Overlapping Ownership and Product Innovation*

Rune Stenbacka[1]

and

Geert Van Moer[2]

April 08, 2022

Abstract

We characterize the effect of overlapping ownership (OO) on investments in product innovation. We analyze two opposing forces: (1) OO induces firms to internalize that success on their own behalf erodes the rivals' business, reducing investments; (2) OO softens competition in the product market, enhancing investments. These forces also determine potential shifts between the stable symmetric investment equilibrium and asymmetric equilibria. Our analysis reveals that the competition-softening effect can induce OO to raise total welfare. We thus present a mechanism different from spillovers as a source for OO to yield welfare benefits.

Keywords: common ownership; cross-ownership; product innovation; competition

JEL Classification: L13, D43, O32

---

*The authors acknowledge comments from David Myatt as well as participants at the MACCI-2020 conference in Mannheim.

[1] Hanken School of Economics and Helsinki Graduate School of Economics, P.O. Box 479, 00101 Helsinki, Finland. E-mail: Rune.Stenbacka@hanken.fi

[2] Hanken School of Economics and Helsinki Graduate School of Economics, P.O. Box 479, 00101 Helsinki, Finland. E-mail: Geert.VanMoer@hanken.fi

1. Introduction

An influential line of research has documented that overlapping ownership, whereby institutional investors hold stakes in competing firms (common ownership) or firms hold stakes in their rivals (cross-ownership), is an increasingly prevalent feature of the economy.[3] Overlapping ownership has the potential to relax competition in the product market, as shown in the pioneering study by O'Brien and Salop (2000).[4] The key mechanism is that managers, through overlapping minority shareholdings, partly internalize the effects of their product market decisions on their competitors' profits. This theoretical mechanism, originally developed by O'Brien and Salop (2000), has subsequently been extensively discussed in, for example, OECD (2017) and Backus, Conlon and Sinkinson (2021). The empirical industry studies Azar, Raina and Schmalz (2021) as well as Azar, Schmalz and Tecu (2019), focusing on banking and airlines, respectively, argue that there is a causal link between common ownership concentration and consumer prices. These empirical results have subsequently been subject to intense discussions among academic researchers as well as in the competition policy community.

In this study, we investigate the effects of overlapping ownership when two firms not only compete *in* the product market, but also compete *for* the market through product innovation. Product innovation is decisively important in many industries featuring overlapping ownership. For example, Branstetter, Chatterjee and Higgins (2016) report that, in their sample of pharmaceutical firms, the ratio between R&D expenditures and sales amounts to about 15%, and these investments typically concern product innovation.[5] As another example, in the software and platform service industries, upfront investments in product innovation are key to determine whether a firm succeeds to obtain a competitive edge.

We present our model of stochastic product innovation with overlapping ownership in section 2. In section 3 we characterize the symmetric investment equilibrium. We show that the effect

[3] Azar (2016), He and Huang (2017), Seldeslachts, Newham and Banal-Estanol (2017), OECD (2017), Schmalz (2018), Backus, Conlon and Sinkinson (2021), and Banal-Estañol, Seldeslachts and Vives (2020) are examples of studies which have presented evidence of the increasing importance of common ownership. Dietzenbacher, Smid and Volkerink (2000), Nain and Wang (2016), and Heim, Hüschelrath, Laitenberger and Spiegel (2019) offer empirical work on cross-ownership.

[4] Early studies on the effects of partial ownership also include, for example, Reynolds and Snapp (1986) and Farrell and Shapiro (1990).

[5] As Newham, Seldeslachts and Banal-Estanol (2018) point out, overlapping ownership is a characteristic feature of several major firms in the pharmaceutical industry.

of overlapping ownership on symmetric investments is determined as the outcome of an interplay between two forces: (1) When a firm increases its R&D investment, it exerts a negative externality on its rival because doing so raises the probability that the rival faces a competitor in the product market. In other words, investing more raises the probability of successfully innovating, which erodes the profit of the rival. With a higher degree of overlapping ownership firms internalize this externality to a higher extent. The internalization of this *business-eroding effect* reduces the equilibrium investments in product innovation. (2) When overlapping ownership softens duopoly competition in the product market, the prospect of increased profits in the product market raises the returns from successfully innovating. This *competition-softening effect* increases incentives for product innovation. Our analysis presents a general characterization of these two counteracting effects. The analysis also presents examples with explicit modes of competition, Cournot competition and Hotelling competition, for which we assess whether a higher degree of overlapping ownership discourages or stimulates investments.

These findings add a new dimension to the recent literature on overlapping ownership and R&D (López and Vives (2019) and Antón, Ederer, Giné and Schmalz (2021a)), which highlights the presence of spillovers as being a necessary factor for overlapping ownership to be able to promote R&D. Contrary to our study, this literature, however, studies process innovation rather than product innovation. With non-drastic process innovation overlapping ownership softens competition in the product market, resulting in lower output, and therefore the incentives to invest in process innovation *decline* unless there are sufficiently strong spillovers. The reason is that process innovations operate through a reduction in firms' marginal costs, so that "the benefit to firms from investing in R&D decreases proportionally with output" (López and Vives (2019), p. 2406). In contrast, we show that with product innovation, there is a competition-softening effect of overlapping ownership which tends to *enhance* (rather than reduce) symmetric investments, by improving the profits associated with duopoly competition. Consequently, a higher degree of overlapping ownership may stimulate investments in product innovation and generate a total welfare expansion, even in the absence of R&D spillovers. Li, Ma and Zeng (2015) as well as Newman, Seldeslachts and Banal-Estañol (2018) study overlapping ownership as an instrument to deter entry in models, where the competition-softening effect of overlapping ownership in the product market is limited.

Further, Zormpas and Ruble (2021) apply a real options approach to study the effect of overlapping ownership on the timing patterns of technology adoption.[6]

Section 4 demonstrates that overlapping ownership may lead to an unstable symmetric investment equilibrium. Focusing on an R&D investment technology characterized by a constant hazard rate, we show that the interplay between the two countervailing forces, the business-eroding effect and the competition-softening effect, is once again crucial. (1) As overlapping ownership causes firms to internalize how their innovation erodes the rival's business, the symmetric investment equilibrium whereby both firms compete in innovation can become unstable. Whenever so, there is an asymmetric investment equilibrium whereby only one firm invests a positive amount, and the other firm invests zero. We show that a shift from the stable symmetric equilibrium to such a corner equilibrium always benefits producers at the expense of total welfare. With degrees of overlapping ownership leading to a corner equilibrium, however, overlapping ownership has no effects on investment or welfare. (2) When overlapping ownership softens competition in the product market to a sufficient extent, it can also make competition in innovation more viable, invalidating the corner equilibria and inducing a shift towards a stable symmetric investment equilibrium. We show that such a shift always hurts producers and raises total welfare. The analysis of the stability condition associated with the symmetric equilibrium thus confirms that the effect of overlapping ownership is determined by the interplay between the business-eroding and competition-softening effect. When the latter dominates the former, overlapping ownership can raise total welfare also in the absence of spillovers.

Even though the recent literature on overlapping ownership has predominantly focused on evaluating how it affects product market competition, our study is not, as we have already pointed out, the first one to explore the effects of overlapping ownership on investments in R&D. The theoretical study by López and Vives (2019) and the primarily empirical study by Antón, Ederer, Giné and Schmalz (2021a) investigate process innovation under Cournot competition or Bertrand competition.[7] These authors highlight that, as overlapping ownership leads to output-reductions in the product market, the incentives for pursuing process

[6] Our analysis also differs from Bayona and López (2018), who study asymmetric silent financial interests in a model where Hotelling competitors can invest in quality improvement. They find that asymmetric investments in quality can result in more consumers buying from the high-quality firm, a potentially welfare-enhancing reallocation effect. In contrast to our analysis, within their framework symmetric financial interests reduce innovation incentives.

