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Abstract

This paper develops a procedure for testing hypotheses on the full set of cointegration parameters of the I(2) model. The proposed test is applied to the analysis of small-country import price determination extending the standard empirical framework to allow for variables integrated of order two. The empirical analysis of Danish data for 1975 to 1995 yields a fully specified I(2) long-run structure in terms of stationary pricing-to-market and inventory relations, a nominal second-order stochastic trend embodied in equal proportions in domestic and foreign price levels, and a real first-order trend driving the relative prices and the real interest rate.

Key words: Pricing-to-Market, Long-Run Price Trends, Cointegration, Second-Order Non-Stationarity

JEL Classification: F40, C32, C51, C52

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1 Introduction

This paper suggests a procedure for testing certain hypotheses on the full set of cointegration parameters of the I(2) model. The hypothesis investigated is that a common second-order stochastic trend loads into a set of variables in known proportions. This is of interest to most analyses of nominal variables which are often found to be I(2) and for which there exist theoretical predictions concerning, *e.g.*, long-run proportionality of money and prices. In the context of import price determination, a model specification is investigated in which a nominal second-order stochastic trend affects domestic and foreign price variables with equal loadings so that *all* cointegrating relations of the I(2) model can be expressed in terms of relative prices.

In the literature on import price determination it has become a stylized fact that domestic variables contribute significantly to import pricing in large economies, even in the longer run, with the import price showing highly persistent variations relative to foreign prices and costs. This is the phenomenon named *pricing-to-market* by Krugman (1987). It has been documented for large economies, in particular the U.S., by a number of empirical studies. For instance, Hooper and Mann (1989) found that American producer prices enter with a weight of .39 in the equation that describes the price of imported manufactures over the period from 1973 to 1988. The imports from Japan show a slightly lower coefficient of .33. See Menon (1995a) for a survey of the empirical literature.

More surprising from a theoretical point of view are recent findings of long-run pricing-to-market effects also in the case of small economies, *e.g.*, Norway, Denmark, and Australia. This appears at odds with fundamental theoretical propositions, including the small open economy assumption. Naug and Nymoen (1996) found that a domestic price measure enters with a weight of .37 into the long-run determination of Norwegian import prices. In the Danish case, Jensen and Knudsen (1992) established a significant role for domestic costs in an import pricing relation, although the weight could not be precisely estimated. Menon (1995b) determined a coefficient of .34 for domestic prices in the case of manufactured imports to Australia.

Modern time-series techniques are used in much of this literature to deal with issues like non-stationarity of the variables and the distinction between short-term adjustment and long-run equilibria. The present paper uses an extended approach that allows for variables being integrated of order two. Empirically, this is in line with recent findings in other areas, *e.g.*, Juselius (1995) and Banerjee, Cockerell, and Russell (1998), that nominal variables are better described as being I(2) than first-order non-stationary. It will also be argued that this extension can partly reconcile the findings of pricing-to-market in small economies with traditional propositions in international economics.

The outline of the paper is as follows: Section 2 reviews the main theoretical ideas in the literature on models of long-run import pricing and examines two alternative interpretations in a cointegration setup. The econometrics of estimation and hypothesis testing in I(2) systems have been developed only recently and a brief account is therefore provided in section 3 which establishes the model and the notation, reviews some recently developed tools and derives a test of certain hypotheses on the cointegration parameters. The empirical findings for the analysis of Danish import prices are reported in section 4. Section 5 offers some conclusions.

2 Import Pricing in the Long Run

This section first outlines a theory of import price determination with price setting firms. The basic so-called pricing-to-market relation is then augmented to allow for demand effects, inflation costs, and inventory costs. The standard time-series interpretation of this model in terms of first-order non-stationary variables is discussed and an alternative interpretation is suggested in anticipation of the empirical results for the Danish case.

2.1 A Theoretical Framework

A prototypical model of import prices is derived by Naug and Nymoen (1996), based on the work of Krugman (1987) and Hooper and Mann (1989), see also Menon (1995b) for a similar approach. The price of imports to the domestic market in period t, p_{mt},[1] is related to c_{ft}, the domestic currency value of foreign exporters' marginal costs, and to p_{bt}, the corresponding domestic price index, according to

$$p_{mt} = (1-\theta)c_{ft} + \theta p_{bt},\ 0 \leq \theta \leq 1. \tag{1}$$

The foreign producer is a price-setter who determines the price for the domestic market as a markup over marginal costs. The domestic price, p_{bt}, is present here due to the assumption that the optimal markup depends positively on the ratio of the price of domestically produced substitutes to foreign production costs.

[1]Lowercase denotes log-transformed variables.

Relation (1) is termed the *pricing-to-market* relation. Formally, it is derived as a relation for goods exported from a representative producer in one foreign country to the particular market in question. In practice, p_{mt} is taken to be an index aggregating the prices of imports from all origins. Moreover, many studies use the foreign export price to all destinations, p_{ft}, as a proxy for the marginal cost variable which is not easily observable.[2] The relative price, $p_{mt} - p_{ft}$, then reflects the extent to which this particular market is being discriminated compared to the average destination, in addition to usual factors such as transportation costs, insurance, and tariffs.

Naug and Nymoen (1996) argue that further variables which provide information to foreign producers on domestic demand conditions potentially affect the pricing decision. The pricing-to-market relation is augmented by a vector of additional variables, z_t,

$$p_{mt} = (1 - \theta)p_{ft} + \theta p_{bt} + \theta_1 z_t. \tag{2}$$

Easily observable cyclical variables such as the rate of unemployment and the rate of inflation, Δp_t,[3] could presumably be included in z_t. However, Naug and Nymoen note that even if z_t is informative about the cyclical stance of the domestic economy, the effect of changing demand conditions on the pricing decision is theoretically ambiguous.

A specific rationale for the first-difference of prices to appear in pricing relations is provided by theories relating the markup over costs to the rate of inflation. Russell, Evans, and Preston (1997) consider a model in which price-setting firms are imperfectly informed and face a comparatively large loss if prices are set too high, *e.g.*, due to a kinked demand curve. In that case, firms will act cautiously and choose relatively low markups. Assuming that uncertainty about prices is increasing with inflation, a lower markup is then associated with a higher rate of inflation. Banerjee, Cockerell, and Russell (1998) find empirical evidence of an inflation cost in their analysis of domestic prices, import prices, and unit labour costs in Australia.

In a foreign trade context it is noted that the markups of domestic as well as foreign producers could be negatively affected by inflation. The relative price, $p_{bt} - p_{ft}$, changes with inflation only to the extent that foreign and domestic firms react differently, *e.g.*, if foreign firms perceive demand to be more elastic with price increases than domestic firms do. In that case, the inflation cost would

[2]See Menon (1995b) for a critical discussion.

[3]Naug and Nymoen (1996) use the consumer price inflation. See below for a different choice more directly related to the import pricing information set.

cause the relative price of foreign goods to decrease with a higher rate of inflation.

The role of import distributors may require separate attention. Dwyer and Lam (1994) and Dwyer, Kent, and Pease (1994) find the distinction between the price of imports "over the docks" and at the retail level important in the case of Australia. In the present data set import prices are measured as importers' sales prices that could be directly affected by domestic factors, *e.g.*, in warehousing, repackaging and transportation. As a main determinant of inventory costs, the rate of interest, R_t, is included in the empirical analysis. A relation of the form

$$p_{mt} - p_{ft} = \rho(R_t - \Delta p_t),\ \rho > 0, \tag{3}$$

is expected to hold from this line of argument and will be denoted an inventory relation.

