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Do Prices Move Together in the Long Run? An I(2) Analysis of Six Price Indices

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Price convergence in the medium and long run. An I(2) analysis of six price indices. *

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Abstract

The paper discusses methodological questions related to econometric time-series modelling of $I(2)$ data. Long-run and medium-run relationships between two general price indices, the $US\ CPI$ and WPI and four commodity prices indices, the WBI, $CRBI$, $GSCI$, and ECI are investigated in a multivariate set-up. The statistical concepts of cointegration and polynomial cointegration are related to long-run and medium-run price homogeneity.

Keywords: Polynomially Cointegrated VAR, I(2) Analysis, Commodity Prices.

1. Introduction

There has recently been an increased interest in the econometric relationship between so called commodity price indices and general price indices, like the CPI and the WPI (See Baillie (1989), Kugler (1991), Trivedi (1995) Granger & Jeon (1996), and Gallo, Marcellino, & Trivedi (1997). The economic background for this interest is the assumption that commodity prices should react faster to inflationary signals, for instance originating from changes in money stock, and hence could act as forward indicators of general price inflation.

Econometrically, these studies differ a lot. Baillie (1989), Kugler (1991) and Granger and Jeon (1996) essentially investigate bivariate relationships. Trivedi

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(1995) and Gallo, Marcellino & Trivedi (1997) investigate multivariate relationships between four commodity price indices and two general price indices as indicators of US inflation. Some of the studies investigate relationships between inflation rates, others relationships between levels, and as in Granger and Jeon (1996) a relationship between a commodity price index in levels and a general price index in differences. The order of integration of the individual time series are discussed and tested using univariate test procedures. In general, the conclusion is that price levels are $I(1)$ or, possibly, $I(2)$. None of the papers test the order of integration (and cointegration) based on a multivariate analysis of all price variables allowing for $I(2)$ features.

The purpose of this paper is twofold. First, to investigate the multivariate relationships between levels, differences and acceleration rates of the six price indices analyzed in Trivedi (1995) and Gallo, Marcellino & Trivedi (1997) based on the polynomially cointegrated VAR model. Since the previously analyzed models are sub-models of this general model one should be able to evaluate some of the puzzling and (sometimes) inconsistent empirical results reported in the above studies. Second, to demonstrate the potential usefulness of the $I(2)$ model for analyzing price movements in the short, medium, and long run. The methodological approach is similar to Juselius (1994, 1995, 1998a) but they use the method to address different problems.

Because the $I(2)$ model has a complicated structure, empirical results are not easily accessible to the nonexpert reader. Therefore, I first give a detailed interpretation of the different components of the $I(2)$ model in terms of long-run and medium-run steady-state relationships, driving forces and short-run adjustment behaviour. I then argue, using the empirical results as illustrations, that an analysis based on sub-sets of the six price indices can lead to inconsistent results and misleading conclusions. For instance, I show that a minimum of three price indices are needed to produce cointegration and that bivariate analyses will, therefore, fail to find evidence on price convergence. I also show that long-run price homogeneity (between levels of prices) only exceptionally implies medium-run price homogeneity (between inflation rates). Hence, differencing the data to avoid the $I(2)$ analysis, i.e. analyzing the inflation rates instead of the prices, is likely to distort the analysis and produce puzzling and implausible results.

The organization of the paper is as follows: In Section 2 the $I(2)$ model is introduced as a parameter restriction on the general VAR model. The different levels of integration and their relationship are discussed and interpreted in terms of adjustment to dynamic steady-state relations and driving trends of first and second order. The concept of long-run and medium-run price homogeneity is formally defined and discussed using the $I(2)$ model structure. In Section 3 the empirical model is introduced and empirically checked for misspecification. The orders of integration and cointegration are tested within the multivariate model. As a sensitivity analysis I investigate in Section 4 how the choice of cointegration rank affects the classification of single price indices as $I(1)$ or $I(2)$, as excludable or non-excludable from the long-run relations, and as adjusting or non-adjusting to the long-run relations. In Section 5 the vector x_t is decomposed into the so called $(\beta, \beta_\perp)$ and $(\alpha, \alpha_\perp)$ directions and interpreted accordingly. Price homogeneity in the long and medium run is also analyzed in this section. Section 6 investigates cointegration relationships between commodity and general price indices and a fully specified overidentified model is estimated. Section 7 summarizes the basic findings.

2. The Statistical Model and its Interpretation

The $I(2)$ model is introduced in Section 2.1 as a parameter restriction on the unrestricted VAR model and its corresponding moving average representation is presented. In Section 2.2 two different ways to classify the components of the model are discussed which relate to the two-step and FIML estimation procedure. The interpretation of the components takes advantage of the dual decomposition into long-run cointegration relations and driving common trends. The long-run relations are classified as

(1) cointegrating relations from $I(2)$ to $I(1)$ i.e. $CI(2,1)$ relations,

(2) cointegrating relations from $I(2)$ to $I(0)$ i.e. $CI(2,2)$ relations, and finally

(3) polynomially cointegrating relations combining $CI(2,1)$ relations with relations between differences.

The driving trends are classified as common driving trends of first and second order. In Section 2.3 this decomposition is used to discuss long-run and medium-run price homogeneity and, hence, price convergence in the long and medium run.

2.1. The I(2) model

The VAR model for x_t, $(p \times 1)$, with a constant term, μ, $(p \times 1)$, and centered seasonal dummies, S_t, $(p \times 3)$, is given by:

$$\begin{gathered} \Delta^2 x_t = \Gamma_1 \Delta^2 x_{t-1} + \Gamma \Delta x_{t-1} + \Pi x_{t-2} + \Phi S_t + \mu + \varepsilon_t, \\ \varepsilon_t \sim N_p(0, \Sigma\ ),\ t = 1, ..., T \end{gathered} \tag{2.1}$$

The parameters $\{\Gamma_1, \Gamma, \Pi, \Phi, \mu, \Sigma\}$ are all unrestricted.

The hypothesis that x_t is $I(2)$ is formulated in Johansen (1991) as two reduced rank hypotheses: $\Pi = \alpha\beta'$ and $\alpha'_\perp \Gamma \beta_\perp = \zeta\eta'$, where α, β are $p \times r$ and ζ, η are $(p-r) \times s_1$ matrices. We need further to decompose $\alpha_\perp = \{\alpha_{\perp 1}, \alpha_{\perp 2}\}$ and

$\beta_\perp = \{\beta_{\perp 1}, \beta_{\perp 2}\}$, where $\alpha_{\perp 1} = \alpha_\perp(\alpha'_\perp\alpha_\perp)^{-1}\zeta$, $\alpha_{\perp 2} = \alpha_\perp\zeta_\perp$, $\beta_{\perp 1} = \beta_\perp(\beta'_\perp\beta_\perp)^{-1}\eta$, $\beta_{\perp 2} = \beta_\perp\eta_\perp$, and $\zeta_\perp, \eta_\perp$ are the orthogonal complements of ζ and η, respectively.

The moving average representation is given by:

$$\begin{aligned} x_t = {} & C_2 \sum_{s=1}^{t}\sum_{i=1}^{s}\varepsilon_i + C_2\tfrac{1}{2}\mu t^2 + C_2\Phi\sum_{s=1}^{t}\sum_{i=1}^{s}S_i + C_1\sum_{s=1}^{t}\varepsilon_s \\ & + C_1\Phi\sum_{s=1}^{t}S_s + (C_1 + \tfrac{1}{2}C_2)\mu t + Y_t + A + Bt, \;\; t = 1, ..., T \end{aligned} \tag{2.2}$$

where Y_t defines the stationary part of the process, A and B are a function of the initial values $x_0, x_{-1}, ..., x_{-k+1}$, and the coefficient matrices satisfy:

$$C_2 = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}\alpha'_{\perp 2}, \quad \beta' C_1 = -\overline{\alpha}'\Gamma C_2, \quad \beta'_{\perp 1}C_1 = -\overline{\alpha}'_{\perp 1}(I - \Psi C_2)$$

where $\Psi = \Gamma\overline{\beta}\overline{\alpha}'\Gamma + I - \Gamma_1$ and the shorthand notation $\overline{\alpha} = \alpha(\alpha'\alpha)^{-1}$ is used. See Johansen (1992, 1995).

From (2.2) it appears that an unrestricted constant in the model allows for linear and quadratic trends in the DGP. Johansen (1991) suggested the decomposition of the constant term μ into the $\alpha, \alpha_{\perp 1}, \alpha_{\perp 2}$ projections:

$$\mu = \alpha'\mu_0 + \alpha'_{\perp 1}\mu_1 + \alpha'_{\perp 2}\mu_2,$$

where

$\mu_0 = (\alpha'\alpha)^{-1}\alpha'\mu$ is related to the intercept of the stationary cointegration relations,

$\mu_1 = (\alpha'_{\perp 1}\alpha_{\perp 1})^{-1}\alpha'_{\perp 1}\mu$ is related to the slope coefficient of linear trends in the variables, and

$\mu_2 = (\alpha'_{\perp 2}\alpha_{\perp 2})^{-1}\alpha'_{\perp 2}\mu$ determines the slope coefficient of quadratic trends in the variables.

In the subsequent empirical analysis I will restrict $\alpha'_{\perp 2}\mu_2 = 0$, i.e. I will allow linear but no quadratic trends in the price levels. The motivation is that price inflation in general should be modelled as a nonzero mean process, but that linear trends in inflation are only reasonable as local phenomena and, therefore, should be modelled stochastically, not deterministically.

