundle x'_i is necessarily better than x_i for all i, since here we work with *potential* rather than actual Pareto improvements.

With Assumption 3 on firms' preferences and Assumption 4 on the well-behavedness of the mechanism replaced by Assumptions 6 and 5, respectively, we still get a counterpart of Theorem 1, reducing equilibria of the economy with imperfect competition to ordinary compensated equilibria of the underlying economy:

THEOREM 4.2 *Let $(\mathcal{E}, \Gamma, (\Phi_j^r)_{j=1}^n)$ be an economy with imperfect competition satisfying Assumptions 1,2,5, and 6, and such that $\theta_{ij} > 0$ for all i and j.*

If (z, p, σ) be is an equilibrium then (z, p) is a compensated equilibrium in $\mathcal{E}$.

PROOF: Let $j \in \{1, \ldots, n\}$ and suppose that

$$y'_j - y_j \in \left(\sum_{i=1}^{m} [P_i(x_i) - \{x_i\}] \right)$$

for some $y'_j \in Y_j$. Then, by Assumption 6(ii), the vector $(0, \ldots, 0, y'_j - y_j, 0, \ldots, 0)$ belongs to $T_{C_j(z,p,\sigma)}(y_1, \ldots, y_n)$. Using the definition of tangents, this means that for any sequence τ_j^ν such for each ν $\pi(\tau_i^\nu, \sigma_{-j})$ contains an allocation-price pair (z^ν, p^ν) with $(y_1^\nu, \ldots, y_n^\nu)$ converging to $(y_1, \ldots, y_n)$ and any sequence $(t^\nu)_{\nu=1}^\infty$ decreasing to 0, there is a sequence $(\tilde{y}_1^\nu, \ldots, \tilde{y}_n^\nu)_{\nu=1}^\infty$ in $(\mathbf{R}^l)^n$ converging to $(0, \ldots, 0, y'_j - y_j, 0, \ldots, 0)$, such that $y_k^\nu + t^\nu \tilde{y}_k^\nu \in C_j(z, p, \sigma)$ for all ν and $k = 1, \ldots, n$.

We claim that there is a choice of sequence $(t^\nu)_{\nu=1}^\infty$ such that

$$\sum_{k=1}^{n} (y_k^\nu + t^\nu \tilde{y}_k^\nu) \in \left(\sum_{i=1}^{m} [P_i(x_i) - \{x_i\}] \right)$$

for some ν; then for each sequence $(\sum_{k=1} \tilde{y}_k^\nu)_{\nu=1}^\infty$ from the complement of $\sum_{i=1}^m [P_i(x_i) - \{x_i\}]$ and each sequence $(t^\nu)_{\nu=1}^\infty$ we would have

$$(\sum_{k=1}^n \tilde{y}_k^\nu) + t^\nu (\sum_{k=1}^n y_k^\nu) \notin \left(\sum_{i=1}^m [P_i(x_i) - \{x_i\}] \right),$$

so that $y_j' - y_j$ must be a tangent of the complement of $\sum_{i=1}^m [P_i(x_i) - \{x_i\}]$ at x_i, a contradiction, which proves our claim.

We conclude that for each $\bar{\nu}$ there is $\nu > \bar{\nu}$, a strategy choice τ_j^ν and allocation-price pair $(z^\nu, p^\nu) \in \pi(\tau_j^\nu, \sigma_{-i})$ such that

$$\sum_{k=1}^n [y_k^\nu - y_k] = \sum_{i=1}^m [x_i^\nu - x_i] \in \sum_{i=1}^m [P_i(x_i) - \{x_i\}].$$

If $p^\nu \cdot (\sum_{i=1}^m x_i^\nu) \leq p^\nu \cdot (\sum_{i=1}^m x_i)$ for all ν, then, arguing as above, we would have that $y_j' - y_j'$ belongs to the complement of $\sum_{i=1}^m [P_i(x_i) - \{x_i\}]$, a contradiction. Thus, there is ν such that

$$p^\nu \cdot (\sum_{i=1}^m x_i^\nu) > p^\nu \cdot (\sum_{i=1}^m x_i).$$

This means that $\pi(\tau_j^\nu, \sigma_{-j})$ contains a price-allocation pair in $\Phi_j^r(z, p)$, contradicting that (z, p, σ) is an equilibrium.