This outline has several implications for the empirical analysis. First, it singles out four variables, p_{mt}, p_{ft}, p_{bt}, and R_t, as a minimal information set for the analysis of import price determination. Second, the pricing-to-market relation (2) and the inventory relation (3) are both homogeneous in prices, which would seem a necessary condition for any model of import prices to be a valid characterization of the behaviour of economic agents in the long run. Finally, the outline suggests some theoretical bounds for the parameters θ and ρ whereas the sign of any effect of inflation either as a demand effect or as an inflation cost cannot be determined *a priori*.

2.2 Alternative Time-Series Interpretations

The issues to be addressed next require a specification of the time-series properties of the variables involved.[4] The first issue is how pricing-to-market agrees with fundamental notions such as the law of one price, the purchasing power parity, and the small open economy assumption. The second and related question is if pricing-to-market occurs as a short-run or a long-run phenomenon. The arguments of Naug and Nymoen (1996) based on first-order non-stationary price variables will be outlined and an alternative in terms of second-order non-stationarity is then suggested.

Suppose that the prices can be characterized as I(1) variables. According to Naug and Nymoen (1996) some basic propositions of international economics can then be related to the stationarity of relative prices. A long-run version of the law of one price holds if p_{mt} and p_{ft} cointegrate from I(1) to I(0) with cointegrating

[4]In doing so, the relations (2) and (3) will be interpreted as holding except for a stationary error term.

parameters of one and minus one. Likewise, purchasing power parity is said to hold as a long-run relation if p_{bt} and p_{ft} cointegrate to I(0), *i.e.*, if the relative price $p_{bt} - p_{ft}$ is stationary.

However, if both propositions are to hold simultaneously there exist two independent cointegrating relations among the price variables. Relative prices would be stationary and (2) would imply "short-run pricing-to-market" behaviour. It is not clear how θ is identified empirically in this case, since any linear combination of the two relative prices would then be stationary.

Alternatively, if only one cointegrating relation existed among the price variables, then at most one of the propositions may hold. Either of the relative prices can be stationary (but not both), or both relative prices can be I(1) and cointegrating with parameters θ and $1 - \theta$ as in (2). The latter situation is dubbed "long-run pricing-to-market". Note that in this case the price levels and the relative prices are characterized by the same order of integration, *i.e.* they are equally persistent in qualitative terms.

The empirical analysis of Danish data below suggests that p_{mt}, p_{ft}, and p_{bt} are well described as I(2) variables. Suppose that prices are indeed second-order non-stationary and assume that R_t is at most I(1). This scenario encompasses a version of the law of one price, the purchasing power parity proposition and long-run pricing-to-market behaviour while allowing for different degrees of persistence of the price levels and the relative prices. The basic assumption needed is long-run price proportionality, that the *same* second-order non-stationary stochastic trend is embodied in the price variables with *identical* coefficients.

A single common I(2) trend then affects the variables with loadings proportional to the vector $b \equiv (1, 1, 1, 0)'$. Linear combinations of the variables defined by vectors orthogonal to b eliminate the trend and thus cointegrate from I(2) to I(1) or to stationarity. These cointegrating relations which can be expressed in terms of the relative prices and the interest rate, satisfy the homogeneity requirement. While prices may in fact diverge permanently, relative prices are less persistent in qualitative terms than the stochastic trend embodied in the price levels.

The theoretical propositions hold in the weak sense of being associated with cointegration from I(2) to I(1). Pricing-to-market behaviour according to (2) could be a long-run phenomenon implying cointegration between first-order non-stationary relative prices. This scenario also leaves open the possibility that the I(1) rate of inflation and the interest rate can have long-run effects in determining prices as emphasized by Banerjee *et.al.* (1998). The small open economy assumption that prices are determined outside the domestic economy translates into a question about the origins of the stochastic trends, in particular the second-order non-stationary trend driving the price levels.

All assumptions underlying this scenario are empirically testable. The following section establishes a set of econometric tools for this purpose.

3 A Tool Kit for the Econometric Analysis

This section serves to introduce the main tools applied in the empirical section. Recent results on the I(2) model, the two-step estimation procedure, and the analysis of exogeneity will be adapted for the particular model specification used here. A simple procedure is then established for inference on the full set of cointegration parameters of the I(2) model.

3.1 The I(2) Analysis

Consider a p-dimensional vector time series X_t which is modelled by a kth order vector autoregression, conveniently parameterized[5] for the I(2) analysis as

$$\begin{aligned}\Delta^2 X_t = \Pi X_{t-1} \quad &- \quad \Gamma \Delta X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} \\ &+ \quad \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \quad t = 1, \ldots, T. \end{aligned} \tag{4}$$

The term ε_t is assumed to be identically and independently distributed $N(0, \Omega)$. The initial observations, $X_{-k+1}, \ldots, X_0$, are taken to be fixed. The model includes constant and linear drift terms and may also include a vector of additional deterministic terms, D_t, *e.g.*, seasonals or intervention dummies. For ease of notation the terms $\Delta^2 X_{t-i}, \quad i = 1, \ldots, k-2$, and D_t are collected in a vector U_t with parameter Ψ.

Some notation will be needed in the following. For a $p \times r$ matrix α of rank r, let $\alpha_\perp$ denote a basis of the $p \times (p-r)$ orthogonal complement and define $\bar{\alpha} = \alpha(\alpha'\alpha)^{-1}$. For the matrix ξ of dimensions $(p-r) \times s$ and rank s, $s < p-r$, define $\alpha_1 = \bar{\alpha}_\perp \xi$ and $\alpha_2 = \alpha_\perp \xi_\perp$. The matrices α, α_1, and α_2 are thus mutually orthogonal. Likewise, for β $(p \times r)$ and η $(p - r \times s)$, $s < p-r$, define $\beta_1 = \bar{\beta}_\perp \eta$ and $\beta_2 = \beta_\perp \eta_\perp$.

The I(2) model is a submodel of (4) with the parameters restricted by the reduced rank conditions

$$\Pi = \alpha\beta' \quad \text{and} \quad \alpha'_\perp \Gamma \beta_\perp = \xi\eta'. \tag{5}$$

[5] See Johansen (1996, Chapter 9) for its relation to the original VAR in levels.

The deterministic specification is that suggested by Rahbek, Kongsted and Jørgensen (1998). It restricts the parameters of the constant and linear drift terms in (4) by

$$\alpha_\perp' \mu_0 = -\xi \eta_0' - \alpha_\perp' \Gamma \bar{\beta} \beta_0' \quad \text{and} \quad \mu_1 = \alpha \beta_0', \tag{6}$$

where η_0' and β_0' are vectors of dimensions s and r. The parameter $\bar{\alpha}' \mu_0$ remains unrestricted.

When the roots of the characteristic polynomial of (4) can be assumed to be either at one or outside the unit circle, the necessary and sufficient conditions for X_t to be I(2) are that $\text{rank}(\Pi) = r < p$, $\text{rank}(\alpha_\perp' \Gamma \beta_\perp) = s < p - r$, and that a further rank condition holds which prevents the variables from being integrated of higher orders, see Johansen (1992). Under these conditions and (6), Rahbek *et. al.* (1998) show that the vector process has linear deterministic trends in all components, including the cointegrating relations. Quadratic trends (or higher) for which there is usually no evidence in the data, are absent by the restrictions.