[7] For example, Lopéz and Vives (2019) focus primarily on Cournot competition, but also present a robustness analysis modelling a Bertrand oligopoly with product differentiation. Vives (2020) discusses the effects of overlapping ownership on market power and innovation in the presence of external effects more broadly.

innovations tend to decline.[8] Our analysis shows that product innovation incentives, in contrast, can *increase*, as a result of higher expected profits in the product market. More generally, our analysis of product innovation captures not only the effect of overlapping ownership on market performance within a given market structure but additionally endogenizes that market structure. For example, we are able to evaluate the effect of overlapping ownership on the probability that there is a product on offer to begin with, an issue which is of primary welfare importance.

An influential research approach in industrial organization, initiated by d'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992), has also emphasized the role of spillovers in arguments to support R&D collaboration between firms. Overlapping ownership differs from R&D collaboration in these models, because it induces softer competition in the product market. This effect gives rise to a mechanism whereby overlapping ownership can raise total welfare that does not rest on the presence of R&D spillovers.

Federico, Langus and Valletti (2018) explore the effect of a merger on product innovation, focusing on interior investment equilibria. Their analysis identifies a related trade-off between an "innovation externality" and a "price-coordination" effect in the context of merger analysis. In section 5 we conduct a comparison of overlapping ownership with a merger. We establish conditions such that the investments with a merger in the two research labs are symmetric. We also show that, for example with Hotelling competition in the product market, the investments in product innovation with overlapping ownership are not a weighted average of the investments without overlapping ownership and those with a merger.

Our results can also be viewed in light of the recent stream of influential studies, for example Gutiérrez and Philippon (2017a, 2017b), demonstrating a decline in investment in the United States and Europe. In particular, Gutiérrez and Philippon (2017a) present a cross-industry empirical analysis focusing on the US and argue that increased common ownership is correlated with lower investments. In this respect they note that "the mechanisms remain to be understood…". We offer a model of product innovation followed by product market competition that details such a mechanism. According to our analysis, a decline in investments is associated with the feature that overlapping ownership does not substantially soften competition in the product market.

[8] Shelegia and Spiegel (2015) study an effect whereby process innovation can increase because of asymmetries in a Bertrand product market.

We proceed as follows. Section 2 presents the model. In Section 3 we present a detailed analysis of the symmetric investment equilibrium with a particular focus on the effects of overlapping ownership on investments as well as on issues related to stability and welfare. Section 4 analyses asymmetric investment equilibria and reports the associated welfare implications. Section 5 compares the investments in a configuration of overlapping ownership with those under a merger. Finally, Section 6 concludes.

2. The Model

We consider an industry with two firms, A and B. The strategic interaction between the firms is modelled by a two-stage game. The first stage focuses on product innovation and models each firm's decision about how much to invest in R&D. The second stage captures product market competition. The product innovation outcomes are subject to uncertainty and determine whether the market structure in the product market is a duopoly, a monopoly, or features no entry.

We model overlapping ownership in reduced form. Each firm makes its decisions based on an objective function that grants positive weight to the rival firm's profit. In particular, the objective function of firm i equals

$$\Gamma_i \equiv (1-\mu)\pi_i + \mu\pi_j, \tag{1}$$

where $0 \le \mu < 1/2$ captures the degree of overlapping ownership and where π_i and π_j denote the expected profits of firms *i* and *j*, respectively ($i, j \in \{A, B\}, i \neq j$). Parameter μ determines the extent to which firm i internalizes rival firm j's profits. It captures either common ownership or cross-ownership, both of which are widespread practices (see, Vives (2020)).[9]

[9] For details regarding the interpretation of μ and its microfoundations with common ownership or cross-ownership and various assumptions regarding the degree of control, we refer to López and Vives (2019). The objective function in their setting is equivalent to (1) and is obtained by replacing μ with $\lambda/(1+\lambda)$ and multiplying (normalizing) the objective function using factor $1+\lambda$. An increase in the value for μ means that firm i's objective function attaches an increased weight on the rival's profit relative to one's own profit. The parameter μ is suitable to conduct comparative statics: a higher degree of common ownership or cross-ownership corresponds to a higher μ.

In the stage of product innovation, each firm i determines its investment in R&D to maximize its objective function (1). Firm i's investment volume is denoted by x_i and it determines the probability $0 \leq f(x_i) < 1$ that firm i's product innovation is successful (henceforth: firm i succeeds). We make the following assumption.

Assumption 1 (Decreasing returns to R&D): *An increased investment volume increases the probability of success, $f(x_i)$, at a decreasing rate. Furthermore, a positive investment volume is necessary for success. Formally: $f'(x_i) > 0$, $f''(x_i) < 0$, and $f(0) = 0$.*

Assumption 1 can be interpreted in the following alternative way: the cost of reaching the probability of success f is increasing and strictly convex, and no investment costs are needed to obtain $f = 0$.

The stage of product innovation can generate four possible states of nature. Table 1 depicts how firms' R&D investments determine the probability that each state of nature occurs and how each state of nature maps into a product market structure.

Probability	**State of nature**	**Product market structure**
$[1-f(x_A)][1-f(x_B)]$	None of the firms succeeds	No entry
$f(x_A)[1-f(x_B)]$	Only A succeeds	A achieves monopoly profit
$[1-f(x_A)]f(x_B)$	Only B succeeds	B achieves monopoly profit
$f(x_A)f(x_B)$	Both firms succeed	A and B achieve duopoly profits

Table 1: *The probability and market structure associated with each state of nature.*

Next, we model and analyse the outcome of the product market stage. With probability $[1-f(x_A)][1-f(x_B)]$, none of the firms succeeds. In that event, the investments do not yield any return. With the investment as a sunk cost, firm A's profit then equals $-x_A$ and firm B's profit equals $-x_B$.

With probability $f\left(x_i\right)\left[1-f\left(x_j\right)\right]$, only firm i succeeds, and it achieves a monopoly position. The monopoly rent excluding investment expenses equals π^M. Firm i's profit then equals $\pi^M - x_i$, whereas firm $j's$ investment does not yield any return so that firm j's profit is $-x_j$. With probability $f\left(x_j\right)\left[1-f\left(x_i\right)\right]$, the state of nature is equivalent except for the roles of firms *i* and *j* being reversed. To rule out that both firms invest zero, we consider sufficiently high monopoly rents.

Assumption 2 (R&D investments attractive): *If the rival invests zero, it is an optimal response to invest a positive amount:* $f'(0)\pi^M > 1$.

Finally, with probability $f\left(x_A\right) f\left(x_B\right)$ both firms succeed, achieving duopoly profits in the product market. We apply a reduced-form representation of the duopoly profits: under duopoly each firm earns a profit equal to $\delta(\mu)\pi^M$. The factor $\delta(\mu)$ captures the intensity of product market competition. This general representation has the attractive feature that it does not require to explicitly specify the mode of competition, which determines the value of the factor $\delta(\mu)$.

Assumption 3 (Duopoly profits): *If both firms succeed in innovation, each of them obtains a strictly positive duopoly profit less than the monopoly rent, or* $0<\delta(\mu)<1$. *Further, duopoly profits are differentiable as a function of* μ*, and overlapping ownership either relaxes or has no effect on product market competition, meaning that* $\delta'(\mu)\geq 0$.

We thus capture the full spectrum of modes of competition except for the extreme cases of pure Bertrand competition[10] or independent markets.