In the subsequent empirical analysis I sometimes need to distinguish between individual vectors and elements of a vector. The following notation will be used:

For a matrix β, $\beta_{.i}$ denotes the i'th column, $\beta_{.ij}$ the j'th element of the i'th column. The r stationary relations $\beta_{.} = \{\beta_{0.}, \beta_{1.}\}$, such that $\beta_{.i}$, $i = 1, ..., r$, indicate CI vectors in the $I(1)$ space, $\beta_{0.i}$, $i = 1, ..., r_0$, indicate directly stationary CI vectors, and $\beta_{1.i}$, $i = 1, ..., r_1$, indicate polynomially stationary CI vectors. The $p-r$ nonstationary relations $\beta_\perp = \{\beta_{\perp 1.}, \beta_{\perp 2.}\}$, such that $\beta_{\perp 1.i}$, $i = 1, ..., s_1$, indicate $CI(2,1)$ vectors that cannot be made stationary by polynomial cointegration, and $\beta_{\perp 2.i}, i = 1, ..., s_2$, indicate the $I(2)$ vectors that do not cointegrate at all.

Table 2.1: Decomposing the price vector using the I(2) model

	$x_t = [\beta, \beta_{\perp 1}, \beta_{\perp 2}]' x_t$		$x_t = [\beta_0, \beta_1, \beta_{\perp 1}, \beta_{\perp 2}]' x_t$
$r = 3$		$r_0 = 1$	$\beta'_{0.1} x_t \sim I(0)$
	$[\beta'_{.1} x_t \sim I(1), \quad \omega'_{.1} \Delta x_t \sim I(1)]$		
	$[\beta'_{.2} x_t \sim I(1), \quad \omega'_{.2} \Delta x_t \sim I(1)]$	$r_1 = 2$	$[\beta'_{1.1} x_t \sim I(1), \quad \kappa'_{1.1} \Delta x_t \sim I(1)]$
	$[\beta'_{.3} x_t \sim I(1), \quad \omega'_{.3} \Delta x_t \sim I(1)]$		$[\beta'_{1.2} x_t \sim I(1), \quad \kappa'_{1.2} \Delta x_t \sim I(1)]$
$s_1 = 1$	$\beta'_{\perp 1} x_t \sim I(1)$		
$s_2 = 2$	$\beta'_{\perp 2.1} x_t \sim I(2)$		
	$\beta'_{\perp 2.2} x_t \sim I(2)$		

Note: Components between [ ] are cointegrating CI(1,1).

A similar notation is used for α.

2.2. Interpretation and estimation

The $I(2)$ model has a rich but complicated structure. It is no easy task to give the intuition for the many relationships and the different levels of integration and cointegration such that they become more easy to interpret. To facilitate the interpretation of the subsequent empirical results within the $I(2)$ model, Table 2.1 decomposes the vector process x_t into the $I(0)$, $I(1)$ and $I(2)$ directions. The first part of the table (column 2) describes the classification of the process into r stationary polynomially cointegrating relations, $\beta' x_t + \omega' \Delta x_t$, and $p - r$ nonstationary relations, $\beta'_{\perp} x$. The nonstationary relations $\beta'_{\perp} x_t$ can further be divided into the s_1 first order nonstationary relations, $\beta'_{\perp 1} x_t$, which are the $CI(2,1)$ relations that cannot become stationary by polynomial cointegration, and the s_2 second order nonstationary relations, $\beta'_{\perp 2} x_t$, which are not cointegrating at all. The second part of the table (column 4) illustrates that for $r > s_2$ the r polynomially cointegrating relations can be further decomposed into $r_0 = r - s_2$ directly cointegrating relations, $\beta'_0 x_t$, and $r_1 = r - r_0 = s_2$ polynomially cointegrating relations, $\beta'_1 x_t + \kappa' \Delta x_t$.

For given values of $\{\beta_0, \beta_1, \beta_{\perp 1}, \beta_{\perp 2}\}$ one can derive $\{\alpha_0, \alpha_1, \alpha_{\perp 1}, \alpha_{\perp 2}\}$. There is an interesting duality between $\{\alpha_0, \alpha_1, \beta_{\perp 1}, \beta_{\perp 2}\}$ and $\{\beta_0, \beta_1, \alpha_{\perp 1}, \alpha_{\perp 2}\}$ in the sense that α_0 and α_1 determine the loadings (i.e. speed of adjustment) to the r_0 directly, $\beta'_0 x_t$, and r_1 polynomially cointegrated relations, $\beta'_1 x_t$, and $\beta_{\perp 1}$ and

$\beta_{\perp 2}$ determine the loadings to the first, $\alpha'_{\perp 1}\Sigma_{s=1}^{t}\varepsilon_s$, and second order stochastic trends, $\alpha'_{\perp 2}\Sigma_{s=1}^{t}\Sigma_{i=1}^{s}\varepsilon_i$.

It appears from Table 2.1 that both $\beta' x_t$ and $\beta'_{\perp 1}x_t$ are $CI(2,1)$ but that they differ in the following sense: The former can become stationary by polynomial cointegration, whereas the latter can only become stationary by differencing. The two-step estimation procedure in Johansen (1995) is based on the polynomial cointegration property of $\beta' x_t$, whereas the $FIML$ procedure in Johansen (1997) is based on the $CI(2,1)$ property of $\beta' x_t$ and $\beta'_{\perp 1}x_t$.

The first step of the two-step procedure is based on the $I(1)$ model and an estimate of β (and α) is obtained by solving the usual $I(1)$ eigenvalue problem, ignoring the reduced rank restriction on Γ. In the second step the estimate of $\beta_{\perp 1}$ is obtained by solving a second reduced rank problem based on the estimates $\{\hat{\beta}, \hat{\alpha}\}$. This is essentially done by deriving an equation which only involves differences by multiplying (2.1) for $\Pi = \hat{\alpha}\hat{\beta}'$ by $\hat{\alpha}'_{\perp}$. These equations can be used to find the $CI(2,1)$ directions $\beta'_{\perp 1}x_t$ by applying cointegration techniques. Johansen (1995) and Paruolo (1998) showed that the two-stage procedure gives asymptotically efficient ML estimates.

The $FIML$ estimates of $\{\beta, \beta_{\perp 1}\}$ are obtained using an iterative procedure that at each step delivers the solution of just one reduced rank problem. In this case the eigenvectors are the estimates of the $CI(2,1)$ relations among the $I(2)$ variables x_t, i.e. they give a decomposition of the vector x_t into the $p-s_2$ directions $(\beta, \beta_{\perp 1})$ in which the process is $I(1)$ and the s_2 directions $\beta_{\perp 2}$ in which it is $I(2)$.

Independently of the estimation procedure the crucial estimates are $\{\hat{\beta}, \hat{\beta}_{\perp 1}\}$, because for given values of these it is possible to derive the corresponding estimates of $\{\alpha, \alpha_{\perp 1}, \alpha_{\perp 2}, \beta_{\perp 2}\}$ and, if $r > s_2$, the further decomposition of $\beta = \{\beta_0, \beta_1\}$ and $\alpha = \{\alpha_0, \alpha_1\}$.

2.3. Long-run and medium-run price homogeneity

As discussed above, the $I(2)$ model can distinguish between the $CI(2,1)$ relations between levels $\{\beta' x_t, \beta'_{\perp 1}x_t\}$, the $CI(1,1)$ relations between levels and differences $\{\beta' x_{t-1} + \omega' \Delta x_t\}$, and finally the $CI(1,1)$ relations between differences $\{\beta'_{\perp 1}\Delta x_t\}$. When discussing the economic interpretation of these components there is a need to modify the generic concept of "long-run" steady-state relations accordingly. I will here use the concept of:

- a static long-run steady-state relation for $\beta'_0 x_t$,
- a dynamic long-run steady-state relation for $\{\beta'_1 x_t + \kappa' \Delta x_t\}$, and
- a medium-run steady-state relation for $\beta'_{\perp 1}\Delta x_t$.

In the subsequent empirical analysis the notion of price homogeneity plays an important role for the analysis of price convergence in the long run and the medium run. Both in the $I(1)$ and the $I(2)$ model long-run price homogeneity can be defined as zero sum restrictions on β. In the $I(2)$ model there is the additional possibility of medium-run price homogeneity defined as homogeneity between the inflation rates. To illustrate the interpretational difficulties in the latter case I follow the ideas in Johansen (1995) and rewrite the levels and difference components of model (2.1) as:

$$\begin{aligned}\Gamma\Delta x_{t-1} + \Pi x_{t-2} &= (\Gamma\overline{\beta})\beta'\Delta x_{t-1} + (\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 1} + \alpha_{\perp 1})\beta'_{\perp 1}\Delta x_{t-1} \\ &\quad + (\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 2})\beta'_{\perp 2}\Delta x_{t-1} + \alpha_0\beta'_0 x_{t-2} + \alpha_1\beta'_1 x_{t-2}\end{aligned} \tag{2.3}$$

The Γ matrix has been decomposed into three parts describing different effects from the lagged inflation rates and the Π matrix has been decomposed into two parts describing the effects from the stationary, $\beta'_0 x_{t-1}$, and the nonstationary, $\beta'_1 x_{t-1}$, cointegration relations. Note that the matrices in parentheses can be interpreted as adjustment coefficients. It is useful to study the order of integration of each component:

$$\begin{aligned}\beta'_{\perp 1}\Delta x_{t-1} &\sim I(0), \quad \beta'\Delta x_{t-1} \sim I(0), \quad \beta'_0 x_{t-2} \sim I(0), \\ \beta'_{\perp 2}\Delta x_{t-1} &\sim I(1), \quad \beta'_1 x_{t-2} \sim I(1)\end{aligned}$$

Because there are only two $I(1)$ components they have to be polynomially cointegrating, i.e. combine to $\alpha_1(\beta'_1 x_{t-2} + \kappa'\Delta x_{t-1})$, where $\alpha_1\kappa' = (\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 2})\beta'_{\perp 2}$.

I will now examine the conditions for medium-run price homogeneity under the assumption of long-run price homogeneity in β. If $R'\beta_i = 0$, $i = 1, 2, ..., r$, where $R' = [1, 1, ..., 1]$, then the first r.h.s component of (2.3), $(\Gamma\overline{\beta})\beta'\Delta x_{t-1}$, gives a homogeneous effect from lagged inflation rates. The interpretation is that prices are adjusting both to the equilibrium error between the prices, $\beta' x_{t-2}$, and to the change in the disequilibrium error, $\beta'\Delta x_{t-1}$.