When X_t is I(2) the linear combinations defined by β, β_1, and β_2 partition the process into parts with different cointegration properties. The matrix β_2 is proportional to the loadings of the common I(2) trends and the $p-r-s$ variables $\beta_2' X_t$ embody the I(2) components of X_t. The $r+s$ linear combinations $(\beta, \beta_1)' X_t$ are cointegrating in the sense of being integrated of an order less than two. They can be divided into the s combinations, $\beta_1' X_t$, which remain I(1), and $\beta' X_t$ which cointegrate to stationarity with the first-difference of the I(2) component in the r polynomially cointegrating relations,

$$S_t = \beta' X_t - \delta \beta_2' \Delta X_t, \tag{7}$$

where $\delta = \bar{\alpha}' \Gamma \bar{\beta}_2$ is $r \times (p - r - s)$. Note that S_t in general is stationary around a linear trend.

A priori, the differences between components in terms of persistence are due to the order of their stochastic trends rather than differences in the deterministic part. Clearly, the linear trend may have zero coefficients in certain directions. In particular, some of the polynomially cointegrating relations may be stationary rather than trend-stationary. However, whether a trend is present does not affect the asymptotic properties of tests and estimators in this model.[6]

The main model features discussed in section 2 have counterparts in terms of the sub-processes of X_t with different cointegrating properties. A very strong degree of persistence exists in the level of the process due to the I(2) trend. Certain

[6]This is a property of asymptotic similarity, see Rahbek *et. al.* (1998).

linear combinations of the levels are still persistent but I(1) rather than I(2) and there is a potential role for first-differences of the variables in the polynomially cointegrating relations.

For estimation of the I(2) model the two-step algorithm of Johansen (1995) will be applied as adapted to the above deterministic trend specification by Rahbek *et. al.* (1998). The first step leaves Γ and μ_0 unrestricted. Estimates of $\beta^* = (\beta', \beta_0')'$ and α are obtained by a so-called reduced rank regression of $\Delta^2 X_t$ on $X_{t-1}^* = (X_{t-1}', t)'$ corrected for ΔX_{t-1}, a constant, and U_t. This is equivalent to the standard I(1) reduced rank analysis with a restricted linear term, see Johansen (1996, Section 6.2). This reduced rank regression also defines a first-step rank test statistic denoted Q_r.

The second step in turn takes the estimates of β^* and α as given. Since a main result of this paper concerns a restriction on second-step parameters more details will be provided on this step. The model is divided into two submodels. The partial model of $\bar{\alpha}'\Delta^2 X_t$ conditional on $\alpha_\perp'\Delta^2 X_t$ has no reduced rank condition and can be estimated by OLS. The marginal model of $\alpha_\perp'\Delta^2 X_t$ which contains the I(2) reduced rank condition, $\alpha_\perp'\Gamma\beta_\perp = \xi\eta'$, is estimated by a reduced rank regression of $\alpha_\perp'\Delta^2 X_t$ on $((\overline{\beta}_\perp'\Delta X_{t-1})', 1)'$ corrected for $\beta^{*\prime}\Delta X_{t-1}^*$ and U_t. This procedure provides estimates of $\eta^* = (\eta', \eta_0')'$ and ξ and the second-step rank test statistic, $Q_{r,s}$.[7] Inference on r and s can be based on the joint rank test statistic, $S_{r,s} = Q_r + Q_{r,s}$, see Rahbek *et. al.* (1998) for the testing procedure.

The analysis of weak exogeneity with respect to the cointegration parameters of the I(2) model, (β, β_1, δ), has been derived by Paruolo and Rahbek (1998). For a case in which $r > 0$ and $s > 0$ the $p - m$ dimensional subset of variables $a'X_t$ is shown to be weakly exogenous for the cointegration parameters if and only if

$$a'(\alpha, \alpha_1, \zeta_1) = 0, \tag{8}$$

where $\zeta_1 = \Gamma\bar{\beta}$ and $m \geq r + s$. The first part, $a'\alpha = 0$, implies that the β-cointegrating combinations and, in effect, the polynomially cointegrating combinations, S_t, do not appear in the equations for $a'\Delta^2 X_t$. The condition ensures asymptotic efficiency of the first-step estimate of β although it is not sufficient for weak exogeneity in the I(2) case. If the second part holds, $a'\alpha_1 = 0$, then also the β_1-cointegrating combinations will be absent from the equations for $a'\Delta^2 X_t$. Conditioning the analysis on the variable $a'X_t$ will be asymptotically efficient for the cointegration parameters and the corresponding error term, $a'\varepsilon_t$, is identified as a source of the I(2) common trends. The final part, $a'\zeta_1 = 0$, implies that the

[7]The second step is comparable to the I(1) analysis of a model with a restricted constant term, see Johansen (1996, Section 6.2).

β-combinations remain absent from the equations for $a'\Delta^2 X_t$ also in the form of first-differences.

Paruolo and Rahbeks suggestion is to examine each part separately and sequentially based on the two-step estimation procedure. The restriction that $a'\alpha = 0$ is analyzed in the first step. Imposing the restriction the statistical analysis considers the model of $A'X_t$ conditional on $a'X_t$, where A is $p \times m$ such that $\text{sp}(A, a) = R^p$.[8] Adapted for (4) – (6) the estimates of α and β^* subject to $\alpha = A\psi$ are obtained by reduced rank regression of $A'\Delta^2 X_t$ on X^*_{t-1} corrected for ΔX_{t-1}, a constant, and U_t. Under the hypothesis that $a'\alpha = 0$ the likelihood ratio test, Q_{a1}, is asymptotically distributed as χ^2 with $(p-m)r$ degrees of freedom, see Paruolo and Rahbek (1998).

The second step uses the restricted estimate of α and the corresponding β^* from the first step. Provided that $\alpha = A\psi$, its orthogonal complement can be represented as

$$\alpha_\perp = (\bar{A}\psi_\perp, a) = (A_1, a). \tag{9}$$

The model is divided into a model of $\bar{\alpha}'\Delta^2 X_t$ conditional on $\alpha'_\perp \Delta^2 X_t$, and a marginal model of $\alpha'_\perp \Delta^2 X_t$. The restriction $a'\alpha_1 = 0$ implies that the second-step loading matrix,

$$\xi = \alpha'_\perp \alpha_1 = (A_1, a)'\alpha_1 = \begin{pmatrix} A'_1\alpha_1 \\ a'\alpha_1 \end{pmatrix}, \tag{10}$$

has zeros in the last $p-m$ rows. Adapted for the deterministic specification in (6), the restricted estimate of ξ is obtained from the conditional model of $A'_1\Delta^2 X_t$ given $a'\Delta^2 X_t$ by reduced rank regression of $A'_1\Delta^2 X_t$ on $((\overline{\beta}'_\perp \Delta X_{t-1})', 1)'$ corrected for $\beta^{*'}\Delta X^*_{t-1}$, U_t, and $a'\Delta^2 X_t$. The likelihood ratio test, Q_{a2}, against the model which remains unrestricted in the second step is asymptotically distributed as χ^2 with $(p-m)s$ degrees of freedom under the condition that $a'(\alpha, \alpha_1) = 0$, see Paruolo and Rahbek (1998).