We are now ready to characterize firm *i*'s objective function (1) as a function of its investment level as follows:

[10] In subsection 3.3 we analyse a Hotelling framework which allows for arbitrarily small transportation costs. Pure Bertrand competition is a limiting case of our analysis. With pure Bertrand competition, since $\mu<1/2$, the pricing decision in the product market places more weight on the firm's own profit than on that of the rival, so that the standard undercutting argument induces $\delta(\mu)=0$ over the whole range. Our analysis in subsection 3.2 and section 4 then predicts that, when f is characterized by a constant hazard rate, overlapping ownership has no welfare effects.

$$\Gamma_i(x_i, x_j) = (1-\mu)\Big(f(x_i)\big[1- f(x_j)\big]\pi^M + f(x_i) f(x_j)\delta(\mu)\pi^M - x_i\Big)$$
$$+\mu\Big(\big[1- f(x_i)\big] f(x_j)\pi^M + f(x_i) f(x_j)\delta(\mu)\pi^M - x_j\Big), \tag{2}$$

reflecting that firm i 's places weight $1-\mu$ on its own profit and weight μ on its rivals's profit. The investment game can have symmetric as well as asymmetric equilibria. In the next section we focus on the symmetric equilibrium, and subsequently analyse potential asymmetric equilibria in section 4.

3. The Symmetric Investment Equilibrium

3.1 The Effects of Overlapping Ownership on Investments and Welfare

Initially we explore the effects of an increased degree of overlapping ownership on the investments in product innovation in a symmetric equilibrium. Based on differentiation of the objective function (2) we find that the necessary first-order condition associated with firm *i*'s optimization is given by

$$\frac{\partial \Gamma_i}{\partial x_i} \;=\; (1-\mu)\, f'(x_i)\,\pi^M \left[1- f(x_j)\right] - f'(x_i)\,\pi^M \left[f(x_j)(\mu-\delta(\mu))\right] - (1-\mu) \;=\; 0.$$

According to this first-order condition, the investment returns depend on the investment of the rival firm. The first term, $(1-\mu)\, f'(x_i)\,\pi^M$, captures the return to firm *i* when the rival's innovation is unsuccessful, which happens with probability $1- f(x_j)$. Under such circumstances, investing increases the probability of being the only successful firm - and earning the monopoly rent - rather than being unsuccessful as well. This effect is positive and stimulates investment. The second term, $-f'(x_i)\,\pi^M \left[f(x_j)(\mu-\delta(\mu))\right]$, captures the scenario where the rival succeeds (with probability $f(x_j)$). Under such circumstances, investing more increases the probability that both firms are successful instead of only the rival firm. Success on behalf of both firms eliminates the rival's monopoly rent (π^M), and thereby with overlapping ownership also the firm's share μ of that monopoly rent, and transforms this into duopoly rents for both firms ($\left[(1-\mu)+\mu\right]\delta(\mu)\pi^M$). The final term in (3), $1-\mu$, captures the direct investment cost facing firm *i*. We can simplify the first-order condition according to

$$\frac{\partial\Gamma_i}{\partial x_i} \;=\; f'(x_i)\,\pi^M\left[1-\mu-f\left(x_j\right)(1-\delta(\mu))\right] \;-\; (1-\mu) \;=\; 0. \qquad (3)$$

Combining (3) with the analogous first-order condition for firm *j* we can conclude that the symmetric investment equilibrium $x_i^* = x_j^* = x^*$ must satisfy the condition

$$f'\left(x^*\right)\pi^M\left[1-\mu-f\left(x^*\right)(1-\delta(\mu))\right] \;-\; (1-\mu) \;=\; 0. \qquad (4)$$

By Assumption 1 the success probability is a strictly concave function ($f''\left(x_i\right)<0$), making it possible for us to express the sufficient second-order condition $\frac{\partial^2\Gamma_i(x^*,x^*))}{\partial x_i^2}<0$ according to

$$1-\mu-f\left(x^*\right)\,(1-\delta(\mu)) \;>\; 0.$$

Further, in light of this inequality the condition for stability of the symmetric equilibrium

$$\frac{\partial^2\Gamma_i(x^*,x^*)}{\partial x_i^2}\frac{\partial^2\Gamma_j(x^*,x^*)}{\partial x_j^2}-\frac{\partial^2\Gamma_i(x^*,x^*)}{\partial x_i\partial x_j}\frac{\partial^2\Gamma_j(x^*,x^*)}{\partial x_j\partial x_i}>0$$

is equivalent to

$$\frac{1-\mu}{1-\delta(\mu)} \;>\; f(x^*) \;-\; \frac{f'(x^*)\,f'(x^*)}{f''(x^*)}. \qquad (5)$$

Notice that (5) implies that the second-order condition $1-\mu-f\left(x^*\right)\,(1-\delta(\mu)) \;>\; 0$ is satisfied. From (5) we can conclude that not only the success probability function, but also the degree of overlapping ownership and the intensity of product market competition are factors affecting whether the stability condition is met.

To interpret (5), note that the technical property leading to stability is that the reaction functions have slopes with absolute value below one. This means that the optimal investment level of a firm should not be too responsive to the investment level of its rival. The responsiveness arises from the feature that duopoly competition erodes monopoly rents.[11] A high value for the

[11] To see this, note that if markets are (almost) independent ($\delta(\mu)\to 1$), the stability condition is always met.

duopoly profit as a fraction of the monopoly profit ($\delta(\mu)$) helps to meet the stability condition of the symmetric investment equilibrium. In other words, when the prospect of duopoly competition is attractive, the symmetric equilibrium such that both firms select positive investment volumes is less prone to instability. In subsection 3.2 we will characterize the effect of overlapping ownership on the stability of the symmetric investment equilibrium. On the one hand, a higher degree of overlapping ownership has the direct effect (taking as given the value for $\delta(\mu)$) of increasing the responsiveness of a firm's optimal investment volume to that of its rival, an effect which pressures the stability condition. The intuitive reason is that a higher investment volume by the rival then makes it is optimal for firm *i* to reduce its own investment, in order not to jeopardize the internalized part of the rival's profit ($\mu\pi_j$). On the other hand, overlapping ownership, by potentially raising $\delta(\mu)$, has the indirect effect of relaxing the stability condition.

We proceed by charactering the effect of an increased degree of overlapping ownership on R&D investments, focusing on a configuration under which a stable symmetric investment equilibrium exists. In the Appendix we establish the following comparative statics property

$$\frac{dx^*}{d\mu} = \frac{1+\pi^M f'(x^*)\left[-1+f(x^*)\delta'(\mu)\right]}{\left(1-\delta(\mu)\right)\pi^M f'(x^*)f'(x^*)-\pi^M f''(x^*)\left(1-\mu-f(x^*)\left(1-\delta(\mu)\right)\right)}, \qquad (6)$$

where the denominator is positive because of Assumption 1 and the stability condition (5). Consequently, a higher degree of overlapping ownership decreases investment if and only if the numerator satisfies

$$1+\pi^M f'(x^*)\left[-1+f(x^*)\delta'(\mu)\right]<0.$$

Using (4) and rearranging we find that a higher degree of overlapping ownership reduces the symmetric equilibrium investment if and only if

$$\frac{1-\mu-f(x^*)\left[1-\delta(\mu)\right]+(1-\mu)\left[-1+f(x^*)\delta'(\mu)\right]}{1-\mu-f(x^*)\left[1-\delta(\mu)\right]} < 0.$$

This condition for a higher degree of overlapping ownership to reduce the symmetric investment can be rewritten as

$$\frac{\delta'(\mu)}{1-\delta(\mu)} < \frac{1}{1-\mu} \ . \tag{7}$$

Condition (7) holds when $\delta'(\mu)$ is limited. This means that the increase in duopoly profit, as a fraction of monopoly profit, induced by an increase in overlapping ownership is not too high. In the remainder of our paper we will refer to this condition as follows.

Definition 1 *An increase in the degree of overlapping ownership does not substantially relax product market competition if (7) holds, whereas it substantially relaxes competition otherwise.*

We can summarize our findings regarding the symmetric equilibrium in the following way.

Result 1 *Assume that the stability condition (5) holds.*

(a) There exists a unique symmetric investment equilibrium satisfying (4).

(b) An increase in overlapping ownership strictly decreases investments in the symmetric equilibrium, if it does not substantially relax product market competition.

(c) An increase in overlapping ownership increases investments in the symmetric equilibrium, if it substantially relaxes product market competition.