Because $\beta' x_t$ is $I(1)$, a homogeneous adjustment of inflation rates is not sufficient for convergence to a stationary steady-state and a non-homogeneous adjustment has to take place. The latter is described by the third component, $(\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 2})\beta'_{\perp 2}\Delta x_{t-1}$. If $R'\beta = 0$, then in most cases $R'\beta_{\perp 2} \neq 0$ and the inflationary effect from the third component is non-homogeneous.

When $R'\beta_{\perp 1} = 0$, there is overall long-run homogeneity between prices and the second r.h.s. component corresponds to a homogeneous effect from lagged inflation rates. If $R'\beta_{\perp 1} \neq 0$, there exists inflation convergence in a non-homogeneous direction, in the sense of $\beta'_{\perp 1}\Delta x_{t-1}$ being a stationary cointegration relation between inflation rates. This case corresponds to a non-homogeneous effect from lagged inflation rates.

To conclude, the condition for overall long-run price homogeneity is $R'\beta = 0$ and $R'\beta_{\perp 1} = 0$. I will use the notion of a weak form for long-run price homogeneity when $R'\beta = 0$ and $R'\beta_{\perp 1} \neq 0$. Note, however, that medium-run price homogeneity is, in general, not possible, even if overall long-run price homogeneity holds. This is because $\beta_1' x_{t-1} \sim I(1)$ needs a non-homogeneous reaction in the inflation rates to achieve convergence to long-run steady state.

3. The empirical model

This section defines the variables of the VAR model and reports some multivariate and univariate residual misspecification tests. The cointegration rank, r, and $I(2)$ trends, s_2, is investigated based on the roots of the characteristic polynomial and the trace tests of the two-step procedure.

3.1. Checking the VAR model

Model (2.1) with three lags and $\alpha_{\perp 2}'\mu = 0$ is the baseline model. Hence, no quadratic trends are allowed in the data. The vector $x_t' = [p1, p2,\ p3,\ p4,\ p5,\ p6]$ is based on quarterly observations for t =1970:4-1993:4 where:

p1 = CPI, the US consumer price index
p2 = WBI, the World Bank commodity index
p3 = CRBI, the Commodity Research Bureau index
p4 = GSCI, the Goldman-Sachs index
p5 = ECI, the Economist commodity index
p6= WPI, the US wholesale price commodity index.

The price indices *p1* and *p6* measure general price movements, whereas *p2-p5* are so called commodity price indices. All variables are in logarithmic values. The definition of the special commodity price indices are given in Appendix A and the graphs of the levels and differences of all six variables are shown in Appendix B.

All estimates are based on the Johansen (1995) two-step procedure and have been calculated using a computer routine developed by C. Jørgensen within the package CATS for RATS (Hansen and Juselius, 1995).

The multivariate tests for residual normality, heteroscedasticity, and first and fourth order residual independence reported in Table 3.1 do not suggest misspecification. Since a single significant outcome can easily "drown" in the multivariate test, the univariate ARCH and Jarque-Bera normality tests are also included. The univariate tests give some indication of ARCH effects and non-normality for $p3$ and $p4$. Inspection of the residuals showed that these effects were mainly related to the turmoil of the break-down of the previous Bretton Woods system in 1973 and the first oil crisis at the end and beginning of 1974.

Table 3.1: Misspecification tests and characteristic roots

Multivariate tests:						
Residual autocorr.LM_1	$\chi^2(36)$	$=$	42.5	p-val.	0.21	
LM_4	$\chi^2(36)$	$=$	43.2	p-val.	0.19	
Normality: LM	$\chi^2(12)$	$=$	16.3	p-val.	0.18	
Univariate tests:	Δcpi	$\Delta crbi$	$\Delta gsci$	Δwbi	Δeci	Δwpi
$ARCH(3)$	2.0	5.6	12.8	7.2	4.4	1.8
$Jarq. - Bera(2)$	4.7	4.0	7.3	6.6	0.5	0.2
R^2	0.80	0.45	0.35	0.42	0.43	0.70
Eigenvalues of the Π-matrix:	0.37	0.29	0.21	0.14	0.06	0.03
The trace test	118.9	76.5	44.7	23.0	8.4	2.8
The asymp. 90 % quant.	89.4	64.7	43.8	26.7	13.3	2.7
6 largest roots of the process:						
Unrestricted model:	0.98	0.98	0.87	0.84	0.84	0.78
$r = 4$	1.00	1.00	0.91	0.91	0.79	0.79
$r = 3$	1.00	1.00	1.00	0.90	0.90	0.80
$r = 2$	1.00	1.00	1.00	1.00	0.87	0.87

As a sensitivity analysis I have re-estimated the model accounting for these effects. All basic results remained unchanged but the model without dummies produced more clear-cut results. Since these shocks were "true" price shocks I have preferred to treat them as "normal" shocks, albeit quite large and all subsequent results are based on the no-dummy model. There were no signs of parameter non-constancy in the model based on the recursive tests procedures reported in Hansen and Johansen (1993).

All subsequent empirical results will be based on the VAR model, which means that current correlations are left unmodelled. To give the reader an impression of the magnitude of these correlations the estimated residual correlation matrix is given below:

$$\hat{\Sigma} = \begin{bmatrix} 1.0 & & & & & \\ 0.3 & 1.0 & & & & \\ 0.2 & 0.7 & 1.0 & & & \\ 0.4 & 0.4 & 0.7 & 1.0 & & \\ 0.1 & 0.8 & 0.7 & 0.4 & 1.0 & \\ 0.6 & 0.4 & 0.6 & 0.7 & 0.4 & 1.0 \end{bmatrix}$$

3.2. Determining the two rank indices

In the $I(2)$ model the choice of cointegration rank r and the number of $I(2)$ components s_2 can be based on the trace tests. In the two-step procedure $r = \overline{r}$ is first determined and $s_1 = \overline{s}_1$ is found by solving another eigenvalue problem. Paruolo (1996) showed that the joint hypothesis (r, s_1) can be tested by combining the two test procedures. He also simulated the asymptotic distributions for the $I(2)$ model with different restrictions on the constant term.

There has recently been an increased interest in the small sample properties of the cointegration tests. For instance, Johansen (1998) derives Bartlett corrections that significantly improve the size of the tests on cointegration relations, but also demonstrates the low power of these tests. Jørgensen (1998) reports similar results based on a broad simulation study. She demonstrates the low power of the trace tests in $I(2)$ or near $I(2)$ models for samples sizes and adjustment coefficients similar to the present study.

Since the null of a unit root is not necessarily reasonable from an economic point of view, the low power is a serious problem. This is a strong argument for basing the choice of r and s_1 on economic theory as well as the statistical information in the data (Juselius, 1998a, 1998b). Economic theory suggests often a prior hypothesis for the number of independent trends, i.e. for $p-r$. For instance, based on the assumption of completely flexible prices, no market regulations or trade barriers, economic theory would suggest just one common stochastic price trend.

This assumption is clearly unrealistic for the price behavior in the investigated period. Therefore, the choice of r and s_1 will be empirically based in this study. I will, however, use all available information, economic as well as statistical, instead of relying exclusively on the trace tests. In particular, I will exploit the information provided by the roots of the characteristic polynomial of the VAR model when there are $I(2)$ or near $I(2)$ components in the data as described below.

The number of unit roots in the characteristic polynomial is $s_1 + 2s_2$, where s_1 and s_2 are the number of $I(1)$ and $I(2)$ components respectively. The intuition is that the additional s_2 unit roots belong to Δx_t, and, hence, to the Γ matrix in (2.1). Therefore, the roots of the characteristic polynomial contain information on unit roots associated with both Γ and Π, whereas the standard $I(1)$ trace test only contain information on unit roots in the Π matrix. If there are no $I(2)$ components the number of unit roots (or near unit roots) should be $p-r$, otherwise $p-r+s_2$. In table 3.1. I report the characteristic roots of the unrestricted VAR for $r = 2, 3, 4$. It appears that there remain two large roots in the model whatever value of r is chosen. This is strong evidence of two stochastic $I(2)$ trends.

The test statistics reported in Table 3.2 are based on the joint determination of (r, s_1) as described in Paruolo (1996) for the model with $\alpha'_{\perp 2}\mu = 0$. The 95%

Table 3.2: Testing the joint hypothesis Q(s_1,r)

p-r	r	$Q(s_1, r)$					
6	0	350.1	279.3	220.7	179.0	148.8	125.9
		240.4	*203.1*	*174.8*	*148.5*	*126.7*	*109.2*
5	1		240.4	180.5	139.8	109.7	82.6
			171.9	*142.5*	*117.6*	*98.0*	*81.9*
4	2			153.7	105.8	**66.7**	**51.2**
				116.3	*91.4*	*73.0*	*58.0*
3	3				96.7	**50.5**	**33.2**
					70.9	*51.4*	*38.8*
2	4					64.5	18.5
						36.1	*22.6*
1	5						13.9
							12.9
s_2		6	5	4	3	2	1

quantiles are given in italics. The test procedure starts with the most restricted model ($r = 0, s_1 = 0, s_2 = 6$) in the upper left hand corner, continues to the end of the first row, and proceeds similarly row-wise from left to right until the first acceptance. This procedure delivers a correct size asymptotically, but does not solve the problem of low power. Because economic theory suggests few rather than many common trends, a reversed order of testing might be preferable from an economic point of view.

It appears that ($r = 2, s_1 = 2, s_2 = 2$) is first accepted, but also that ($r = 3, s_1 = 1, s_2 = 2$) is equally acceptable. In terms of economic interpretation the two cases are quite different. In the first case $r_0 = r - s_2 = 2 - 2 = 0$ implies no stationary cointegration relation between price indices that satisfies long-run price homogeneity, whereas in the second case there exists one stationary homogeneous steady-state relation. Because of the low power and the lack of a strong economic prior the choice between the two cases is not straightforward. Nevertheless, the presence of three rather than four common trends seem more likely in a reasonably deregulated economy like the US. Moreover, the case ($r = 3, s_1 = 1, s_2 = 2$) allows for a much richer description of interrelations and dynamic interactions between the price indices and is, therefore, the preferred choice.