The last part of (8), $a'\zeta_1 = 0$, involves further parameters of the marginal model of $a'\Delta^2 X_t$. Imposing $a'(\alpha, \alpha_1) = 0$, the likelihood ratio test, Q_{a3}, of $a'\zeta_1 = 0$ is constructed in Paruolo and Rahbek (1998).

[8] A and a need not be orthogonal but can be chosen this way without loss of generality, see Paruolo and Rahbek (1998). Assume therefore also that $a'A = 0$.

3.2 Restrictions on the Cointegration Parameters

This section shows how the two-step algorithm can be adapted to analyzing certain hypotheses on the cointegration parameters. Specifically, a hypothesis is considered that *all* cointegrating relations of the I(2) model are subject to $p-q$ linear restrictions as defined by the $p \times (p-q)$ matrix b,

$$b'(\beta, \beta_1) = 0, \tag{11}$$

with $q \geq r+s$. The theoretical interest in (11) is evident in view of the discussion of pricing relationships in section 2. Moreover, the restriction is equivalent to

$$sp(b) \subseteq sp(\beta_2), \tag{12}$$

where β_2 is the matrix by which the I(2) common trends load into the variables. Thus, (11) is the hypothesis that the known matrix b defines (a subset of) the I(2) loadings.

The restriction (11) has the same structure as (8). The first part, $b'\beta = 0$, can be parameterized as

$$\beta = B\varphi, \quad B = b_\perp, \tag{13}$$

for some $q \times r$ matrix φ of freely varying parameters. The orthogonal complement can then be constructed as

$$\beta_\perp = (\bar{B}\varphi_\perp, b) = (B_1, b), \tag{14}$$

and for the second-step parameter, η, it holds that

$$\eta = \beta_\perp' \beta_1 = (B_1, b)'\beta_1 = \begin{pmatrix} B_1'\beta_1 \\ b'\beta_1 \end{pmatrix}. \tag{15}$$

With both parts of (11) imposed, η can be parameterized as $\eta = H\kappa$ with $H = (I_{q-r}, 0)'$ and κ being a $(q-r) \times s$ matrix of parameters which vary freely under the hypothesis.

The statistical analysis of (11) is based on the two-step procedure, treating the restrictions on β and β_1 sequentially and separately. In the first step, the model is estimated first unrestrictedly and then subject to the restriction on β. The calculations are equivalent to a standard I(1) estimation, see Johansen (1996, Section 7.2.1). The likelihood ratio test of the restriction on β, denoted Q_{b1}, is asymptically distributed as χ^2 with $(p-q)r$ degrees of freedom subject to (13).

This follows from Rahbek *et. al.* (1998, Theorem 4.2).

The second step proceeds by first estimating η unrestrictedly and then imposing $b'\beta_1 = 0$ in (15). Both estimations are conditional on the first-step restricted estimate of β and the corresponding estimate of α. Interest remains in the equation containing the I(2) reduced rank condition which is obtained by premultiplying (4) by $\alpha'_\perp$. Using the identity $I_p = \bar{\beta}\beta' + \bar{\beta}_\perp\beta'_\perp$ and imposing (5) and (6) the equation reduces to

$$\alpha'_\perp \Delta^2 X_t = -\xi(\eta', \eta'_0)\begin{pmatrix} \bar{\beta}'_\perp \Delta X_{t-1} \\ 1 \end{pmatrix} - \alpha'_\perp \Gamma \bar{\beta}\beta^{*\prime}\Delta X^*_{t-1} + \alpha'_\perp \Psi U_t + \alpha'_\perp \varepsilon_t. \tag{16}$$

The second-step unrestricted estimates are obtained from (16) by reduced rank regression of $\alpha'_\perp\Delta^2 X_t$ on $((\bar{\beta}'_\perp \Delta X_{t-1})', 1)'$ corrected for $\beta^{*'}\Delta X^*_{t-1}$ and U_t. With the product moment matrices denoted by $M_{ij\cdot\beta}$, $i, j = \alpha_\perp, \beta_\perp$, see Rahbek *et. al.* (1998) for their exact definitions, this amounts to solving the $(p - r + 1)$-dimensional eigenvalue problem,

$$|\zeta M_{\beta_\perp\beta_\perp\cdot\beta} - M_{\beta_\perp\alpha_\perp\cdot\beta} M^{-1}_{\alpha_\perp\alpha_\perp\cdot\beta} M_{\alpha_\perp\beta_\perp\cdot\beta}| = 0 \tag{17}$$

for eigenvalues $1 > \zeta_1 > \ldots > \zeta_{p-r} > 0$, $\zeta_{p-r+1} = 0$, and eigenvectors $V^* = (v^*_1, \ldots, v^*_{p-r+1})$. The unrestricted estimates are $\eta^* = (\eta', \eta'_0)' = (v^*_1, \ldots, v^*_s)$ and $\beta_1 = \bar{\beta}_\perp \eta$.

Next, the restriction $b'\beta_1 = 0$ is imposed. With $\eta = H\kappa$ and the definition

$$H^* = \begin{pmatrix} H & 0 \\ 0 & 1 \end{pmatrix} \tag{18}$$

the restricted equation reads

$$\alpha'_\perp \Delta^2 X_t = -\xi(\kappa', \eta'_0)H^{*'}\begin{pmatrix} \bar{\beta}'_\perp \Delta X_{t-1} \\ 1 \end{pmatrix} - \alpha'_\perp \Gamma \bar{\beta}\beta^{*\prime}\Delta X^*_{t-1} + \alpha'_\perp \Psi U_t + \alpha'_\perp \varepsilon_t. \tag{19}$$

This yields a reduced rank regression of $\alpha'_\perp\Delta^2 X_t$ on $H^{*'}((\overline{\beta}'_\perp \Delta X_{t-1})', 1)'$ corrected for $\beta^{*'}\Delta X^*_{t-1}$ and U_t. The restricted estimates are obtained by solving the $(q - r + 1)$-dimensional eigenvalue problem,

$$|\zeta_b H^{*\prime} M_{\beta_\perp\beta_\perp\cdot\beta} H^* - H^{*\prime} M_{\beta_\perp\alpha_\perp\cdot\beta} M^{-1}_{\alpha_\perp\alpha_\perp\cdot\beta} M_{\alpha_\perp\beta_\perp\cdot\beta} H^*| = 0 \tag{20}$$

for eigenvalues $1 > \zeta_{b1} > \ldots > \zeta_{bq-r} > 0$, ζ_{bq-r+1}, and eigenvectors $V^* = (v^*_{b1}, \ldots, v^*_{bq-r+1})$. The restricted estimates are $(\kappa', \eta'_0)' = (v^*_{b1}, \ldots, v^*_{bs})$, $\eta = H\kappa$, and $\beta_1 = \bar{\beta}_\perp \eta$.

Finally, given the first step estimates of β^* and α, the likelihood ratio test of $b'\beta_1 = 0$ is

$$Q_{b2} = T \sum_{i=1}^{s} \ln \left(\frac{1-\zeta_{bi}}{1-\zeta_i} \right), \tag{21}$$

which is asymptotically distributed as $\chi^2((p-q)s)$ subject to $b'(\beta, \beta_1) = 0$. This follows by the fact that η^* is mixed Gaussian, see Rahbek *et. al.* (1998, Theorem 4.3).