We conclude that

$$[Y_j - \{y_j\}] \cap \left(\sum_{i=1}^m [P_i(x_i) - \{x_i\}] \right) = \emptyset$$

for every j. By separation of convex sets together with the fact that $P_i(x_i) - \{x_i\}$ is uniquely supported at 0 by p (according to Assumption 2), we have that y_j maximizes $p \cdot y$ on Y_j. Since j was arbitrary, we conclude that (z, p) is a compensated equilibrium in $\mathcal{E}$. $\square$

The result shows that equilibria with imperfect competition will result in compensated equilibria also under shareholder real wealth maximization, even with the weaker assumptions on the mechanism. What is shown by this result and that of the previous section is that under imperfect competition (of the "objective demand" type considered here), the allocation-price pairs obtainable are well-studied compensated equilibria of the economy considered, a result which indicates that the theory may have little to add to our understanding of the functioning of economies under other conditions than perfect competition (since our version of imperfect competition just simulates the well-known case).

On the other hand, the results depend on our assumptions, among which in-particular those specifying properties of the mechanism (Assumptions 4 and 5) are difficult to interpret in general. Therefore, we turn to a more general formulation of the previous equivalence results, where we may avoid assumptions on the mechanism.

5 A general equivalence result

The conclusions of Theorems 1 and 3, stating that equilibria of imperfect competition economies with objective demand are actually ordinary compensated equilibria, are quite strong, reducing as they do the class of allocations obtained in imperfectly competitive equilibria to a subset of those obtained in ordinary competitive equilibria, but on the other hand they depend heavily on the assumptions, some of which are rather restrictive. Thus, we would like to avoid the universal shareholder property which is in the assumptions of both Theorem 2 and Theorem 3 in order to cover also economies with a more restricted ownership of the firms. And, as already mentioned, the assumptions on the mechanism, whether stated as Assumption 4 or as Assumption 5, are posing rather heavy restrictions on the setup of the ecconomies with imperfect competition.

In the present section, we present a generalized version of the theorem in the previous section. The two restrictive features of Theorem 1 mentioned above are done away with. This of course does not come for free, and the result obtained is not a straightforward generalization of Theorem 1; instead of identifying different sets of allocation-price pairs in the same underlying economy, we shall introduce another economy $\hat{E}$ associated with the original economy $\mathcal{E}$, and then the equilibria with imperfect competition of $\mathcal{E}$ are ordinary compensated equilibria of $\hat{\mathcal{E}}$. However, the main message is retained – equilibria with imperfect competition are not new objects but belong to the well-studied class of competitive equilibria, when the choice sets are defined suitably.

We need an extension of the standard general equilibrium model, so that the sets of feasible production plans available to each firm may depend on the actual allocation. Formally, an economy with production externalities is an array $\hat{\mathcal{E}} = (X_i, \succsim_i, \omega_i)_{i=1}^m, \hat{A}, (V_j)_{j=1}^n((\theta_{ij})_{i=1}^m)_{j=1}^n)$, where

- (as previously) for $i \in \{1, \dots, m\}$, the triple $(X_i, \succsim_i, \omega_i)$ denotes a consumer with consumption set $X_i \subset \mathbf{R}^l$, preferences $\succsim_i \subset X_i \times X_i$, and initial endowment $\omega_i \in \mathbf{R}^l$, for $j \in \{1, \dots, n\}$,
- a set $\hat{A} \subset \mathbf{R}^{l(m+n)}$ of feasible allocations $(x_1, \dots, x_m, y_1, \dots, y_n)$, and for each $j \in \{1, \dots, n\}$
  - $V_j : \hat{A} \to \mathbf{R}^l(m+n)$ is a correspondence assigning to each feasible allocation a set of displacements of final allocation available to firm j,
  - for $i \in \{1, \dots, m\}$, $\theta_{i,j} \in [0, 1]$ denotes the profit share of consumer i in firm j, where $\sum_{i=1}^m \theta_{ij} = 1$ for all j.

In an economy with production externalities, the result of a change in the choice of production by some firm j is a change in allocation as described by the correspondences V_j.

We shall need a counterpart of the well-behavedness assumption for the general case.