Because the choice of $r = 2$ or 3 leads to quite different decompositions of the data and the evidence from the test procedure was partly inconclusive, I report sensitivity analyses of the two choices of r in the next section.

4. Sensitivity analyses

To investigate the consequences of choosing $r = 2$ or 3 three different types of sensitivity tests are performed. In the $I(1)$ model the tests are called tests of stationarity, long-run exclusion, and weak exogeneity (Hansen and Juselius, 1995). In the $I(2)$ model both their formulation and interpretation change. The first two hypotheses now involve restrictions on $\beta_{\perp 1}$ besides β, and the weak exogeneity hypothesis involves additional restrictions on $\alpha_{\perp}$ and $\beta_{\perp}$ (Rahbek and Paruolo, 1998).

For reasons related to the estimation procedure discussed in Section 2.2., it is difficult within the two-stage procedure to derive LR tests for hypotheses involving restrictions on $\beta_{\perp 1}$ and $\alpha_{\perp 1}$. The $FIML$ procedure of Johansen (1997) provides a more unified framework for inference in the $I(2)$ model. Because of its novelty, well-tested computer programs are not yet available. Therefore, I apply the three test procedures exclusively to β and α and discuss how the interpretation has to be modified accordingly.

Section 4.1 investigates whether any of the price indices is empirically an $I(1)$ variable by testing whether any single price index corresponds to a unit vector in the cointegration space β. Section 4.2 investigates whether any of the price indices can be excluded from the β space, implying no long-run relationship with the remaining variables. Section 4.3 investigates absence of long-run levels feedback on any of the price indices.

4.1. Are any of the prices I(1)?

Univariate test procedures are usually adopted when testing for the order of integration of each of the variables. Here I will investigate the order of integration using the more complete information of the multivariate model.

The test whether a variable is at most $I(1)$ can be formulated as $\{\beta, \beta_{\perp 1}\} = \{b, \psi_1\}$, where b is a unit vector and ψ_1 is a $p \times (r + s_1 - 1)$ vector of unrestricted coefficients. Accepting H_0 implies that the variable in question is $I(1)$, or alternatively $I(0)$ if b is in $sp(\beta_0)$. Since tests involving restrictions on $\beta_{\perp 1}$ cannot be performed within the two-stage procedure I test instead $\beta = (b, \psi)$ where ψ is a $p \times (r - 1)$ vector of unrestricted coefficients. If the $I(1)$ hypothesis is accepted in $sp(\beta)$ it would equally be accepted in $sp(\beta, \beta_{\perp 1})$, but if the hypothesis is rejected in $sp(\beta)$ it might nevertheless be accepted in $sp(\beta, \beta_{\perp 1})$.

The test statistics are reported in Table 4.1. To improve readability, insignificant values have been indicated in bold face and the preferred case $r = 3$ in italics. The smaller the value of r, the more "conservative" the test, the larger r the more "permissive" the test. For $r = 3$ the hypothesis is rejected for all

Table 4.1: Stochastic properties of the data

r	ν	$\chi^2(\nu)$	*p1*	*p2*	*p3*	*p4*	*p5*	*p6*
					Tests of $I(1)$:ness			
2	4	9.49	21.70	11.01	17.09	16.05	16.80	18.70
3	3	*7.81*	*14.50*	*8.22*	*9.89*	**7.08**	*11.53*	*12.39*
					Tests of long-run exclusion			
2	2	5.99	8.33	**4.56**	8.59	16.10	**5.89**	13.66
3	3	*7.81*	*15.05*	*11.10*	*9.10*	*20.68*	*8.61*	*20.14*
					Tests of zero rows in α			
2	2	5.99	**1.69**	6.39	**0.50**	**3.25**	**5.89**	7.44
3	3	*7.81*	**1.86**	*9.54*	**0.72**	**7.28**	**6.88**	*12.28*

variables, possibly with the exception of *p4* for which there is weak evidence of acceptance. For $r = 2$ there is strong rejection of the unit vector hypothesis.

I conclude that all price indices are best approximated as $I(2)$ variables.

4.2. Long-run exclusion

Here I ask whether some of the price indices can be excluded from the long-run analysis. For instance, if any of the $I(2)$ price indices is long-run unrelated with the remaining indices, then one of the two $I(2)$ trends would be exclusively related to that price index. For instance if p_1 can be long-run excluded, then:

$$\beta' = \begin{bmatrix} 0,0,0 \\ *,*,* \\ *,*,* \\ *,*,* \\ *,*,* \\ *,*,* \end{bmatrix}, \beta'_{\perp 1} = \begin{bmatrix} 0 \\ * \\ * \\ * \\ * \\ * \end{bmatrix} \Longrightarrow \beta'_{\perp 2} = \begin{bmatrix} *,0 \\ 0,* \\ 0,* \\ 0,* \\ 0,* \\ 0,* \end{bmatrix}.$$

In this case the exclusion of p_1 would reduce the number of stochastic $I(2)$ trends and hence simplify the analysis considerably. At the same time the tests can be used as a check on the empirical adequacy of previously selected sets of price indices.

The hypothesis of long-run exclusion can be expressed as a zero row in $\{\beta, \beta_{\perp 1}\}$ or, equivalently, a unit vector in $\beta_{\perp 2}$. This hypothesis involves restrictions on both β and $\beta_{\perp 1}$, alternatively $\beta_{\perp 2}$ and cannot be tested with the two-step procedure.

The modified hypothesis $\beta = H\phi$, alternatively $H_{\perp}'\beta = 0,$ where $H_{\perp}$ is a unit vector leaves $\beta_{\perp 1}$ unrestricted. If long-run exclusion is rejected in β it would also be rejected in $\{\beta, \beta_{\perp 1}\}$, but if long-run exclusion is accepted in β, it might, however, be rejected when tested on $\beta_{\perp 1}$.

The LR test statistic is approximately distributed as $\chi^2(r)$. For $r = 3$ all test statistics are significant on the 5 % level, and none of the variables can be excluded from $sp(\beta)$. For $r = 2$ there is weak evidence of long-run exclusion of $p2\,(WBI)$ and $p5\,(ECI)$.

We conclude that there is no convincing evidence that any of the price indices are unrelated with the other indices in the long run. Hence, reducing the set of variables under study is likely to leave out important information on price convergence in the long run.

4.3. Hypotheses on the adjustment coefficients.

The hypothesis of zero restrictions on a row of α is usually interpreted in terms of weak exogeneity of the corresponding variable w.r.t. the long-run parameters of interest β. As discussed in Paruolo and Rahbek (1998) this is no longer the case in the $I(2)$ model. The test of weak exogeneity involves complicated restrictions on the parameters $\{\alpha_{\perp 1}, \beta_{\perp 1}\}$. Since I do not intend to make inference in a partially specified model, weak exogeneity *per se* is not important. It is, however, of interest to test hypotheses on the strength of adjustment of each price index to the long-run relations, $\beta' x_t$. This can be investigated within the two-step procedure.

The test is of the form $R'\alpha = 0$, where R is a unit vector. The LR test statistics, approximately distributed as $\chi^2(r)$, are reported in the lower part of Table 4.1. Independently of the choice of r there is clear evidence of no adjustment for $p1$ and $p3$ and of weak adjustment for $p4$ and $p5$. Because the single hypotheses tests are not independent I also test the joint hypothesis of no adjustment for different sets of prices indices. Since no more than r of the variables can be jointly non-adjusting, a maximum of three indices are jointly tested. The hypothesis of no adjustment for $p1$ and $p3$ based on $r = 3$ was strongly accepted with a test statistic of 2.82, approximately distributed as $\chi^2(6)$. Adding any of the other price indices significantly increased the test statistic.

Altogether I conclude that $p1(CPI)$ and $p3(CRBI)$ are not adjusting to the long-run relations.

5. The components of the I(2) model

Based on the rank tests in Section 3 and the sensitivity analyses in Section 4, I continue the analyses with the preferred case $(r = 3, s_1 = 2)$ and the estimates

Table 5.1: Decomposing the process into the I(0), I(1), and I(2) directions

	$\hat{\beta}_0$	$\hat{\beta}_{1.1}$	$\hat{\kappa}_1$	$\hat{\beta}_{1.2}$	$\hat{\kappa}_2$	$\hat{\beta}_{\perp 1.1}$	$\hat{\beta}_{\perp 2.1}$	$\hat{\beta}_{\perp 2.2}$
p1 (CPI)	-0.81	0.03	-5.10	-0.55	1.84	-3.10	-0.14	-3.87
p2 (WBI)	-0.51	-0.16	1.90	0.05	-1.94	1.41	1.29	4.08
p3 (CRBI)	0.19	1.00	0.36	-0.17	-1.11	0.35	0.98	2.34
p4 (GSCI)	-0.04	-0.55	2.80	-0.24	-1.85	-4.27	0.91	3.89
p5 (ECI)	0.10	-0.56	-4.13	-0.13	0.71	3.16	0.65	-1.50
p6 (WPI)	1.00	0.26	-2.69	1.00	0.56	-2.35	0.33	-1.18
$\Sigma coef.$	0.07	0.01	-6.86	-0.04	-1.79	4.80		
	$\alpha_{0.1}$	$\alpha_{1.0}$		$\alpha_{1.1}$		$\hat{\alpha}_{\perp 1.1}$	$\hat{\alpha}_{\perp 2.1}$	$\hat{\alpha}_{\perp 2.2}$
p1 (CPI)	-0.011	-0.004		-0.013		-0.02	-0.02	**0.12**
p2 (WBI)	**0.441**	0.213		-0.107		**-0.06**	0.02	0.00
p3 (CRBI)	-0.071	0.046		0.111		**-0.07**	**-0.11**	-0.03
p4 (GSCI)	-0.262	**0.491**		**0.235**		-0.03	**0.05**	0.01
p5 (ECI)	0.083	**0.306**		0.052		**0.11**	**-0.07**	-0.01
p6 (WPI)	**-0.052**	0.022		**-0.090**		-0.04	**-0.07**	-0.04

reported in Table 5.1 are for this case. In Section 5.1 I interpret the estimates of β and α, and in Section 5.2 of $\beta_\perp$ and $\alpha_\perp$. Finally, in Section 5.3 I investigate long-run price homogeneity in β, and medium-run homogeneity in Γ.