An overall test of (11) is constructed by considering a rejection region which is the union of the regions for the individual tests, *i.e.* the overall hypothesis is rejected if any of the individual tests are rejected. The size of each individual test can be chosen as $\nu/2$ in which case the true but unknown size of the overall test is between $\nu/2$ and ν.

3.3 A Restricted VAR

A case of special interest is $q = r + s$ which implies that β_2 is known up to a normalization. Its practical importance derives from the fact that when the matrix of I(2) loadings is completely identified, the analysis can proceed using standard I(1) methods. One example is of course the case when there is $p-r-s = 1$ common I(2) trend and long-run price proportionality holds.

From the properties of the different components of X_t it follows that the process $\breve{X}_t = (X_t'\beta, X_t'\beta_1, \Delta X_t'\beta_2)'$ is I(1). The polynomially cointegrating relations, S_t, are embodied as standard I(1) cointegrating vectors,

$$S_t = \beta' X_t - \delta\beta_2'\Delta X_t = (I_r, 0, -\delta)\breve{X}_t. \tag{22}$$

Under the hypothesis that $b'(\beta, \beta_1) = 0$, the processes $\breve{X}_t$ and $\tilde{X}_t = (X_t'B, \Delta X_t'b)'$ are related by the non-singular transformation,

$$\tilde{X}_t = \begin{pmatrix} A^{-1\prime} & 0 \\ 0 & I_{p-r-s} \end{pmatrix} \breve{X}_t, \tag{23}$$

where $A = (\varphi, \tilde{B})$, see Kongsted (1998) for the expression for $\tilde{B}$, and b is chosen as a valid basis for β_2. Then, also $\tilde{X}_t$ is I(1) and its cointegrating relations are

$$\tilde{\beta} = \begin{pmatrix} A & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} I_r \\ 0 \\ -\delta' \end{pmatrix} = \begin{pmatrix} \varphi \\ -\delta' \end{pmatrix}. \tag{24}$$

Inference on $\tilde{\beta}$, including δ, can therefore be obtained from the I(1) analysis of the process $\tilde{X}_t$. A vector autoregression for $\tilde{X}_t$ specified as

$$\Delta\tilde{X}_t = \tilde{\alpha}\tilde{\beta}'\tilde{X}_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_i \Delta\tilde{X}_{t-i} + \tilde{\mu}_0 + \tilde{\alpha}\tilde{\beta}_0' t + \tilde{\Phi} D_t + \tilde{\varepsilon}_t \tag{25}$$

corresponds to the I(2) model under the restriction (11).[9] Applying the unrestricted I(1) reduced rank regression to this model ignores the restriction on μ_0 in (6) and allows an additional lag in $b'\Delta X_t$. While analyzing the model without imposing the restrictions is convenient, it leads to a loss of efficiency and of one observation.

4 The Empirical Analysis

This section contains the empirical analysis[10] of the long-run relations governing import pricing. The data is presented and a vector autoregression is established as an adequate representation of the data. The issue of exogeneity is investigated and the long-run price proportionality hypothesis is tested. Some tentative evidence is offered on the nature of the polynomially cointegrating relations and the common trends.

4.1 The Data

The variables $X_t = (p_{mt}, p_{ft}, p_{bt}, R_t)'$, are taken from the database of the Danish Central Bank macromodel, Mona, see Christensen and Knudsen (1992).[11] The sample period is first quarter 1975 to last quarter 1995 less initial values.

The import price measure, p_{mt}, is the one used in Mona, a broad index of importers' sales prices, see the discussion in section 2.1. The after-tax nominal

[9] See Kongsted (1998) for an account of the relation between the I(2) model and the transformed model (25).

[10] The analysis used Cats in Rats and Clara Jørgensen's I(2) programme with the author's extensions.

[11] See the Appendix for the exact data definitions.

rate of interest, R_t, is included in order to model the costs of holding stocks of imports.[12] The measure of domestic prices, p_{bt}, is a broad-based index including *e.g.* domestic transportation services. Thus, it captures the price of competing domestic goods as well as elements of costs to importers. The foreign price variable, p_{ft}, is a weighted average of the price of exports from 17 countries in Danish currency. Finally, the term D_t consists of a single impulse dummy, $D922_t$, which is included in order to correct for a change in the structure of taxation from employer's contributions to VAT in the second quarter of 1992. The change has primarily affected p_{bt}.

Figure 1 highlights a number of features of the Danish case. The nominal variables, p_{mt}, p_{ft}, and p_{bt} are clearly non-stationary, showing similar trends until the mid-eighties. Then, p_{mt} and p_{ft} tend to slow down significantly compared to the domestic price index. The relative prices, $p_{mt} - p_{ft}$ and $p_{mt} - p_{bt}$, also appear non-stationary but with a degree of persistence significantly lower than the price levels. The average quarter-to-quarter rate of inflation in annual terms defined as $\Delta p_t \equiv \frac{4}{3}(\Delta p_{mt} + \Delta p_{ft} + \Delta p_{bt})$ does not appear stationary yet it seems to have no significant deterministic trend. R_t has broadly similar features.

4.2 The Vector Autoregression

The unrestricted VAR with five lags is relied upon as a sufficiently general starting point for the empirical analysis. Likelihood ratio tests of sequentially excluding lags from the general model indicate that $k = 2$. A number of misspecification tests with respect to serial correlation, general heteroskedasticity, non-normality, and autoregressive conditional heteroskedasticity show the VAR(2) to be in accordance with the model assumptions.

Evidence on the dynamic properties of the unrestricted VAR(2) is provided by the eigenvalues of the companion matrix which are

$$(.947 \pm .043i, .781, .500, .288 \pm .368i, -.394, -.004). \tag{26}$$

The roots of the characteristic polynomial are the inverses of these eigenvalues. The roots remain outside the unit circle in the complex plane but two or perhaps three seem close to one. No roots appear to correspond to seasonal unit roots.

A formal test on the rank indices, r and s, and the associated number of unit roots is based on the joint rank statistic, $S_{r,s}$. It is reported in table 1 with the

[12] See Kongsted (1996) for an analysis that also included domestic unit labour costs.