The intuitive explanation of the effect of the degree of overlapping ownership on the symmetric investment volume proceeds as follows. Investing exerts a negative externality on the rival's expected profit π_j by increasing the probability that it faces a competitor in the product market. A higher degree of overlapping ownership increases the extent to which firms internalize this negative externality. This effect tends to reduce the equilibrium investments. However, if an increased degree of overlapping ownership also stimulates duopoly profits, the improved prospects of succeeding in innovation act as a countervailing force. When overlapping ownership substantially relaxes product market competition, this countervailing force dominates and induces an investment expansion.

We next characterize the welfare effects of overlapping ownership. Let CS^M ($CS^D(\mu)$) denote consumer surplus with monopoly (duopoly) so that $W^M = \pi^M + CS^M$ and $W^D(\mu) = 2\delta(\mu)\pi^M + CS^D(\mu)$ capture total gross surplus with monopoly and duopoly, respectively. We assume that $W^M \leq W^D(\mu)$, meaning that the total gross surplus when only one of the firms succeeds in innovation is lower than total gross surplus when both firms succeed. In the symmetric investment equilibrium the total welfare induced by the degree μ of overlapping ownership is given by

$$W(\mu) = 2f(x^*)(1 - f(x^*))W^M + f(x^*)^2 W^D(\mu) - 2x^*. \tag{8}$$

We can make a decomposition of the effect of a higher degree of overlapping ownership on total welfare in the following way.

$$\frac{dW}{d\mu} = \underbrace{f(x^*)^2 W^D{}'(\mu)}_{\text{static effect}} + \underbrace{2\frac{dx^*}{d\mu}\left[f'(x^*)\left[(1 - f(x^*))W^M + f(x^*)(W^D(\mu) - W^M)\right] - 1\right]}_{\text{dynamic effect}}. \tag{9}$$

The first term represents the **static effect** of an increased degree of overlapping ownership. Specifically, it captures the allocative welfare effect of a change in μ when there is a duopoly market structure. Since overlapping ownership tends to soften duopoly competition, it is plausible to assume that the static effect is weakly negative ($W^D{}'(\mu) \leq 0$)[12]. As overlapping ownership tends to raise profits in the product market (by Assumption 3, $2\delta'(\mu)\pi^M \geq 0$), we can conclude that it decreases consumer welfare ($CS^D{}'(\mu) \leq 0$).

The second term in (9) represents the **dynamic effect**[13] of overlapping ownership on welfare. It captures how a change in μ induces a change in investment level, and hence in the probabilities of reaching a market structure with monopoly or duopoly. To investigate the dynamic effect, it is useful to decompose $W^M = \pi^M + CS^M$ and $W^D(\mu) = 2\delta(\mu)\pi^M + CS^D(\mu)$, so that the dynamic effect can be written as

$$\begin{aligned}&2\frac{dx^*}{d\mu}\left[f'(x^*)\left[(1 - f(x^*))CS^M + f(x^*)(CS^D(\mu) - CS^M)\right]\right]\\ &+2\frac{dx^*}{d\mu}\left[f'(x^*)(1 - 2f(x^*)(1 - \delta(\mu)))\pi^M - 1\right].\end{aligned}$$

[12] The explicit modes of competition we analyse in subsection 3.3 indeed satisfy this property.
[13] A comment clarifying the terminology might be justified. The static effects refer to pure product market effects, whereas the dynamic effects refer to innovation effects channelled through the investment volumes. Our terminology does not refer to any timing considerations in a literal sense.

The top (bottom) line represents the dynamic welfare effect on consumers (producers). An induced increase in investments has an ambiguous effect on consumer welfare.[14] Next, we show that producers suffer if an increase in overlapping ownership induces an expansion of the investment volume. Using the first-order condition (4), which states that $f'(x^*)\pi^M = \frac{1-\mu}{1-\mu-f(x^*)(1-\delta(\mu))}$, we can rewrite the dynamic effect on producers as

$$2\frac{dx^*}{d\mu}\underbrace{\left[\frac{f(x^*)(1-\delta(\mu))}{1-\mu-f(x^*)(1-\delta(\mu))}(2\mu-1)\right]}_{<0}.$$

We sign the last term using the second-order condition and $\mu < 1/2$. Consequently, expanded investments make firms worse off.

We can summarize our findings regarding the decomposition of the welfare effects of overlapping ownership according to the following general result.[15]

Result 2

(a) The static effect of overlapping ownership hurts consumers and benefits producers.

(b) The dynamic effect of overlapping ownership hurts (benefits) producers if overlapping ownership substantially relaxes (does not substantially relax) product market competition.

The total welfare effect of a higher degree of overlapping ownership follows from (9) and equals the sum of its static effect and its dynamic effect. As the static effect is weakly negative, a higher degree of overlapping ownership can only raise total welfare when the dynamic effect is positive. In subsection 3.3 we will show that, for example with Hotelling competition, a higher degree of overlapping ownership can indeed raise total welfare.

[14] For example, the Hotelling model, which we will analyse in subsection 3.3, has the feature that, when the degree of overlapping ownership is high, consumer welfare with a monopoly market structure exceeds consumer welfare with a duopoly market structure. In combination with a high value for $f(x^*)$, it is then possible for an induced increase in investments to harm consumers.

[15] For the purpose of completeness we include the static welfare effects in the result, although the static effects follow straightforwardly from our assumption $W^D{}'(\mu) \leq 0$ and Assumption 3.

3.2 Stability

We next analyse in greater detail the stability of the symmetric equilibrium when the R&D production function is characterized by a constant hazard rate $\frac{f'(x_i)}{1-f(x_i)}=\gamma$, a feature which forms the basis of many influential studies in various fields including the economics of innovation. The associated probability of success function equals $f(x_i)=1-e^{-\gamma x_i}$ such that $f'(x_i)=\gamma e^{-\gamma x_i}$ and $f''(x_i)=-\gamma f'(x_i)$. The stability condition (5) can now be simplified according to the following result.

Result 3 *With a constant hazard rate the stability condition of the symmetric equilibrium is*

$$\mu<\delta(\mu). \tag{10}$$

Result 3 means that the symmetric investment equilibrium is stable whenever the degree of internalisation (left-hand-side in (10)) is smaller than the duopoly profit as a fraction of the monopoly profit (right-hand-side in (10)).

In Figure 1 the segment above the 45-degree line captures the combinations $(\mu,\delta(\mu))$ such that the symmetric equilibrium is stable. By Assumption 3, the symmetric equilibrium is stable for sufficiently small values of μ. Further, as Figure 1 illustrates, fixed points of the function $\delta(\mu)$, i.e. levels of overlapping ownership satisfying the equation $\delta(\mu)=\mu$, capture transformations between the stable and unstable region.

Figure 1 illustrates potential transformations between the stable and unstable region in response to increases in the degree of overlapping ownership. In particular, we can make the following observations. A shift in the nature of the symmetric equilibrium from stable to unstable requires that there exists a degree of overlapping ownership satisfying $\delta(\mu)=\mu$ and $\lim_{\delta(\mu)\to\mu^+}\delta'(\mu)<1$, meaning that the curve $\delta(\mu)$ approaches the 45-degree line from above with a slope lower than one. By (7), this can take place only when overlapping ownership does not substantially relax product market competition. Conversely, a shift from an unstable configuration to a stable one requires that $\delta(\mu)=\mu$ and $\lim_{\delta(\mu)\to\mu^-}\delta'(\mu)>1$, meaning that the curve $\delta(\mu)$ approaches the 45-degree line from below in the unstable region with a slope higher than one. This cannot happen unless an increase in the degree of overlapping ownership substantially relaxes product market competition.

As (6) and (7) make clear, the function $\delta(\mu)$ determines how investments respond to an increase in overlapping ownership and whether the symmetric equilibrium is stable or not. This means that mode of competition in the product market is decisive. We will therefore in the next subsection explore the effects of overlapping ownership in detail for two specific configurations: (1) Cournot competition with linear inverse demand in a homogenous product market, and (2) Hotelling competition with linear transportation costs in a differentiated product market.