The economy with production externalities

$$\hat{\mathcal{E}} = ((X_i, \succsim_i, \omega_i)_{i=1}^m, \hat{Y}, (V_j)_{j=1}^n, ((\theta_{ij})_{i=1}^m)_{j=1}^n)$$

satisfies the following:

(i) For each consumer i, X_i is nonempty, closed, convex, bounded from below, and satisfies $X_i + \mathbf{R}_+^l \subset X_i$, and $\succsim_i$ is complete preorder on X_i which is continuous, monotonic, and convex;

(ii) the set $\hat{A}$ is closed, convex, and nonempty, and

(ii.a) for each i and $z = (x_1, \ldots, x_m, y_1, \ldots, y_n) \in \hat{A}$, $x_i \in X_i$,

(ii.b) for each $j \in \{1, \ldots, n\}$ and each feasible allocation $z \in \hat{A}$, the set $V_j(z)$ is closed, convex, and contains 0.

We define $\hat{F}$ as the set of allocation-price pairs (z, p) in $\hat{\mathcal{E}}$ such that $z \in \hat{A}$ and $x_i \in \xi_i(p, p \cdot x_i)$ for each i.

Having defined the notions of allocations and feasible allocations in an economy $\hat{\mathcal{E}}$, the introduction of economies with imperfect competition is straightforward: Indeed, following the approach in Section 2 we now define a (generalized) economy with imperfect competition as a triple $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$, where

– $\hat{\mathcal{E}}$ is an economy with production externalities,

– Γ a mechanism for choice of allocation in $\mathcal{E}$ (that is, $\Gamma = (\Sigma_1, \ldots, \Sigma_n; \pi, \hat{\mathcal{F}})$, where Σ_j are nonempty strategy sets, $j = 1, \ldots, n$, and $\pi : \Sigma \to \hat{\mathcal{F}}$ is a correspondence assigning to each strategy array $\sigma \in \Sigma = \prod_{j=1}^n \Sigma_j$ a nonempty set of feasible allocation-price pairs in $\hat{\mathcal{E}}$), and

– for each j, Φ_j is a preference correspondence on $\hat{\mathcal{F}}$.

It is easily seen that the definition of an equilibrium in $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$ may be defined exactly as in Definition 1.

Definition 5.1 *Let $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$ be an economy with imperfect competition. An equilibrium in $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$ is an array (z, p, σ), where*

(1) $(z, p) \in \pi(\sigma)$,

(2) σ is a Nash equilibrium of $(\Gamma, (\Phi_j)_{j=1}^n)$ in the sense that there is no $j \in \{1, \ldots, n\}$ and $(z', p') \in \Phi_j(z, p)$ such that

$$(z', p') \in \pi(\tau_j, \sigma_{-j})$$

for some $\tau_j \in \Sigma_j$ where (τ_i, σ_{-j}) is the strategy array obtained from σ by replacing σ_j with τ_j.

The concept of a compensated equilibrium in $\hat{\mathcal{E}}$, on the other hand, differs slightly from the corresponding concept defined in $\mathcal{E}$, since the production externalities play a role in the behavioral constraints; however, the changes necessary are rather obvious – instead of choosing a profit maximizing production plan the firm will be in equilibrium when no change from the actual production plan will yield greater profits. Formally, a compensated equilibrium in $\hat{\mathcal{E}}$ is an allocation-price pair (z, p) such that

(1) $(z, p) \in \hat{\mathcal{F}}$,

(2) $p \cdot v_{ij} \leq 0$ for each $j \in \{1, \ldots, n\}$, $i \in \{1, \ldots, m\}$ with $\theta_{ij} > 0$, and each $v_j \in V_j(z)$ (recall that $v_j \in \mathbf{R}^{l(m+n)}$ specifies a displacement of allocation).

The reason for our introduction of the production externality, which for a superficial view is a phenomenon with no relation to imperfect competition, is that there is nevertheless an intimate connection, as showed by the following theorem. The theorem states that for any generalized economy with imperfect competition $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$ there is an associated economy with production externalities $\hat{\mathcal{E}}'$ such that all equilibria in $(\hat{\mathcal{E}}, \Gamma, (\phi_j)_{j=1}^n)$ are compensated equilibria in $\hat{\mathcal{E}}'$. The initial economy $\hat{\mathcal{E}}$ may or may not have production externalities; however, the associated economy $\hat{\mathcal{E}}'$ will typically be one with nontrivial production externalities.