5.1. Interpreting β and α

The stationary component $\beta_0' x_t$ seems to describe a homogeneous steady-state relation between *p1* and *p6* with some additional effects from *p2* and *p3.* The two polynomially cointegrating relations $\beta_{1.i}' x_{t-1} + \kappa_i' \Delta x_t$, $i = 1, 2$ are not uniquely determined in terms of stationarity in the sense that the transformation $\alpha_1 M M^{-1\prime}(\beta_1', \kappa')$ where M is a non-singular 2×2 matrix, leaves the likelihood function unchanged. However, choosing M such that $\beta_{1.11} = 0$, $\beta_{1.22} = 0$ and a normalization at $\beta_{1.13}$ and $\beta_{1.26}$ hardly changes the estimates at all. Therefore, the above estimates can be given an economic interpretation under this identifying assumption.

The first dynamic steady-state relation can now be described as a relation between all price levels excluding *p1* (CPI) and all six price differences, whereas the second is between all price levels excluding $p2\,(WBI)$ and all six price differences. The results suggest long-run price homogeneity for $\beta' x_t$, but probably not for $\beta_\perp' x_t$. Consistent with the results of Section 2.3 the κ coefficients do not sum to zero.

All three cointegration relations are significant in some of the price equations, which provides additional support for the choice of $r = 3$. The strength of the adjustment of each variable on itself in each equation can be inferred from the product coefficients $\alpha_{k.ij}\beta_{k.ij}$, $k = 0$, $i = 1,...,6$, $j = 1$ and $k = 1, i = 1,...,6$, $j = 1,2$ reported below. Significant adjustment coefficients are given in bold face. For the directly cointegrating relation the result is:

$$\alpha_{0.i1}\beta_{0.i1} = [0.008, -\mathbf{0.225}, -0.0133, 0.0105, 0.008, -\mathbf{0.052}]$$

The results indicate that only $p2$ and $p6$ adjust significantly to this relation. For the polynomially cointegrating relations the $\alpha_{1.k}$ coefficients relate both to the levels $(\beta_{1.k})$ and the differences $(\omega_{1.k})$ of the process. Since the adjustment to the levels seems more important we focus on the former effect. For the first and second relation the result is:

$$\begin{aligned} \alpha_{1.i1}\beta_{1.i1} &= [-0.000, -0.034, 0.046, -\mathbf{0.270}, -\mathbf{0.171}, 0.005] \\ \alpha_{1.i2}\beta_{1.i2} &= [0.007, -0.005, -0.019, -\mathbf{0.056}, -0.006, -\mathbf{0.090}] \end{aligned}$$

which suggests that $p4$ and $p5$ adjusts significantly to the first relation, whereas $p6$ and $p4$ adjust to the second relation. Consistent with the results of Section 4.3, there is no sign of significant adjustment to any of the three relations in $p1\,(CPI)$ and $p3\,(CRBI)$, suggesting that shocks to these two price indices act as the main driving forces within this data set.

5.2. Interpreting $\alpha_{\perp}$ and $\beta_{\perp}$

The individual vectors $\{\alpha_{\perp 1.i}, \alpha_{\perp 2.j}\}$ and $\{\beta_{\perp 1.i}, \beta_{\perp 2.j}\}$, $i = 1,...,s_{1,}\, j = 1,...,s_2$, are often difficult to interpret unless identifying restrictions are imposed. In the present case $\alpha_{\perp 2}$ and $\beta_{\perp 2}$ are of dimension $p \times 2$, but since the unrestricted estimates were both interesting and perfectly interpretable I saw no need to rotate the vector space. From (2.2) it appears that $\alpha_{\perp 2}$ determines the second order stochastic trends, $\alpha'_{\perp 2}\Sigma_{s=1}^{t}\Sigma_{i=1}^{s}\varepsilon_i$, and $\beta_{\perp 2}$ the loadings of the trends in each variable, with the qualification that the weight matrix $(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$ can be referred to either the common trends or the loadings. The estimates in table 5.1 are based on $\widetilde{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$ and $a_{\perp 2} = \alpha_{\perp 2}$.

The estimates of the two $I(2)$ trends, $\hat{\alpha}'_{\perp 2.j}\Sigma\Sigma\hat{\varepsilon}_i$, $j = 1,2$, where $\hat{\varepsilon}_i$ is the vector of estimated residuals from (2.1), suggest that the first $I(2)$ trend derives from the twice cumulated disturbances of $p3$, $p4$, $p5$, and $p6$, i.e. primarily to the commodity prices, whereas the second $I(2)$ trend is almost completely determined by the twice cumulated disturbances of $p1$. The corresponding estimate of $\beta_{\perp 2}$ indicates that all price indices are influenced by the second $I(2)$ trend (the CPI

trend), and that all price indices, except the CPI are influenced by the first $I(2)$ trend. This is strong evidence of the dominant role of the CPI index. Moreover, judging from the magnitude of the estimated coefficients, $\beta_{\perp 2}$ does not seem to have any zero row, implying that all price indices are $I(2)$, consistent with the test results in Section 4.2.

The estimate of $\alpha'_{\perp 1}\Sigma_{s=1}^{t}\varepsilon_s$ determines the "autonomous" stochastic $I(1)$ trend and suggests that it is essentially a weighted average of the cumulated disturbances of the commodity indices. The estimate of $\beta_{\perp 1}$ determines the $CI(2,1)$ relation that cannot be made stationarity by cointegration between prices and inflation rates, only by cointegration between inflation, i.e $\beta'_{\perp 1}\Delta x_t \sim I(0)$. The estimate suggests the existence of a non-homogeneous medium-run steady-state relation between commodity price inflation and general price inflation. Econometrically, it is an interesting result because it demonstrates the danger of differencing the price variables to get rid of the $I(2)$ problem. One will usually fail to find a homogeneous relation between inflations rate, though such a relation is strongly present between the price levels.

A tentative economic interpretation of $\beta'_{\perp 1}\Delta x_t \sim I(0)$ is that in the medium run a change in a commodity price, say, results in other prices changing, but not necessarily such that a sustainable long-run relationship is achieved. One could, for instance, think of $\beta'_{\perp 1}x_t$ as a "rational expectations" medium-run steady-state relation, as opposed to a "neoclassical" long-run steady-state relation $\beta' x_t$. The former can be interpreted as a steady-state relation for a fixed institutional set-up, and the latter as a long-run sustainable steady-state relation consistent with structural changes if institutions.

5.3. Long-Run Price Homogeneity.

It appears from table 5.1 that $\Sigma_j\beta_{ij} \approx 0$, $i = 1, 2, 3$, suggesting long-run price homogeneity in the r cointegrating relations $\beta' x_t$. The test of the hypothesis $R'\beta = 0$, was 1.86. It is approximately distributed as $\chi^2(3)$ and long-run homogenenity between the price indices in $sp(\beta)$ is strongly supported by the data. Hence, the weak form of long-run price homogeneity seems satisfied.

The hypothesis of overall long-run homogeneity involves restrictions on $\beta_{\perp 1}$ as well, i.e. $(\beta, \beta_{\perp 1}) = H\psi$ where ψ is $(p-1) \times (r + s_1)$. This hypothesis cannot be formally tested within the two-step procedure. But, as already discussed, the requirement $\Sigma_j\beta_{\perp 1.j} = 0$ does not seem to be satisfied and I conclude that weak, but not overall, long-run price homogeneity is present in the data.

As discussed in Section 2.4. empirical support for long-run price homogeneity in levels by no means implies medium-run price homogeneity in differences. Only in the direction of a homogeneous static long-run relation is medium-run price homogeneity likely to be found. Since it is of considerable economic interest to understand price movements both in the long and the medium run, I will further examine this question in Section 6.2.

6. Which price indices are related in the long-run?

To investigate the relationship between individual price indices I have proceeded in the following way. I first ask whether the two general price indices cointegrate, and if not, which other variable(s) have to be added. I then investigate cointegration properties between combinations of the special indices. Finally, I report the estimates and the joint test for three (overidentified) relations; one between the *CPI, WPI,* and *WBI,* another between the *CRBI, WBI,* and *WPI,* and finally one between the four commodity indices.

6.1. Partial cointegration tests

All hypotheses reported in Table 6.1 test whether a single restricted relation is in $sp(\beta)$ and are of the form $\beta = \{H\phi, \psi\}$, where H is a $p \times m$ design matrix imposing $p - m$ restrictions on one of the relations, ϕ is a $m \times 1$ vector of free parameters, and ψ is a $(p-1) \times r$ matrix of unrestricted coefficient. For derivation of the test procedures, see Johansen and Juselius (1992). I have grouped the hypotheses such that $\mathcal{H}_1$–$\mathcal{H}_5$ test hypotheses about the general price indices CPI and WPI relative to the commodity price indices, $\mathcal{H}_6$–$\mathcal{H}_{13}$ test hypotheses about the special commodity indices, and $\mathcal{H}_{14}$–$\mathcal{H}_{20}$ test hypotheses about the special commodity indices relative to one of the general price indices.

In the first group I find that CPI and WPI do not cointegrate by themselves, but that a combination with either WBI or $CRBI$ is strongly cointegrating. In the second group I find that none of the special commodity prices indices cointegrate bivariately, but that the combination of WBI, $CRBI$, and ECI is strongly cointegrating. The result that there is no bivariate cointegration is consistent with the results in Table 5.1, where the two $I(2)$ trends seemed to influence all price variables. In the third group I find that WBI and $CRBI$ are strongly cointegrating with either CPI or WPI.