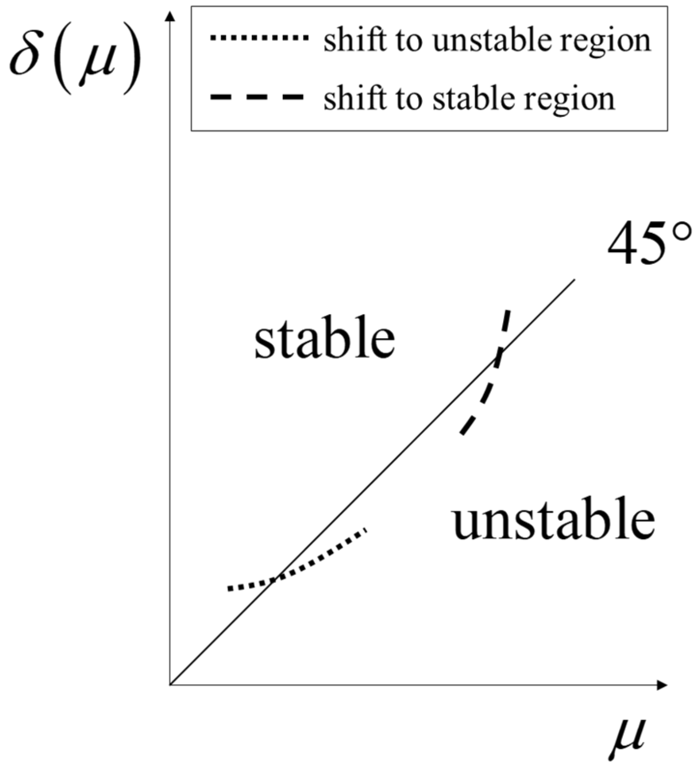


Figure 1 *Stability of the symmetric equilibrium when the hazard rate is constant.*

3.3 Explicit Modes of Competition: Cournot and Hotelling Competition

(1) Cournot competition in a homogeneous industry

The first mode of competition we focus on is Cournot competition with linear inverse demand according to $p = a - b(q_A + q_B)$, where q_i denotes the production by firm *i*. We assume that the firms face constant marginal production costs equal to c.[16]

With successful innovations on behalf of both firms, firm i decides on its production to maximize profits according to

$$\max_{q_i} (1-\mu)\left(a - b\left(q_i + q_j\right) - c\right)q_i + \mu\left(a - b\left(q_i + q_j\right) - c\right)q_j .$$

The profit function does not display the investment expenditures, as these are considered sunk once firms have reached the product market stage. We formulate the first-order condition and apply symmetry to obtain that the equilibrium production is given by

$$q_i^* = \frac{(1-\mu)(a-c)}{3b - 2b\mu} .$$

The associated equilibrium profits are $\left(a - 2bq_i^* - c\right)q_i^*$. Substituting the equilibrium quantity into this expression we find that the Cournot equilibrium profits are

$$\delta(\mu)\pi^M = \frac{(a-c)^2(1-\mu)}{b(3-2\mu)^2} .$$

Further, the monopoly profits π^M equal the standard expression $\frac{(a-c)^2}{4b}$. Consequently, as a fraction of monopoly profits the Cournot equilibrium profits equal

$$\delta(\mu) = \frac{4(1-\mu)}{(3-2\mu)^2} . \tag{11}$$

(2) Hotelling competition in a differentiated industry

The second mode of competition we focus on is the conventional Hotelling model. The two competing firms, *A* and *B*, have zero marginal costs and are located at the endpoints of the unit interval with A located at $x = 0$ and B located at $x = 1$. A continuum of consumers with a

[16] Antón, Ederer, Giné and Schmalz (2021b) explore an effect whereby overlapping ownership endogenously affects the marginal cost in the product market.

mass normalized to one is uniformly distributed on the unit interval. Faced with price p_A charged by A and p_B charged by B, a consumer located at $x \in [0,1]$ has a utility function capturing linear transportation costs according to

$$u_x = \begin{cases} v - p_A - \tau x & \text{when buying from } A \\ v - p_B - \tau(1-x) & \text{when buying from } B \\ 0 & \text{when not buying,} \end{cases}$$

where v denotes the reservation utility and τ denotes the transportation cost parameter. Each consumer is interested to buy one unit of the product.

We assume that the transportation cost is not too high such that $\tau < v/2$. Doing so guarantees that a monopolist has incentives to fully cover the market, meaning that each consumer buys one unit. The interpretation is that we capture product innovations such that access to the product is of primary importance for consumers relative to their preference for purchasing from a specific supplier. Furthermore, we normalize the reservation utility so that $v = 1$. The resulting monopoly rent equals

$$\pi^M = 1 - \tau .$$

We first analyse duopoly competition when the possibility of incomplete market coverage does not constrain the price equilibrium. This condition is satisfied when both prices are strictly below $1 - \frac{1}{2}\tau$. The indifferent consumer is then characterized by location x such that $1 - p_A - \tau x = 1 - p_B - \tau(1-x)$, which simplifies to $x = \frac{1}{2} + \frac{p_B - p_A}{2\tau}$. Firm i thus maximizes the following profit function

$$(1-\mu)\, p_i \left(\frac{1}{2} + \frac{p_j - p_i}{2\tau} \right) + \mu p_j \left(\frac{1}{2} + \frac{p_i - p_j}{2\tau} \right).$$

We construct the first-order condition and apply symmetry to obtain that the symmetric equilibrium price equals $\frac{(1-\mu)\tau}{1-2\mu}$. The condition for the market coverage constraint not to bind is that $\frac{(1-\mu)\tau}{1-2\mu} < 1 - \frac{\tau}{2}$, which holds when $\mu < \mu^*$, where $\mu^* \equiv \frac{2-3\tau}{4(1-\tau)}$.

When $\mu \geq \mu^*$ the equilibrium price equals $1-\frac{\tau}{2}$. Specifically, we show that charging $p=1-\frac{\tau}{2}$ is a best response when the rival charges $1-\frac{\tau}{2}$. By symmetry we can focus on firm A. Crucially, we need to show that it is unprofitable to deviate with a price $p_A > 1-\frac{\tau}{2}$.[17] When charging $p_A \geq 1-\frac{\tau}{2}$, the indifferent consumer has an outside option surplus equal to zero, as it is not worthwhile to purchase from firm B (which charges $1-\frac{\tau}{2}$). Thus, with the upward deviation there is incomplete market coverage. The consumer x indifferent between buying or not is characterized by $1-x\tau-p_A=0$, simplified as $x=\frac{1-p_A}{\tau}$. Firm A's objective function becomes $(1-\mu)\,p_A\frac{1-p_A}{\tau}+\mu\frac{1-\frac{\tau}{2}}{2}$. Notice that the part of B's profit which A internalizes ($\mu\pi_B$) is independent of p_A as the upward deviation implies losing consumers that do not have an interest in purchasing from firm B. The first-order derivative equals $(1-\mu)\frac{1-2p_A}{\tau}$ and the second order derivative $-2\frac{1-\mu}{\tau}$. Deviating is unprofitable when the first-order derivative is non-positive, or $1\leq 2p_A$. Since $p_A \geq 1-\frac{\tau}{2}$, it is sufficient to show that $1\leq 2\left(1-\frac{\tau}{2}\right)$ or $\tau\leq 1$, which always holds.