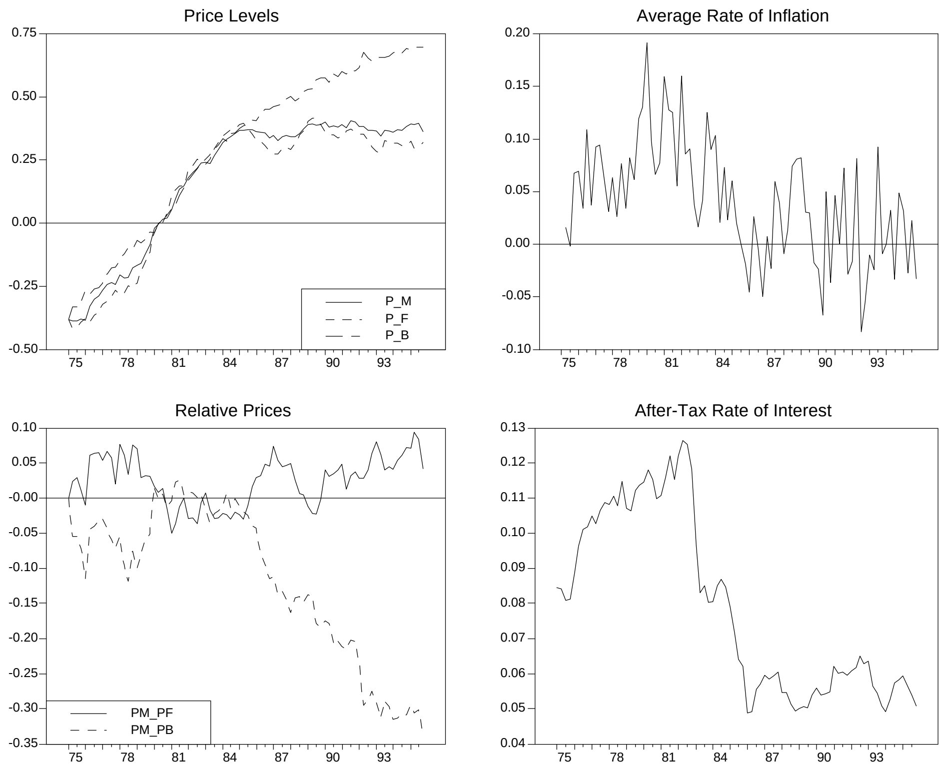

Figure 1: The Data (Logarithms of price variables, inflation in annual terms. To facilitate graphing, constants were added to the logarithms of the variables).

95 per cent quantiles of the asymptotic distribution.[13] A principle will be applied that a model is rejected only if all submodels are also rejected, see Rahbek *et. al.* (1998).

The models having $r = 0$ can be firmly rejected. A narrow non-rejection occurs for the case $(r, s) = (1, 2)$. This case would imply four unit roots in the characteristic polynomial, one polynomially cointegrating relation, and one stochastic I(2) trend. As the number of unit roots appears excessive in view of (26), the case will be considered a rejection although the p-value is slightly above 5 per cent.

The next non-rejected case is $(r, s) = (2, 1)$ which also has one I(2) trend but an additional polynomially cointegrating relation, corresponding to three unit

[13]The test relies on asymptotic distributions which may not provide very good approximations to the actual distributions for the present sample size. It is noted that the reported quantiles do not take into account the presence of the intervention dummy.

Table 1: A Formal Test of I(1) and I(2) Cointegrating Ranks

r	$S_{r,s}$				Q_r
0	343.5	238.8	161.9	122.8	101.0
	137.0	*113.0*	*92.2*	*75.3*	*62.6*
1		187.5	96.9	51.2	48.5
		86.7	*68.2*	*53.2*	*42.2*
2			75.1	27.6	24.5
			47.6	*34.4*	*25.5*
3				13.0	10.4
				19.9	*12.4*
$p-r-s$	4	3	2	1	0

Note: Italics are the 95 per cent quantiles from Rahbek, Kongsted, and Jørgensen (1998) for $S_{r,s}$ and from Johansen (1996, table 15.4) for Q_r.

roots in the process. It clearly cannot be rejected having a p-value in excess of 20 per cent. Imposing the rank deficiency of Π and $\alpha'_\perp \Gamma \beta_\perp$, the eigenvalues of the companion matrix are

$$(\ 1,\ 1,\ 1, .520, .277 \pm .310i, -.451, -.047). \tag{27}$$

The other eigenvalues remain close to their values in the unrestricted case (26).[14]

The empirical evidence of a single I(2) trend is unambiguous. The number of polynomially cointegrating relations is most likely two in view of the supporting evidence provided by the eigenvalues. The case $(r, s) = (2, 1)$ is therefore maintained in the following.

The validity of conditional analysis is considered next. Table 2 reports Paruolo and Rahbek's (1998) sequential test of weak exogeneity for the cointegration parameters. The table can be interpreted as applying an individual test size of 1.67 per cent in order to limit the size of the overall test to the usual level of 5 per cent.

The test rejects decisively that two of the variables, p_{mt} and p_{bt}, are weakly exogeneous already from examining the first sub-hypothesis, $e_i'\alpha = 0$ where e_i denotes a unit vector with one in the ith position. Rejection occurs even if the focus is narrowed and a less conservative level, *e.g.*, of 5 per cent is applied to

[14]The fact that the third unit root is related to an I(2) trend and not to the rank deficiency of Π can be seen from imposing $p - r = 3$ in which case the eigenvalues become $(1, 1, 1, .969, -.398, .283 \pm .220i, -.254)$, *i.e.*, a further near-unit root emerges.

Table 2: The Sequential Test of Weak Exogeneity for (β, β_1, δ)

Statistic	ν	p_{mt}	p_{ft}	p_{bt}	R_t
Q_{a1}	2	19.64[.000]	5.73[.057]	22.58[.000]	1.59[.452]
Q_{a2}	1	-	4.16[.041]	-	31.43[.000]
Q_{a3}	2	-	21.87[.000]	-	-

Note: Numbers in brackets are p-values according to $\chi^2(\nu)$.

Q_{a1}. For R_t the first hypothesis is accepted by a very broad margin whereas the second sub-hypothesis, $e_4'\alpha = 0$, is strongly rejected. The foreign price variable, p_{ft}, is found to satisfy the sub-hypotheses that $e_2'(\alpha, \alpha_1) = 0$ although with fairly small p-values. The final sub-hypothesis, $e_2'\zeta_1 = 0$, is strongly rejected in this case.

The overall conclusion is that no variable can be considered weakly exogenous for the cointegration parameters of the I(2) model. The evidence strongly suggests a lack of any level feedback to R_t. The results also indicate that the innovation to the foreign price variable, $\varepsilon_{p_{ft}}$, could be the source of the I(2) common trend although the p-values are much smaller in this case. This would be in accordance with a small open economy presumption, but of course any interpretation must take account of the fact that no restrictions have been imposed on the innovation correlations.

4.3 Testing Long-Run Price Proportionality

The present section examines the hypothesis of long-run proportionality between the price variables, *i.e.*, the hypothesis that the loadings of the common I(2) trend are proportional to $b = (1, 1, 1, 0)'$.

The unrestricted estimate of β_2 obtained from the two-step estimation procedure is

$$\beta_2 = (1, 1.122, 0.617, -0.002)'$$

when normalized on p_{mt}. The point estimate of the loading to domestic prices is relatively small whereas the remaining coefficients appear close to the theoretical values. Whether the deviations are significant is addressed next by the sequential test proposed in section 3.2.

Table 3: The Sequential Test of Long-Run Proportionality

Statistic	ν	α unrestricted	$e_4'\alpha = 0$
Q_{b1}	2	4.91[.086]	5.20[.074]
Q_{b2}	1	9.27[.002]	5.69[.017]

Note: Numbers in brackets are p-values according to $\chi^2(\nu)$.

The long-run proportionality hypothesis implies that all cointegrating relations defined either by β or β_1 can be expressed as linear combinations of the relative prices and the interest rate variable,

$$\text{sp}(\beta, \beta_1) = \text{sp}(B), \quad B' = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{28}$$

B is easily checked to be a valid choice of $b_\perp$ in (13).