6.2. A complete specification

The joint hypothesis $\{\mathcal{H}_2, \mathcal{H}_{12}, \mathcal{H}_{14}\}$ was first tested but was strongly rejected based on a test statistic of 23.4 distributed as $\chi^2(6)$. For the derivation of the test procedure see Johansen and Juselius (1994). The main reason for rejection was that $\{\mathcal{H}_2, \mathcal{H}_{12}, \mathcal{H}_{14}\}$ long-run excludes $p4$, i.e. the $GSCI$ index. Adding $p4$ to $\mathcal{H}_{14}$ resulted in a test statistics of 1.73, distributed as $\chi^2(5)$, and, hence,

Table 6.1: Cointegration Properties

	p1	*p2*	p3	*p4*	*p5*	*p6*	$\chi^2(\nu)$	*p.val.*
General versus commodity price indices								
$\mathcal{H}_1$	1	0	0	0	0	-1	15.16(3)	0.00
$\mathcal{H}_2$	*-0.68*	*-0.32*	*0*	*0*	*0*	*1*	*1.34(2)*	*0.51*
$\mathcal{H}_3$	*-0.62*	*0*	*-0.38*	*0*	*0*	*1*	*1.32(2)*	*0.52*
$\mathcal{H}_4$	-0.73	0	0	-0.27	0	1	4.97(2)	0.08
$\mathcal{H}_5$	1	0	0	0	-0.88	-0.12	9.77(2)	0.01
Commodity price indices								
$\mathcal{H}_6$	0	1	-1	0	0	0	6.24(3)	0.10
$\mathcal{H}_7$	0	1	0	-1	0	0	6.00(3)	0.11
$\mathcal{H}_8$	0	1	0	0	-1	0	14.52(3)	0.00
$\mathcal{H}_9$	0	0	1	-1	0	0	12.04(3)	0.01
$\mathcal{H}_{10}$	0	0	0	1	-1	0	14.51(3)	0.00
$\mathcal{H}_{11}$	0	1	-0.65	-0.35	0	0	3.62(2)	0.16
$\mathcal{H}_{12}$	*0*	*-0.78*	*1*	*0*	*-0.22*	*0*	*1.71(2)*	*0.43*
$\mathcal{H}_{13}$	0	0	1	-0.61	-0.39	0	5.66(2)	0.06
Commodity versus general price indices								
$\mathcal{H}_{14}$	*0*	*-0.68*	*1*	*0*	*0*	*-0.32*	*0.03(2)*	*0.98*
$\mathcal{H}_{15}$	0	0	1	-0.67	0	-0.33	11.01(2)	0.00
$\mathcal{H}_{16}$	0	0	-0.05	0	-0.95	1	12.08(2)	0.00
$\mathcal{H}_{17}$	0	0	0	-0.28	-0.72	1	10.83(2)	0.00
$\mathcal{H}_{18}$	0	1	0	-0.89	0	-0.11	5.83(2)	0.05
$\mathcal{H}_{19}$	0	-0.19	0	0	-0.81	1	12.03(2)	0.00
$\mathcal{H}_{20}$	*-0.33*	*-0.77*	*1*	*0*	*0*	*0*	*0.01(2)*	*0.99*

the modified joint hypothesis is clearly acceptable. The estimates are reported in Table 6.2 together with the adjustment coefficients. Adjustment coefficients being significant with a p-value > 0.05 are indicated in bold face and with a p-value of approximately 0.10 in italics.

The first relation is very similar to the directly stationary cointegrating relation $\beta_0'x$ in Table 5.1 and $p6$ (WPI) is significantly adjusting to it as well as $p2$ (WBI), albeit less significantly. The next relation is between $p3$ $(CRBI)$, $p2$ (WBI) and $p6$ (WPI), and there is significant adjustment in $p6$ (WPI), and $p4$ $(GSCI)$, and a less significant adjustment in $p5$ (ECI). The last relation ties all the special commodity price indices together, and shows significant adjustment in $p4\,(GSCI)$ and $p5$ (ECI), and weak adjustment in $p2$ (WBI).

Note, however, that the three $\beta'x$ relations are strictly speaking $I(1)$ (though the first one is probably $I(0)$). A test of joint restrictions on the levels and the

Table 6.2: A complete specification of the cointegration relations

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
p1	-0.71	0	0	-0.02	0.01	-0.01
p2	-0.29	-0.67	-0.39	*0.30*	0.02	*0.32*
p3	0	1.00	1.00	-0.08	-0.13	0.14
p4	0	0	-0.33	-0.19	**-0.60**	**1.02**
p5	0	0	-0.28	0.09	*-0.25*	**0.57**
p6	1.0	-0.33	0	**-0.12**	**0.05**	-0.03

Table 6.3: The estimates of Γ and Π

Var.	$\Delta p1$	$\Delta p2$	$\Delta p3$	$\Delta p4$	$\Delta p5$	$\Delta p6$	
		The Γ *matrix*					*sum*
$\Delta p1$	-0.01	-0.07	0.08	-0.04	0.05	-0.13	-0.12
$\Delta p2$	**1.28**	**-0.99**	0.07	-0.54	**0.93**	**0.70**	**1.45**
$\Delta p3$	**-2.94**	-0.42	-0.54	-0.62	0.62	**3.72**	-0.18
$\Delta p4$	0.05	**-0.96**	**0.83**	**-2.12**	**1.07**	**4.78**	4.01
$\Delta p5$	**2.15**	-0.30	0.09	-0.56	0.16	**1.49**	3.03
$\Delta p6$	**0.84**	-0.19	0.17	-0.14	0.20	**-0.89**	0.01
	p1	*p2*	*p3*	*p4*	*p5*	*p6*	
		The Π *matrix*					*sum*
p1	0.02	0.00	-0.00	0.00	0.00	-0.02	0.0
p2	*-0.22*	**-0.22**	**0.33**	*-0.10*	*-0.09*	*0.30*	0.0
p3	0.05	0.06	0.01	-0.05	-0.04	-0.03	0.0
p4	0.14	0.06	**0.42**	**-0.33**	**-0.29**	0.00	0.0
p5	-0.07	-0.08	**0.32**	**-0.18**	**-0.16**	0.17	0.0
p6	**0.08**	0.01	0.03	0.01	0.01	**-0.14**	0.0

differences is not yet available. Instead, I report the estimates of the levels matrix $\Pi = \alpha'\beta$ and the differences matrix Γ in Table 6.3. The estimates of Π are based on α and β in Table 6.2. and Γ is estimated under the reduced rank restriction $\alpha_{\perp}'\Gamma\beta_{\perp} = \zeta\eta'$.

The Γ matrix shows that $p1\,(CPI)$ does not seem to react on any relation between the lagged differences, hence confirming the role of CPI as the main driving force in this system, whereas $p3\,(CRBI)$ seems to react quite strongly to changes in the CPI and the WPI. Medium-run price homogeneity seems only to be present for $p3$ and $p6$ (and $p1$). Because $p3$ does not adjust to any of the long-run relations $\beta' x_t$ and $p6$ primarily adjusts to the static steady-state relation $\beta_0' x_t$ this is consistent with the results of Section 2.3. Altogether, the price adjustment seems to take place primarily in $p2$, $p4$, and $p5$, while $p1$ seems to be pushing all the other indices.

An interesting result is that the major part of the medium-run effects seems to derive from changes in the CPI and the WPI. There are essentially no effects from changes in the commodity indices on the general price indices.

7. Summary of results

Based on the cointegrated VAR model I found convincing evidence for all six prices being $I(2)$, which motivated the decomposition of the vector process x_t into the $I(0)$, $I(1)$, and, $I(2)$ directions. These were interpreted in terms of pushing and pulling forces. I demonstrated that the statistical concept of directly cointegrating, polynomially cointegrating and difference cointegrating relations can be given a natural economic interpretations as static long-run, dynamic long-run, and medium-run steady-state relations. Using the rich structure of the $I(2)$ model I was able to formally address the question of long-run price homogeneity (between price levels) and medium-run price homogeneity (between inflation rates). I showed that even under assumption of long-run price homogeneity, medium-run price homogeneity can only exceptionally be present. Hence, analyzing price homogeneity based on inflation rates instead of price levels is likely to give misleading results.

Several interesting empirical results emerged from this study:

First, the usefulness of calculating the roots of the characteristic polynomial for the choice of cointegration rank indices was pointed out. In particular, when there are $I(2)$ or near $I(2)$ components in the data this turned out to be a valuable diagnostic tool. As a further help in choosing the cointegration indices (r, s_1) the paper demonstrated the use of sensitivity analyses as a complement to the formal tests.

Second, long-run price homogeneity was found to be present in $\beta' x_t$, but probably not in $\beta'_{\perp 1} x_t$. Consistent with the theoretical model medium-run price homogeneity between the differences was present in one of the long-run relations, the directly $CI(2,2)$ cointegrating relation, but not in the others. Among the six price variables medium-run price homogeneity seemed to be present for the CPI, WPI, and $CRBI$, i.e. in those of the price indices that primarily act as driving forces in this system.

Third, the question whether commodity price indices can be used as forward indicators of the permanent part of price inflation measured by the CPI or the WPI index did not receive much empirical support. I found no significant effects on the CPI from the cointegrating relations, neither in levels nor in differences. The finding that the CPI was the driving force within the present system was a very strong result that carried through in all different tests.

Forth, I found three common stochastic trends among the six price indices, of which two were of second order and one of first order. One of the second order trends seemed to derive exclusively from permanent shocks to the CPI index, whereas the other derived from shocks to the commodity indices. All six price indices were affected by the two $I(2)$ trends and a minimum of three price indices were, therefore, needed for cointegration. This result explains previous findings of no cointegration in bivariate cointegration analyses and points to the importance choosing a sufficiently large set of price indices for this kind of analysis.