In light of our calculations above, the equilibrium profits with Hotelling competition are given by

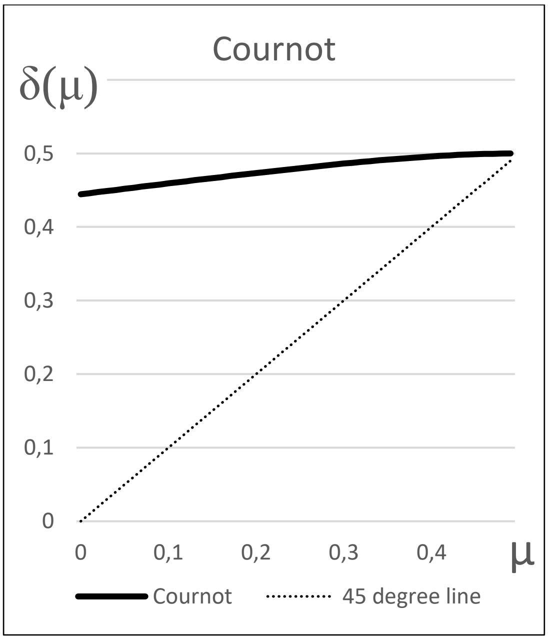
$$\delta(\mu)\pi^M=\begin{cases}\dfrac{(1-\mu)\tau}{2(1-2\mu)} & \text{when } \mu<\mu^* \\ \dfrac{1-\frac{\tau}{2}}{2} & \text{when } \mu\geq\mu^*.\end{cases}$$

[17] The proof that it is unprofitable to charge less is more standard and is omitted for brevity. The indifferent consumer satisfies $1-p_A-\tau x=1-p_B-\tau(1-x)$ and one can use $\mu\geq\mu^*$ to show that the deviation profit is increasing in p_A until $p_A=1-\frac{\tau}{2}$.

As a share of the monopoly profit the equilibrium profit with Hotelling competition is continuous and equals

$$\delta(\mu)=\begin{cases}\dfrac{(1-\mu)\tau}{2(1-2\mu)(1-\tau)} & \text{when } \mu<\mu^* \\ \dfrac{1-\dfrac{\tau}{2}}{2(1-\tau)} & \text{when } \mu\geq\mu^*.\end{cases} \tag{12}$$

Figure 2 depicts $\delta(\mu)$ under Cournot competition (11) as well as Hotelling competition (12). Under Hotelling competition we highlight the role of the intensity of product market competition by distinguishing a low transportation cost parameter from that of a high transportation cost parameter ($\tau=0.15$ and $\tau=0.3$).



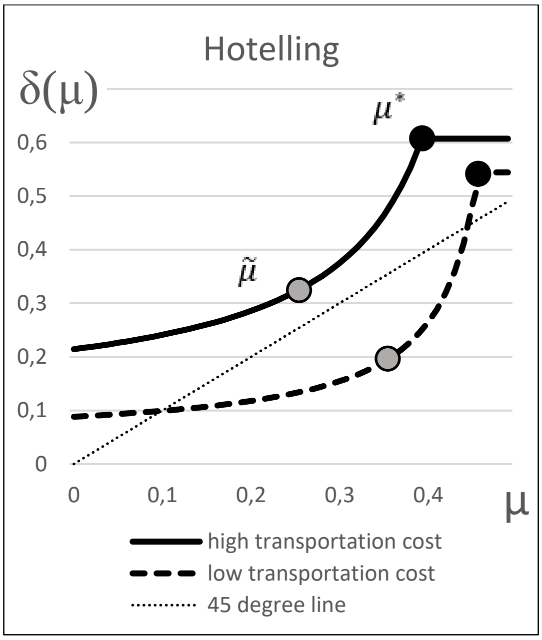


Figure 2 *The duopoly profit as a fraction of monopoly profit with Cournot and Hotelling competition.*

In the Hotelling model we define the threshold value $\tilde{\mu}\equiv\dfrac{(2-3\tau)-\sqrt{\tau(1-\tau)}}{4-5\tau}$. It can be shown algebraically that this value satisfies $0<\tilde{\mu}<\mu^*$. We show in our next Result that the threshold $\tilde{\mu}$ denotes the degree of overlapping ownership at which investments in the symmetric equilibrium shift from a decreasing to an increasing function of μ. As illustrated in the right-hand panel in Figure 2, this shift takes place when the competition-softening effect of μ

becomes sufficiently strong. The degree of overlapping ownership μ^* is the threshold above which $\delta(\mu)$ is constant and determined by the incentives for firms to fully cover the market.

The following result, proven in the Appendix, formally characterizes the effect of overlapping ownership on investments in the symmetric equilibrium.

Result 4 *A higher degree of overlapping ownership*

(a) decreases symmetric equilibrium investments with Cournot competition,

(b) decreases symmetric equilibrium investments with Hotelling competition when $\mu < \tilde{\mu}$, *increases them when* $\tilde{\mu} < \mu < \mu^*$, *and decreases them when* $\mu^* < \mu$.

From Result 4 we can see that the qualitative nature of the effect of overlapping ownership on investments depends on the mode of product market competition. With Cournot competition, overlapping ownership does not substantially relax product market competition. Thus, with Cournot competition the symmetric equilibrium has the feature that a higher degree of overlapping ownership monotonically induces a reduction of the investments in product innovation. In contrast, with Hotelling competition, the effect is non-monotonic. More precisely, with Hotelling competition low degrees of overlapping ownership also reduce investment volumes, but for an intermediate interval of overlapping ownership shares the relationship is reversed. In Figure 2 this feature is illustrated by the intermediate interval with fast growth for the curve $\delta(\mu)$, meaning strong competition-softening effects of overlapping ownership. For sufficiently high degrees of overlapping ownership the investment volumes are again a declining function of overlapping ownership, as reflected by the flat segment of the curve $\delta(\mu)$ in Figure 2 with Hotelling competition.

It is interesting to relate our results to Bindal's (2019) empirical study of the effects of common ownership. She finds that firms with more homogenous products report higher price increases and lower R&D investments in response to an increase in common ownership in comparison to firms with more differentiated products. An interpretation of these empirical findings in light of our analysis is that, with homogenous products, the competition-softening effect of common ownership does not seem to be sufficiently strong so as to induce a positive relationship between common ownership and R&D investments in the dataset considered. More generally, these empirical findings are consistent with our analysis showing that the effects of overlapping ownership depend qualitatively on the mode of product market competition.

Independently of whether we focus on Cournot or Hotelling competition we can draw the conclusion that, if the degree of overlapping ownership is sufficiently low, the relaxation of product market competition induced by overlapping ownership is not sufficiently strong to overturn condition (7). Under such circumstances the equilibrium investment decreases with overlapping ownership. This indicates that a higher degree of overlapping ownership reduces investments in product innovation and simultaneously softens product market competition for sufficiently low degrees of overlapping ownership. In this respect there is no Schumpeterian tradeoff for sufficiently low degrees of overlapping ownership.

Next, we analyse the condition for the symmetric investment equilibrium to be stable under a constant hazard rate for Cournot as well as Hotelling competition. Recall that the associated condition for stability, given by (10), is $\delta(\mu) > \mu$. We conduct a formal analytical analysis for both modes of competition and make use of Figure 2 for illustrative purposes.

(1) Cournot competition in a homogeneous industry

With Cournot competition, we have that $\delta(\mu)$, represented by (11), is independent of model parameters a and c. In that respect the function depicted in Figure 2 is general. In order to show universal stability we establish that $\delta(\mu) > \mu$ over the whole range of feasible values of μ by making use of (11). Based on straightforward differentiation it is established that $\delta'(\mu) = \dfrac{4-8\mu}{(3-2\mu)^3} > 0$ and $\delta''(\mu) < 0$. These properties combined with the observations that $\delta(0) = \dfrac{4}{9}$ and $\delta\left(\dfrac{1}{2}\right) = \dfrac{1}{2}$ show that $\delta(\mu) > \mu$ for $\mu \in \left[0, \dfrac{1}{2}\right[$, implying that the symmetric investment equilibrium is always stable with Cournot competition.