Two cases are considered in which the first step of the estimation either has α unrestricted or is subject to the restriction $e_4'\alpha = 0$ for which firm support was found above. Table 3 collects the evidence on long-run proportionality for these cases. The critical values for Q_{b1} and Q_{b2} are 7.38 and 5.03, respectively, chosen as the 97.5 per cent quantiles in order for the size of the overall test to be limited to 5 per cent.

Without restrictions on α, the first part of the sequential test can be accepted but the overall hypothesis is thoroughly rejected due to Q_{b2} which has a p-value of .002. Imposing the restriction $e_4'\alpha = 0$ in the first step, the β restriction is again accepted and the β_1 test is now only slightly above the critical value. While it appears that the empirical evidence is critical to the long-run proportionality hypothesis, the rejection is due to the β_1 part which is not considered in most analyses. The difference between the cases having α unrestricted and $e_4'\alpha = 0$ indicates that the rejection is in part related to the interest rate equation.

For the purpose of gaining some insight into the nature of the polynomially cointegrating relations the theoretical presumption is tentatively maintained in the following subsection. The analysis of the transformed VAR will provide evidence to suggest that indeed the transformation is succesful.[15]

[15] On a comparison of methods for testing long-run proportionality in I(2) systems, see Kong-

4.4 A Long-Run Model of Import Pricing

The aim of the final part is to derive a fully identified long-run structure, including the polynomially cointegrating relations. The restriction of no levels feedback to R_t is imposed throughout.

Under the hypotheses of one common I(2) trend and long-run proportionality in prices, the polynomially cointegrating relations can be analyzed based on a transformed I(1) VAR as suggested in section 3.3. The vector of variables analyzed is

$$\tilde{X}_t = \begin{pmatrix} B'X_t \\ b'\Delta X_t \end{pmatrix} = (p_{mt} - p_{ft}, p_{mt} - p_{bt}, R_t, \Delta p_t)'. \tag{29}$$

The mean rate of change of prices, Δp_t, is measured in annual terms to make it comparable to R_t, as defined in section 4.1.[16] The eigenvalues of the companion matrix corresponding to the transformed VAR are $(1, 1, .546, -.376 \pm .132i, .339 \pm .195i, .207)$. It appears that the transformation has successfully accounted for all roots at or near unity which is contradicting the above rejection of long-run proportionality.

Interpreting the original data vector X_t as second-order non-stationary and polynomially cointegrating finds strong support in the transformed model. The hypothesis of Δp_t being stationary in which case p_t would only be I(1), can be strongly rejected with a likelihood ratio test statistic of 19.1 to be compared with the $\chi^2(3)$ distribution. Excluding Δp_t from the cointegration space of the transformed VAR which is equivalent to $\delta = 0$, is also firmly rejected with a test of 46.8 to be compared to the $\chi^2(2)$ distribution. Polynomial cointegration is a very significant feature of the data.

Part A of Table 4 shows an exactly identifying representation of the polynomially cointegrating relations in terms of pricing-to-market and inventory-type relations. Both relations allow for a linear trend as discussed in section 3.1. The first relation is identified as an inventory-type relation excluding the term that involves the domestic prices. Ignoring the linear trend and the stationary terms the relation can be rewritten as

$$p_{mt} - p_{ft} = 1.841(R_t - \Delta p_t) - 1.349\Delta p_t. \tag{30}$$

It relates the ratio of import prices over foreign prices to the real interest rate and

sted (1998).

[16] Alternative representations of the first-differenced term are valid if defined as $v'\Delta X_t$ in terms of a matrix v for which $|v'\beta_2| \neq 0$, see Rahbek *et. al.* (1998).

Table 4: An Inventory/Pricing-to-Market Structure on $\tilde{\beta}^*$

A. Exactly Identifying Structure

Relation	$p_{mt} - p_{ft}$	$p_{mt} - p_{bt}$	R_t	Δp_t	$t/100$
		φ'		$-\delta$	$\tilde{\beta}_0'$
Inventory	1	0	-1.841	3.190	.146
			(.826)	(.338)	(.074)
Pricing-to-Market	.730	.270	0	.050	.080
	(.060)	(.060)		(.097)	(.026)

B. Overidentifying Restrictions

Hypothesis	LR test	ν	p-value
H_1: $\delta_1 = 0$	6.81	1	.01
H_2: $\delta_2 = 0$	.01	1	.91
H_3: $\delta_1 = -\varphi_{31}$	3.36	1	.07

C. Overidentifying Structure

$\delta_1 = -\varphi_{31}$, $\delta_2 = 0$, $\tilde{\beta}_{01} = 0$. *LR test* $= 3.94$, $\nu = 3$, *p-value* $= .27$

Relation	$p_{mt} - p_{ft}$	$p_{mt} - p_{bt}$	R_t	Δp_t	$t/100$
		φ'		$-\delta$	$\tilde{\beta}_0'$
Inventory	1	0	-3.025	3.025	0
			(.311)	(.311)	
Pricing-to-Market	.726	.274	0	0	.075
	(.057)	(.057)			(.026)

Note: Numbers in parentheses are the conditional standard errors of the above coefficient.

the rate of inflation. The interest rate coefficient, although not very significant, has a sign which is consistent with the inventory interpretation. The negative inflation effect is consistent with (30) being a model of an importer's markup over the main element of costs, the foreign price, with an inflation cost relative to domestic substitutes.

The second relation is identified as a pricing-to-market relation by excluding the interest rate variable. Import prices are related to a weighted average of foreign and domestic prices and to an inflation term. Foreign price levels are seen to play a dominant role whereas the inflation term is insignificant.

On the basis of this representation some overidentifying restrictions on the coefficients are examined in part B. From the tests of hypotheses H_1 and H_2 it is clear that only the inventory relation has any polynomial cointegration. H_3 is a restriction across β and δ. It appears that the coefficients of R_t and Δp_t can be restricted to be equal with opposite signs, albeit with a fairly low p-value. The last term of the inventory relation (30) can in fact be excluded so that significant inflation effects occur only via the real rate of interest.

Part C of Table 4 reports an overidentifying structure. Hypotheses H_2 and H_3 have been imposed and the linear trend excluded from the inventory relation. As the likelihood ratio test is 3.94 with a p-value of .27 when compared to the $\chi^2(3)$ distribution the structure appears data consistent. It specifies a directly cointegrating, trend-stationary pricing-to-market relation among p_{mt}, p_{ft} and p_{bt}, and a polynomially cointegrating inventory relation between $p_{mt}-p_{ft}$ and $R_t-\Delta p_t$ with no trend.

The inventory/pricing-to-market structure and the tentatively maintained hypothesis of long-run price proportionality allow β_1 to be determined (up to a constant factor of proportionality). Using the fact that it is orthogonal to β_2 as well as to β and applying a normalization on p_{bt},

$$\beta_1 = \left(-(1-\gamma), -\gamma, 1, \frac{2\gamma-1}{\rho}\right)', \ \gamma = \frac{\theta+1}{2-\theta}.$$

The implied estimate $\beta_1 = (-.262, -.738, 1, .157)'$ is obtained upon inserting the estimates $\rho = 3.025$ and $\theta = .274$. The admissible range for θ under the pricing-to-market interpretation is $0 \leq \theta \leq 1$. The parameter γ depends positively on θ and the corresponding lower and upper bounds are .5 and 2. Under the inventory interpretation that $\rho > 0$, the coefficient of the interest rate variable, $(2\gamma-1)/\rho$, is non-negative and less than $3/\rho$.