As a complement I performed a similar analysis (not reported in the paper) exclusively based on the four commodity price indices. The results were on the whole inconclusive and disappointing: only weak evidence of $I(2)$ effects, no evidence of long-run price proportionality, only weakly significant adjustment to the cointegrating relations, *etc.*, strengthening the conclusion that the complex relationships between price indices would be difficult to trace within a smaller set of variables.

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Appendix A:

The composition of the four commodity price indices (from Gallo, Marcellino,and Trivedi (1997).

Commodity Index	CRBI	GSCI	WBI	ECI
Energi	14.3	51.3	-	-
Livestock	14.3	13.6	-	47.4
Crops	42.8	25.4	53.2	-
Misc.[a]	9.5	-	19.7	19.3
Base metals[b]	4.8	6.6	27.1	33.3
Precious metals	14.3	3.2	-	-

a: For the CRBI this component includes orange and lumber juice; for the WBI it includes agricultural nonfood items - cotton, jute, tobacco, and rubber.

b: The CRBI includes only copper; the GSCI includes aluminium, copper, zinc, nickel, lead and tin; the WBI further includes phosphate rock and iron ore.

Appendix B:

The graphs of the data in levels and differences.

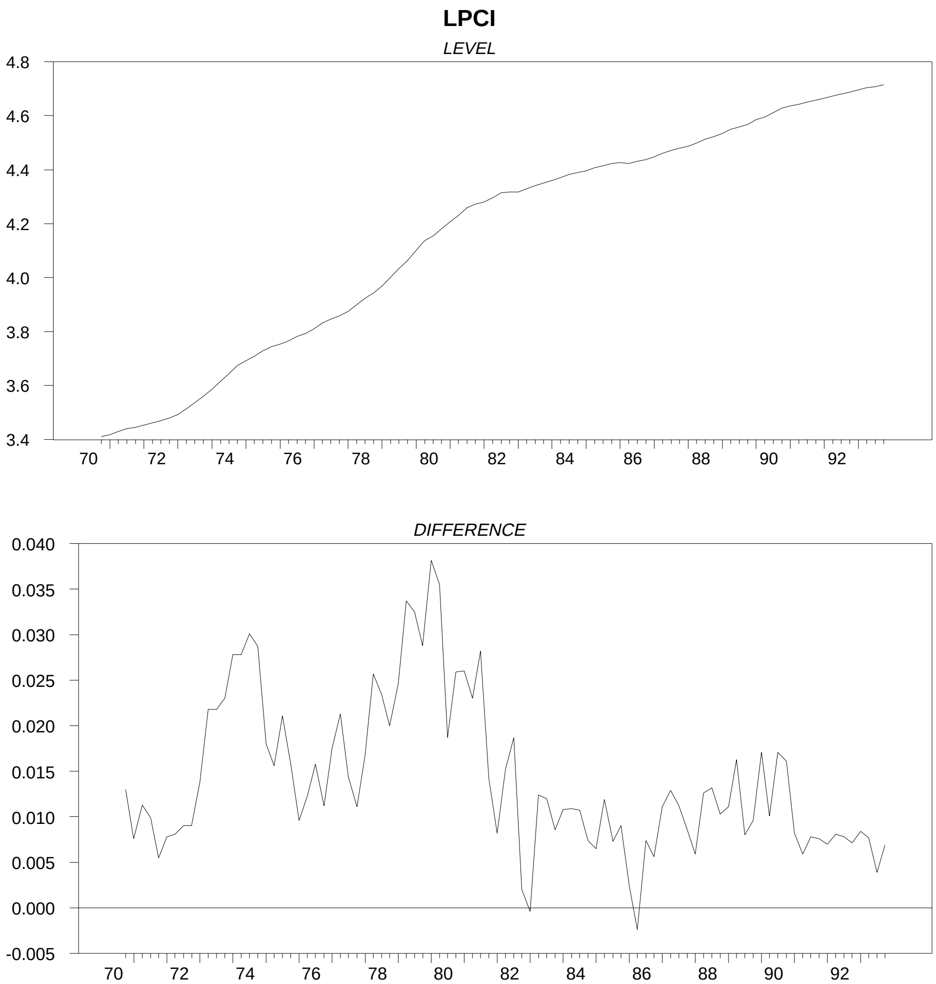

Figure 7.1: The log of CPI in levels and first differences

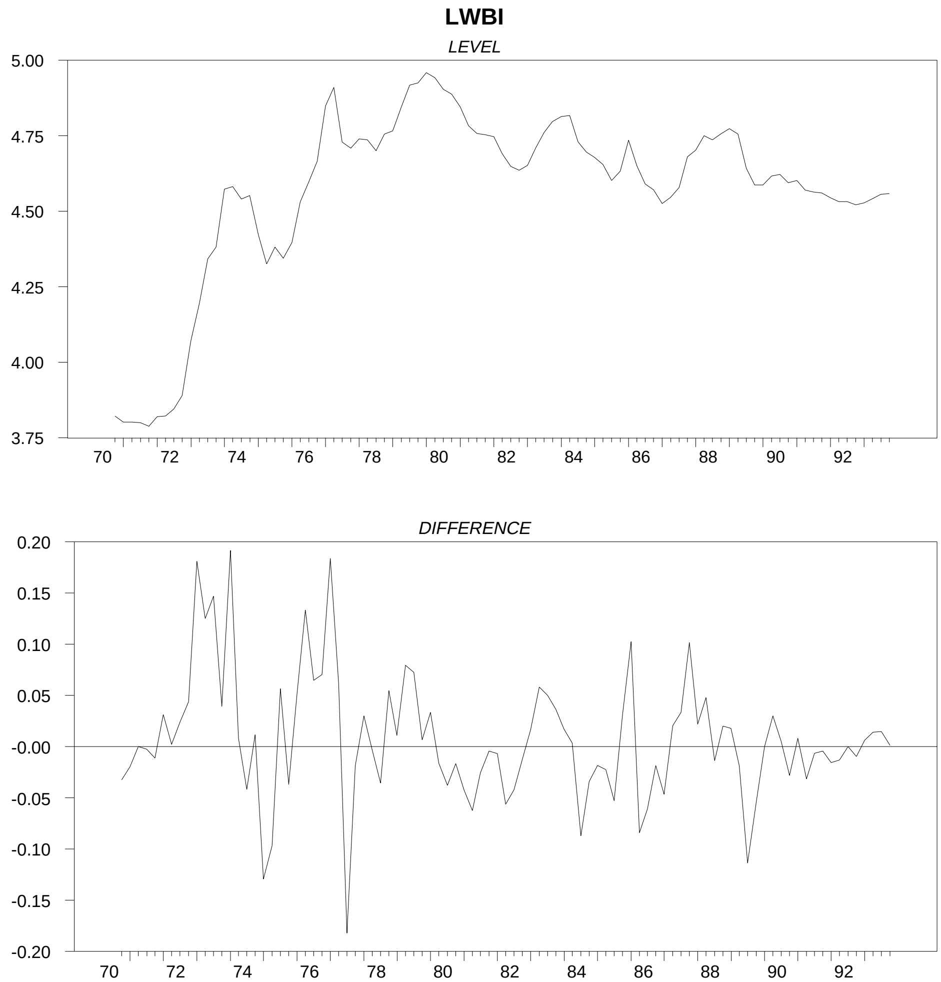

Figure 7.2: The log of the world bank commodity price index in levels and differences.

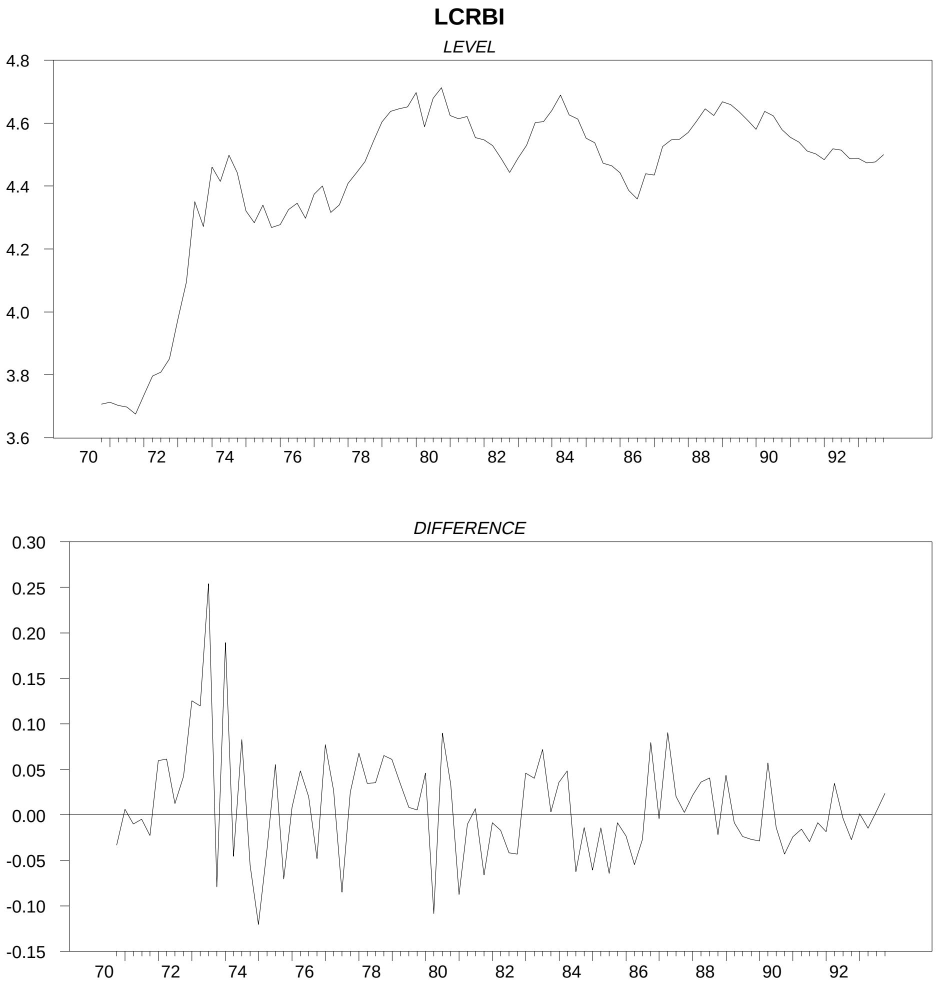

Figure 7.3: The log of the Commodity Research Bureau index in levels and differences.