(2) Hotelling competition in a differentiated industry

With Hotelling competition, Figure 2 indicates that the stability of the symmetric equilibrium depends on the intensity of product market competition. Formally, differentiation of (12) shows that

$$\left.\frac{\partial\delta(\mu)}{\partial\tau}\right|_{\mu<\mu^*}=\frac{1-\mu}{2(1-2\mu)(1-\tau)^2}>0 \text{ and } \left.\frac{\partial\delta(\mu)}{\partial\tau}\right|_{\mu\geq\mu^*}=\frac{1}{4(1-\tau)^2}>0. \qquad (13)$$

This means that $\partial\delta(\mu)/\partial\tau>0$ independently of whether $\mu<\mu^*$ or $\mu\geq\mu^*$. Softer competition in the product market (higher τ) thus makes it more likely that the symmetric equilibrium is always stable. However, as Figure 2 illustrates, for sufficiently low values of τ there is an intermediate interval of degrees of overlapping ownership such that the symmetric equilibrium is unstable.

When $\mu<\mu^*$, by (10), (12) and (13), the threshold $\tilde{\tau}$ above which the symmetric investment equilibrium is stable is determined by the equation $\delta(\mu)=\mu$, which is equivalent to $\frac{(1-\mu)\tilde{\tau}}{2(1-2\mu)(1-\tilde{\tau})}=\mu$. This means that the condition for stability is determined by the inequality $4(1-\tau)\mu^2-(2-\tau)\mu+\tau>0$ which is satisfied whenever the discriminant characterizing the root of the associated quadratic equation is negative. The discriminant is negative whenever $(2-\tau)^2-16\tau(1-\tau)<0$, which is equivalent to $\tau>\tilde{\tau}=\frac{1}{17}\left(10-\sqrt{32}\right)\approx 0,256$.

When $\mu\geq\mu^*$, we see that $\delta(\mu)$ is independent of μ. Since $\delta(\mu)>1/2$ for $\mu\geq\mu^*$ (by (12)) we know that the symmetric equilibrium is stable whenever $\mu\geq\mu^*$. Notice also that, as $\partial\mu^*/\partial\tau<0$ we can infer that we enter the region with a flat curve $\delta(\mu)$ sooner in response to an increase in τ.

We report these findings characterising the stability of the symmetric investment equilibrium in the following result.

Result 5 *With a constant hazard rate*

(a) the symmetric investment equilibrium is always stable with Cournot competition,

(b) the symmetric investment equilibrium is always stable whenever product market competition is sufficiently soft with Hotelling competition, more precisely when $\tau > \tilde{\tau} = \frac{1}{17}\left(10 - \sqrt{32}\right) \approx 0,256$.

Our analysis of Cournot and Hotelling competition supports the general conclusion that a sufficiently low intensity of duopoly competition in the product market competition induces strategic stability with respect to the investment decisions.

We conclude this section by illustrating how overlapping ownership affects total welfare. To simplify notation, we define $z^* \equiv e^{-\gamma x^*}$. The success probability in the symmetric equilibrium can then generally be written as $f\left(x^*\right) = 1 - z^*$ such that that $f'\left(x^*\right) = \gamma z^*$. Consequently, the first-order condition (4) can be simplified as $\gamma z^* \pi^M \left[1 - \mu - \left(1 - z^*\right)\left(1 - \delta(\mu)\right)\right] - \left(1 - \mu\right) = 0$, or $\left(z^*\right)^2 - 2Az^* - B = 0$, whereby $A \equiv \frac{\mu - \delta\left(\mu\right)}{2\left(1 - \delta\left(\mu\right)\right)}$ and $B \equiv \frac{1 - \mu}{\gamma \pi^M (1 - \delta\left(\mu\right))}$. This is a quadratic equation. Since $z^* > 0$ its solution is $z^* = A + \sqrt{A^2 + B}$, and the associated equilibrium investment equals $x^* = \frac{-\ln\left(z^*\right)}{\gamma}$.

Using these calculations in combination with (8), we illustrate that overlapping ownership can raise total welfare by raising investments. We focus on our numerical configuration with Hotelling competition. With Hotelling competition, the competition-softening effect of overlapping ownership in the product market results in a welfare transfer from consumers to producers, but it does not generate any static welfare loss. This means that the total welfare effect of overlapping ownership is driven entirely by its effect on investments.

We specify the values for the transportation cost parameter τ and the hazard rate γ as follows. First, we select our setup with a high transportation cost $\tau = 0.3$, which has the feature that the symmetric equilibrium is always stable.[18] Second, we set the hazard rate not too high, equal to $\gamma = 3$. We interpret the limited value of the hazard rate as reflecting that product innovation is sufficiently complex, which restricts the probability that a successful innovation merely duplicates the success of the rival. This feature induces the property that an increase in investments is beneficial from a total welfare point of view. It holds generally whenever the equilibrium probability of success ($f\left(x^*\right)$) is not too high. Nevertheless, we note that there exist values for γ such that investments may be excessive from the perspective of total welfare. In that case an increase in investments induced by overlapping ownership can decrease total welfare. Figure 3 displays the relationship between overlapping ownership and total welfare for our selected numerical parameter combination.

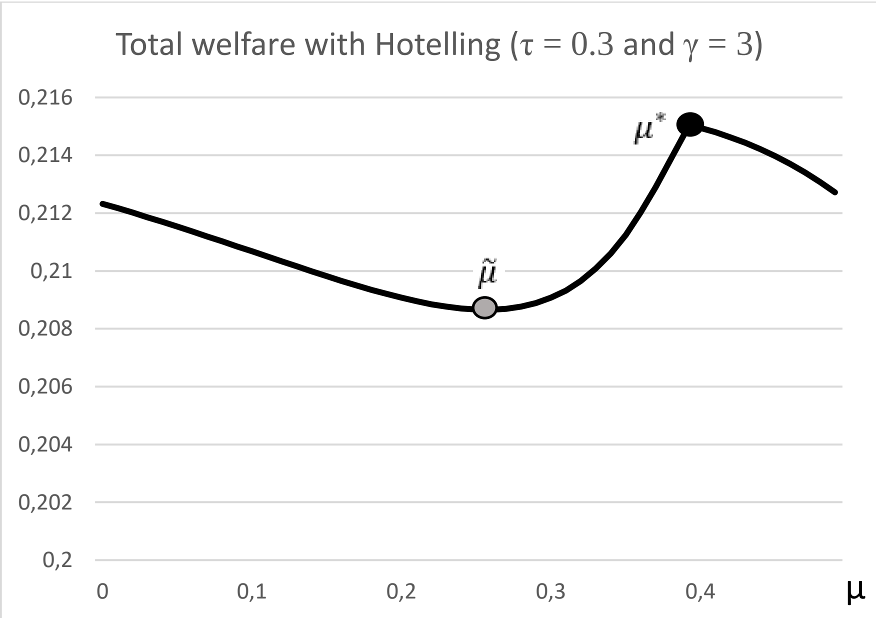


Figure 3 *Total welfare with Hotelling competition when the transportation cost is high.*

In the numerical simulation illustrated in Figure 3, when $\mu < \tilde{\mu}$, investments decline as a function of overlapping ownership, thereby reducing total welfare. For higher degrees of overlapping ownership such that $\tilde{\mu} < \mu < \mu^*$, raising overlapping ownership relaxes product market competition to such an extent that it stimulates investments. The increase in investments delivers benefits to consumers that outweigh the associated losses to the firms, thereby raising

[18] In section 4 we will display total welfare when the transportation cost is low, in which case there is an intermediate interval of overlapping ownership with an asymmetric investment equilibrium.

total welfare. Total welfare reaches a global maximum when $\mu = \mu^*$. When $\mu > \mu^*$, an increase in overlapping ownership does no more relax product market competition, thereby again reducing investments and total welfare.

In light of our welfare analysis we can draw the conclusion that, when the competition-softening effect is sufficiently strong and the product innovation is sufficiently complex, overlapping ownership can raise total welfare. This means that spillovers do not constitute a necessary condition for innovation-based total welfare gains from overlapping ownership.