In order to give an interpretation to this estimate, it is useful to note the role of β_1 in the I(2) model. First, β_1 is the remaining cointegrating vector defining a linear combination of the variables which takes the order of integration from 2 to 1 with no further cointegration to I(0). Specifically, the sum of the coefficients of the price variables is zero, thereby eliminating the common I(2) trend with identical loadings to each price variable. Second, under the set of rank indices established above there are two common trends, one second-order non-stationary and one first-order non-stationary trend. The latter is eliminated by the β-combinations whereas it remains in the combinations defined by β_1, see Johansen (1992).

The inventory/pricing-to-market structure is consistent with the following interpretation of the trends. A nominal second-order trend is driving the price levels with identical coefficients and reappears in the transformed model in first-differences as the only trend in Δp_t. It is shared with equal coefficients by the interest rate variable which in addition has the original first-order trend. The real rate of interest thus remains non-stationary. In order to allow cointegration in the inventory relation and the pricing-to-market relation, both relative prices are driven solely by the first-order trend. Being shared by the real rate of interest and two relative prices it seems natural to label it a real trend.

Other interpretations of the evidence are clearly possible. The fact that R_t is not very significant in the inventory relation under the scheme proposed in part A of Table 4 indicates that empirical identification is relatively weak. Excluding R_t from the unrestricted cointegrating space yields a likelihood ratio test of 4.38 with 2 degrees of freedom and a p-value of .11. Maintaining long-run price proportionality would then imply that $\mathrm{sp}(\beta_1) = \mathrm{sp}(0,0,0,1)'$, the unit vector picking out R_t. Under this scheme, the real rate of interest is still non-stationary whereas the nominal trend in Δp_t is solely responsible for any trends in relative prices.

While the second scheme can be seen as evidence of inflation cost/demand effects in import prices, it allows no role for a real trend in the determination of relative prices. In view of this implication, the inventory/pricing-to-market interpretation will be preferred although neither scheme can be ruled out empirically from the data. To conclude the analysis figure 2 presents the linear combinations that result from the partioning into the directions $(\beta, \beta_1, \beta_2)$ as estimated from the inventory/pricing-to-market structure: The component $b'X_t$ which embodies the I(2) trend,[17] the cointegrating relation $\beta_1'X_t$ which remains I(1) and in general contains both the first-differenced nominal trend and the real trend, and the deviations from the pricing-to-market and inventory relations.

[17] In general, $b'X_t$ also contains first-order non-stationary and stationary processes as well as deterministic components.

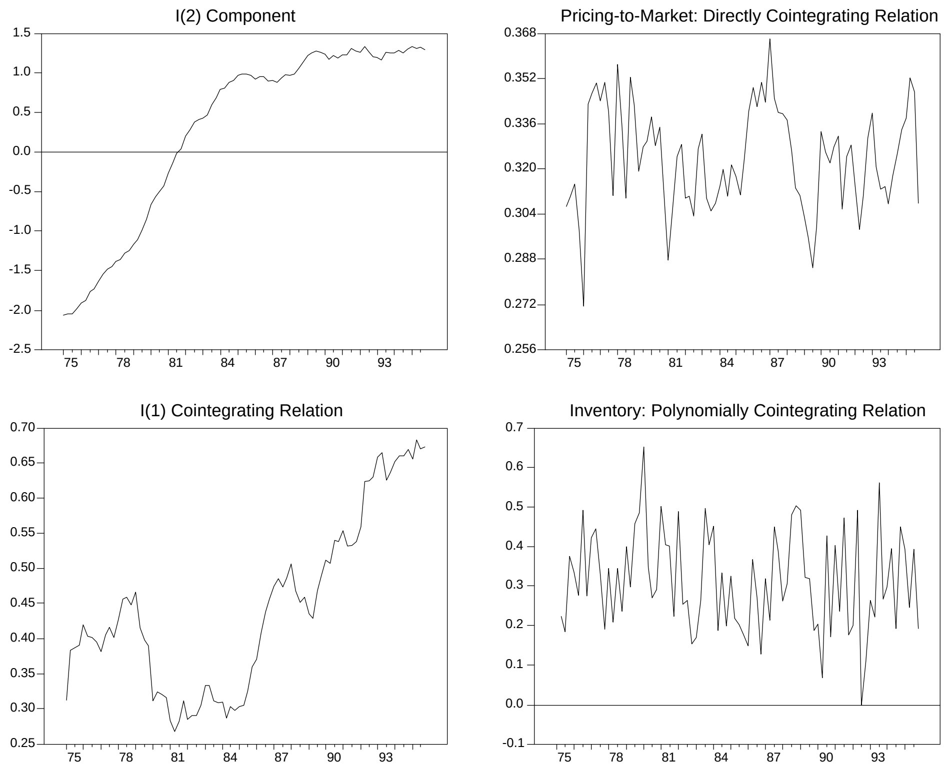

Figure 2: A Structural Partitioning Based on the Inventory/Pricing-to-Market Model.

5 Conclusions

This paper suggests a re-interpretation of long-run import price determination which allows for variables being integrated of order two. By the assumption of long-run price proportionality, domestic and foreign prices are driven by one common I(2) trend with identical coefficients. The small open economy assumption then suggests that this trend has foreign origins.

The empirical analysis of Danish data for the period 1975 to 1995 gives ample support to the I(2) interpretation. There is mixed evidence on the proportionality hypothesis. It is narrowly rejected by the sequential test proposed in this paper, but the transformed VAR which results from applying the theoretical transformation shows no signs of any remaining I(2) trend. The small open economy hypothesis that the price levels are driven by innovations to foreign prices is accepted albeit with a fairly low p-value.

A fully identified long-run structure is obtained. The preferred model is consistent with the inventory/pricing-to-market view of long-run import price determination, a nominal second-order stochastic trend driving domestic and foreign prices, and a real first-order stochastic trend being shared by the relative prices and the real rate of interest.

The estimate of the long-run pricing-to-market parameter, θ, is .274, which is somewhat lower than the estimates obtained for Norway and Australia but in line with results on Danish data for an earlier sample, see Kongsted (1996). The parameter estimate must be interpreted in the light of the measurements adopted for the price variables. In particular, it may partly reflect the domestic content of importers' costs.

Clearly, caution must be exercised in generalizing from evidence based on a single country and a relatively limited period of time. Future research will include applications of the framework and the techniques proposed to other countries to test their robustness.

Appendix

All variables have been derived from the database of Mona, the quarterly model of the Danish Central Bank, see Christensen and Knudsen (1992). Variables in Mona have been seasonally adjusted where applicable.

- p_{mt} is the log of the price of imported goods (excluding energy-related goods, ships, and aeroplanes) (Mona-variable `pmvx`).

- p_{ft} is the log of an import-weighted average of foreign export prices of goods (excluding energy-related goods, ships, and aeroplanes) from 17 countries, see Kongsted (1996) for details (constructed as `pmudl/efkrks`).

- p_{bt} is the log of the domestic deflator of value added in the private sector (excluding energy-related goods, housing, and agriculture) (Mona-variable `pyfbx`).

- R_t is the after-tax rate of interest (constructed as `(1-tax) ibz`).

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