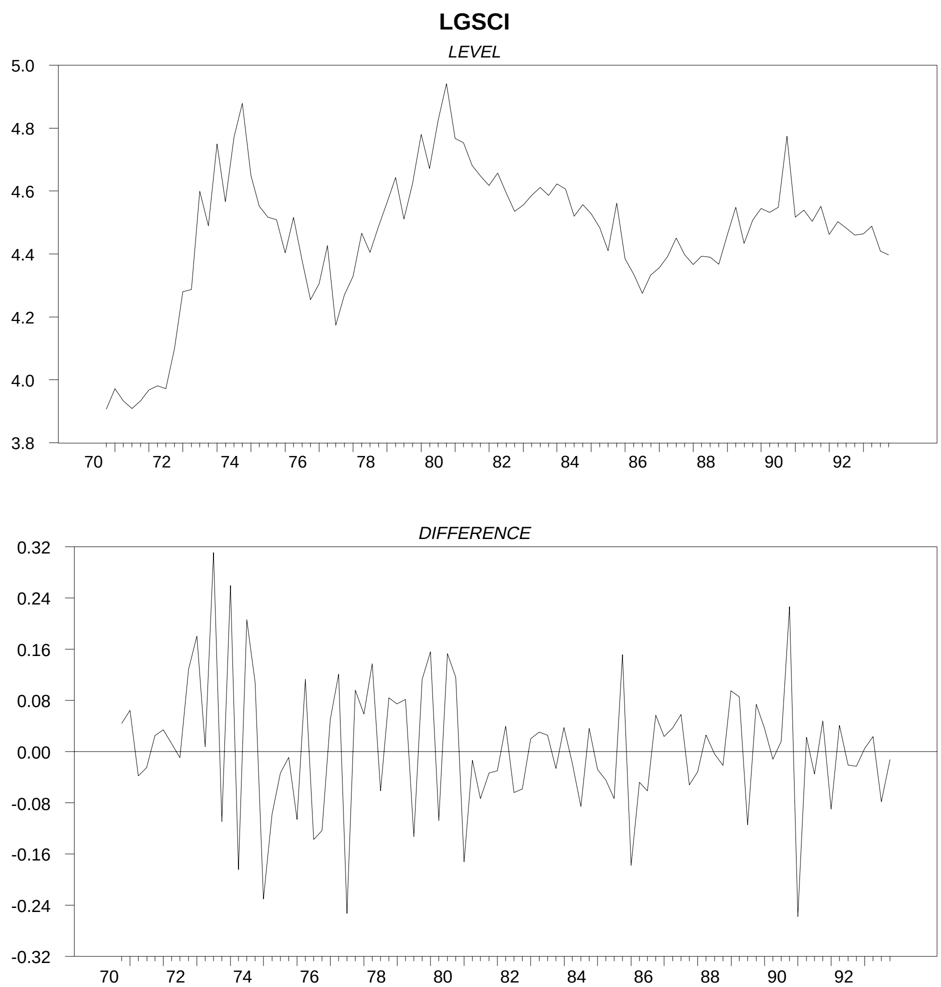

Figure 7.4: The log of the Goldman Sachs commodity price index in levles and differences.

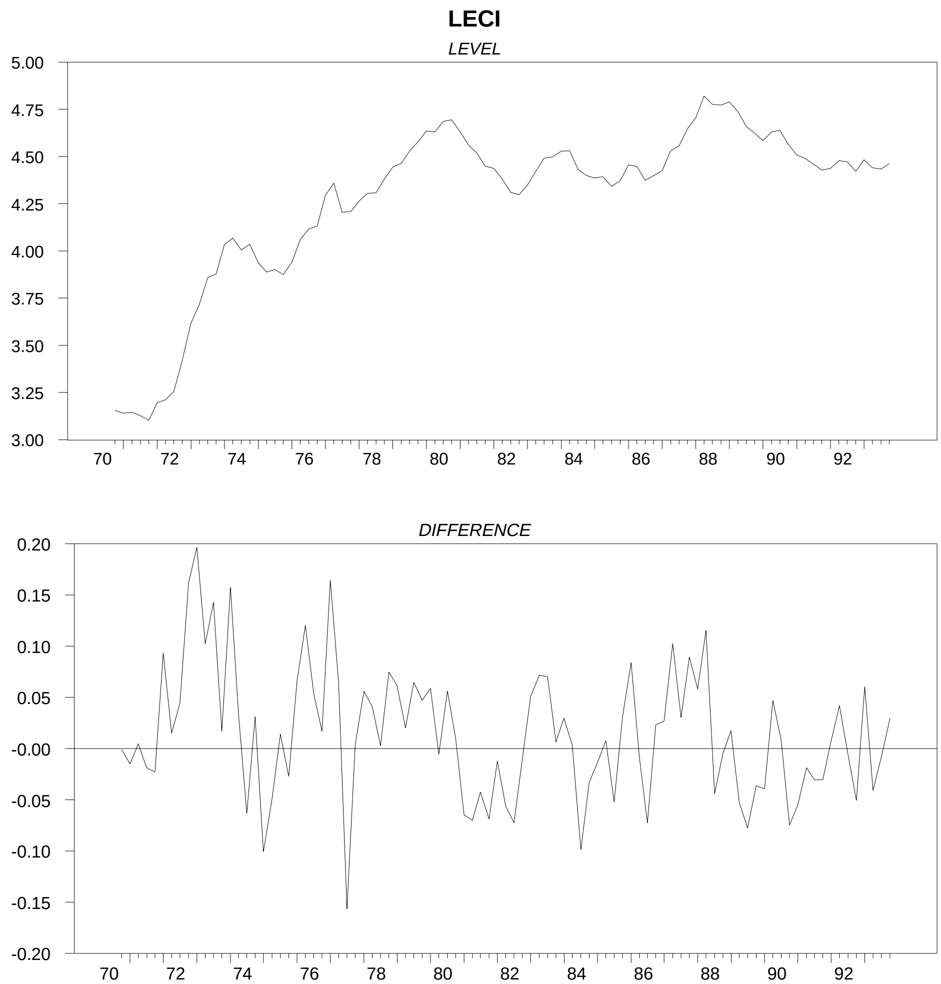

Figure 7.5: The log of the Economist's commodity price index in levels and differences.

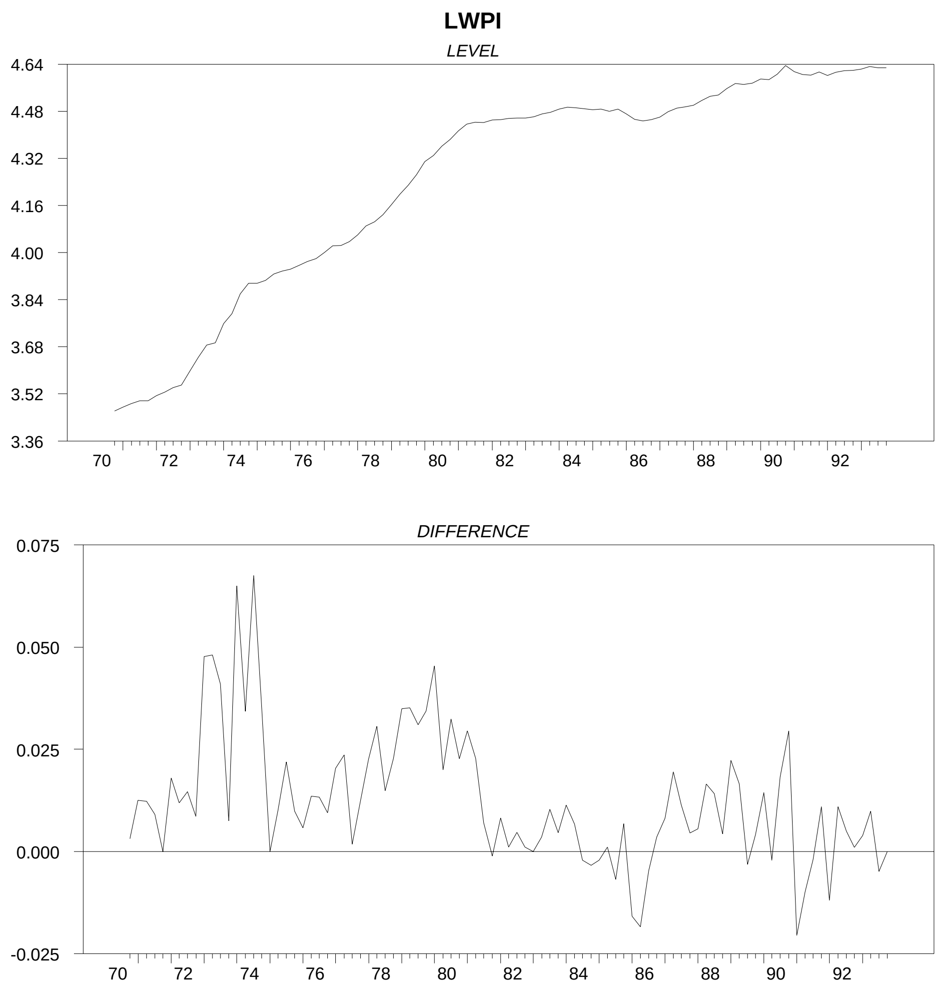

Figure 7.6: The log of wholesale price index in levels and differences