THEOREM 5.2 *Let $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$ be a generalized economy with imperfect competition satisfying Assumptions 2,6, and 7. Then there is an economy $\hat{\mathcal{E}}'$ with production externalities satisfying Assumptions 2 and 7 such that each allocation-price pair $(z, p) \in \hat{\mathcal{F}}$ belonging to an equilibrium (z, p, σ) of $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$ for some $\sigma \in \Sigma$ is a compensated equilibrium of the economy $\hat{\mathcal{E}}$.*

Proof: First of all, we define the economy $\hat{\mathcal{E}}'$ with production externalities associated with $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$. The set $\mathcal{A}$ of feasible allocations in $\hat{\mathcal{E}}'$ coincides with that of $\hat{\mathcal{E}}$. To define the correspondences V_j, we proceed as follows: For each producer j and each equilibrium (z, p, σ) of $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$, the region of feasible consumption displacements of j's shareholders is defined as the set

$$W_j(z, p, \sigma) = \left\{ v \in \mathbf{R}^l \;\middle|\; \exists \tau_j \in \Sigma_j : v \in \sum_{i:\theta_{ij}>0} (\pi_i(\tau_j, \sigma_{-j}) - x_i) \right\}$$

(here, $\pi_i(\tau_j, \sigma_{-j})$ denotes the component corresponding to bundles of consumer i in $\pi(\tau_j, \sigma_{-j})$).

Next, assume that (z', p', σ') is another equilibrium in $\hat{\mathcal{E}}$ such that $(z', p') \neq (z, p)$. If $z = z'$ then, since (z, p') and (z, p) both belong to $\hat{\mathcal{F}}$, there is a unique supporting hyperplane of each set $\{x_i' \mid x_i' \succsim_i z_i + \omega_i\}$ at $z_i + \omega_i$ (by the smoothness property given by Assumption 2), and we conclude that $p = p'$. Thus, $z \neq z'$. It follows that either $z \neq z'$ or $(z', p') = (z, p)$. Consequently, the set

$$W_j(z) = \cup\{W_j(z, p, \sigma) \mid (z, p, \sigma) \text{ equilibrium in } (\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)\}$$

is well-defined whenever z belongs to some equilibrium (z, p, σ) in $(\mathcal{E}, \Gamma, (\Phi_j)_{j=1}^n)$.

Now, for each j, $j = 1, \dots, n$, we may define the production displacement correspondence $V_j : \hat{\mathcal{A}} \to \mathbf{R}^l$ by

$$V_j'(z) = \begin{cases} T_{W_j(z)}(0) & \text{if } (z, p, \sigma) \text{ is an equilibrium for some } p,\ \sigma, \\ \mathbf{R}^l_- & \text{otherwise} \end{cases}$$

(where, in accordance with our earlier notation, $T_{W_j(z)}(0)$ denotes the tangent cone of $W_j(z)$ at 0). It is easily checked that the economy with production externalities $\hat{\mathcal{E}}' = ((X_i, \precsim_i, \omega_i)_{i=1}^m, \hat{A}, (V_j')_{j=1}^n, (\theta_{ij})_{i=1}^m{}_{j=1}^n)$ defined by replacing the correspondences V_j by V_j', each j, satisfies Assumptions 2 and 7.

Let (z, p) be an allocation-price pair belonging to an equilibrium of $\hat{\mathcal{E}}$. To prove that (z, p) is a compensated equilibrium of the economy with production externalities $\hat{\mathcal{E}}' = ((X_i, \precsim_i, \omega_i)_{i=1}^m, \hat{A}, (V_j')_{j=1}^n, (\theta_{ij})_{i=1}^m{}_{j=1}^n)$, we need only show that $p \cdot v_j' \leq 0$ for all $v_j' \in V_j'(z)$. Suppose on the contrary that there is some j and $v_j' \in V_j'(z)$ such that $p \cdot v_j' > 0$. Then, by our definition of $V_j'(z)$, there is some u_j in the tangent cone of $W_j(z)$, and by compactness of Σ, in the tangent cone of $W_j(z, p, \sigma)$ for (z, p, σ) an equilibrium in $(\hat{\mathcal{E}}, \Gamma, (\Phi_j)_{j=1}^n)$, such that

$$u \in \sum_{i:\theta_{ij}>0} [P_i(z_i + \omega_i) + \omega_i].$$

Reasoning as in the proof of Theorem 3, we get that this is a contradiction, and we conclude that (z, p) is a compensated equilibrium in $\hat{\mathcal{E}}'$. □

6 Concluding comments

In the present paper, we have presented a general equilibrium model of an economy with imperfect competition, where the decisions of the firms are taken with consideration to their effects in the economy as a whole, given the choices of the other firms. In the sense that firms anticipate the impact of their actions on the allocations and prices established in the market, the model is one of objective demand.

What has mainly concerned us in this paper was the objective of the firm acting on behalf of its shareholders. Since profit maximization makes little sense in a world where the actions of the firms change the price system and thereby the meaning of the profit to be maximized, other objectives must be considered, and we have focussed on two, namely (1) Pareto-improvements for shareholders, and (2) shareholder real wealth maximization. Our main result was that in case of each of these two objectives, the allocations obtained in equilibrium are rather well-behaved; actually, the final outcome in the imperfectly competitive economy looks as if it has been obtained by the usual competitive mechanism, possibly with a redistribution of incomes.

The results may be interpreted in several ways; since the final allocations are perhaps too well-behaved as compared with what we expect intuitively, it may be the case that the objectives of the firm have not yet been specified in a proper way; shareholder unanimity or shareholder real wealth maximization releives the model of some inconsistencies inherent in other specifications of the firm's objective to be encountered in the literature, but are not by themselves very convincing, and if this holds also for the outcomes resulting from these objectives, they should probably be reconsidered.

Another line of explanation has to do with the objective demand approach. It is perhaps not quite reasonable to assume that the firm can trace the full impact of its choices on the total economic situation of its shareholders. After all, the subjective demand approach, specifying what the firms expect rather than assuming full knowledge, may be a better approach. Needless to say, some of the problems of formulating a reasonable objective of the firm on behalf of the shareholders will remain.

A third – and much simpler – explanation of the results would be obtained by pointing to the restrictiveness of the assumptions. In the objective demand model, the firm can trace the result for its shareholders of its actions, and if the aggregate displacement of shareholders' bundles are not two different from displacements of the chosen production plan, then the equilibria will correspond closely to ordinary compensated equilibria, as indeed it was shown in Theorems 1 and 3. This however hinges on an assumption of appropriate closeness of the two alternative sets of displacements, assumptions which it was not easy to defend in their full generality. Needless to say, under less restrictive assumptions the equilibria with imperfect competition need not coincide with compensated equilibria of the underlying economy.

The problems with the restrictiveness of these assumptions led us to the abstract approach in the previous section, showing that the relationship between equilibria under imperfect competition and compensated equilibria found in the first part of the paper is a particular instance of a much more general result, which on the one hand does not need the restrictive assumptions, but which on the other is mor abstract, involving the notion of a production externality. This concept, which plays a certain unifying role, may well merit a closer study, which among other things could clarify the equilibrium existence problem. This will however be a theme for future research.

7 References

Clarke, F.H. (1983), Optimization and nonsmooth analysis, Wiley, New York.
Debreu, G. (1959), Theory of value, Wiley, New York.

Dierker, E. and B. Grodal (1996) The price normalization problem in imperfect competition and the objective of the firm, Discussion Paper 96/05,University of Copenhagen, Institute of Economics, 1996.

Gabszewicz, J. and J.P. Vial (1972), Oligopoly *à la* Cournot in general equilibrium analysis, Journal of Economic Theory 4, 381 – 400.

Green,J, A.MasColell, H.Whinston (1995)

Hart, O.D. (1985), Imperfect competition in general equilibrium: an overview of recent work, in: K.J. Arrow and S. Honkapohja (eds.), Frontiers of economics, Basil Blackwell, Oxford, 100 – 169.

Negishi, T. (1961), Monopolistic competition and general equilibrium, Review of Economic Studies 28, 196 – 201.