4. Asymmetric Investment Equilibria

In this section we study configurations with asymmetric investment equilibria. We focus on corner solutions whereby one firm invests in innovation, whereas the other firm does not. Without loss of generality we denote the investing firm as firm i and the rival firm as j.

We start by characterizing the investment volume in a corner solution and the condition for a corner solution to be an equilibrium. When the rival firm j invests zero, its success probability is zero (by Assumption 1). Consequently, firm i's optimal investment, now denoted $\hat{x}$, is given by the necessary first-order condition (3). This simplifies to

$$f'(\hat{x})\pi^M = 1. \tag{14}$$

The left-hand side shows firm i's marginal benefit of investing in a corner solution. It equals the increased probability of success multiplied by the resulting monopoly profit. The right-hand side depicts the monetary cost of investing an additional unit. Notice that firm i's second-order condition is satisfied by Assumption 1.

We now investigate the condition for firm j to optimally invest zero when facing competition from firm i investing according to (14). From (3), the first derivative of firm j's objective function equals

$$f'(x_j)\pi^M G(\mu) - (1-\mu), \tag{15}$$

where we define the function $G(\mu) \equiv 1-\mu - f(\hat{x})(1-\delta(\mu))$. When $G(\mu) \le 0$, the derivative is negative, meaning that it is always optimal to invest zero. When $G(\mu) > 0$, notice that firm j's profit is concave with respect to its investment, by Assumption 1. Consequently, it is optimal to invest zero whenever

$$f'(0)\pi^M G(\mu) - (1-\mu) \le 0. \tag{16}$$

We can draw the following conclusion.

Result 6 *Assume that condition (16) holds.*

(a) The corner solutions, with one firm investing according to (14) and the other firm investing zero, are an equilibrium.

(b) In a corner solution, the investment volume $\hat{x}$ is invariant to the degree of overlapping ownership μ.

It follows that, within the segment that gives rise to a corner solution, a marginal change in the degree of overlapping ownership has no effect on consumer welfare or profits. Total welfare in a corner equilibrium equals $f(\hat{x})W^M - \hat{x}$ and is invariant to the degree of overlapping ownership.

It is worthwhile to point out that intensified product market competition (a lower value of $\delta(\mu)$), through its effect on $G(\mu)$, makes it more likely that condition (16) is satisfied. Intensified competition thus enhances the possibility of a corner equilibrium.

We next focus on the configuration with a constant hazard rate. Using that $f'(0) = \gamma$ and $f(\hat{x}) = 1 - \frac{1}{\gamma\pi^M}$ we can rewrite the condition for a corner equilibrium (16) according to

$$\gamma\pi^M \left(1-\mu-\left(1-\frac{1}{\gamma\pi^M}\right)(1-\delta(\mu))\right) - (1-\mu) \le 0,$$

which simplifies to $(\gamma\pi^M - 1)(1-\mu) - (\gamma\pi^M - 1)(1-\delta(\mu)) \le 0$. By Assumption 2, we can then express the condition for a corner solution to be an equilibrium as

$$\delta(\mu) \leq \mu .$$

This condition, together with Result 3, allows us to conclude that it is impossible for a corner solution to be an investment equilibrium under circumstances when the symmetric equilibrium is stable, i.e. (10) holds. Furthermore, when the symmetric equilibrium is unstable, the corner solutions are always an equilibrium. The following result reports these findings.

Result 7 *With a constant hazard rate, the corner solutions are an equilibrium if and only if the symmetric investment equilibrium is unstable (* $\delta(\mu) \leq \mu$ *).*

We next analyse the welfare effects of potential transformations between a stable symmetric equilibrium and a corner solution. By Results 3 and 7, with a constant hazard rate, such transformations occur at the fixed points $\delta(\mu) = \mu$. At the fixed points, the characterization of the symmetric investment level, given by (4), simplifies to

$$f'(x^*)\,\pi^M \left[1 - f(x^*)\right] \quad - \quad 1 \quad = \quad 0 . \tag{17}$$

In combination with (14), we obtain that, at the fixed points $\delta(\mu) = \mu$, the investments in the symmetric equilibrium and corner solution are related to each other as follows:

$$f'(x^*) \left[1 - f(x^*)\right] \quad - \quad f'(\hat{x}) \quad = \quad 0 .$$

With a constant hazard rate, we can rewrite the equality as $\gamma e^{-\gamma x^*} e^{-\gamma x^*} - \gamma e^{-\gamma \hat{x}} = 0$, or

$$2x^* = \hat{x} . \tag{18}$$

We thus obtain that potential transformations between a stable symmetric equilibrium and a corner solution do not affect the total investment volume. Furthermore, at the fixed points, the probability of success in a corner solution can be written, using (18), as

$$f(\hat{x}) = f(2x^*) = 1 - e^{-\gamma 2x^*} = 1 - e^{-\gamma x^*} e^{-\gamma x^*} = 1 - \left(1 - f(x^*)\right)\left(1 - f(x^*)\right), \tag{19}$$

and is therefore identical to the probability that at least one of the firms succeeds in the symmetric equilibrium. This invariance result can be understood based on the following intuitive explanation. There are two countervailing effects. On the one hand, the presence of diminishing returns to R&D indicates that it is efficient to distribute the given investment symmetrically across the two investment units. On the other hand, however, allocating all investments towards a single investment unit is beneficial as it avoids the probability of duplicated success. With a constant hazard rate, these two countervailing effects exactly offset each other.

We are now ready to analyse how a shift from the stable symmetric equilibrium to a corner equilibrium affects consumer welfare. As shown in subsection 3.2, such a shift can only occur when overlapping ownership does not substantially relax product market competition. The difference in consumer welfare can generally be characterized as

$$f(\hat{x})CS^M - \left[f(x^*) f(x^*) CS^D(\mu) + 2f(x^*)(1-f(x^*))CS^M \right],$$

rewritten as

$$\underbrace{f(\hat{x})CS^M - \left[1-(1-f(x^*))(1-f(x^*))\right]CS^M}_{=0} - f(x^*) f(x^*)\left(CS^D(\mu) - CS^M\right).$$

By (19) the first two terms cancel each other out. Consequently, a shift from the stable symmetric equilibrium to a corner equilibrium hurts (benefits) consumers if and only if consumer welfare in a duopoly market structure ($CS^D(\mu)$) exceeds consumers welfare in a monopoly market structure (CS^M).[19]

Next, we analyse how a shift from the stable symmetric equilibrium to a corner equilibrium affects the producing sector. The change in profits equals

$$f(\hat{x})\pi^M - \hat{x} - \left[f(x^*) f(x^*) 2\delta(\mu)\pi^M + 2f(x^*)(1-f(x^*))\pi^M - 2x^* \right],$$

which can be rewritten as

[19] This condition is not always satisfied. For example, with Hotelling competition, our analysis in subsection 3.3 has established that the monopoly price equals $1-\tau$. When $\mu \geq \mu^*$, the equilibrium prices in a duopoly market structure are higher and equal $1-\frac{\tau}{2}$. It can be verified that consumer welfare then only equals $\tau/4$, whereas it equals $\tau/2$ under monopoly.

$$\underbrace{-\hat{x}+2x^*}_{=0}+\underbrace{f(\hat{x})\pi^M-\left[1-\left(1-f\left(x^*\right)\right)\left(1-f\left(x^*\right)\right)\right]\pi^M}_{=0}-f\left(x^*\right)f\left(x^*\right)\left(2\delta(\mu)\pi^M-\pi^M\right)>0.$$

The first four terms cancel each other out by (18) and (19). Furthermore, at the fixed points, we have that $\delta(\mu)=\mu<1/2$. Consequently, a shift from the stable symmetric equilibrium to a corner equilibrium increases profits.

Adding the effects on consumers and producers, we find that the effect of a shift from the symmetric equilibrium to a corner equilibrium on total welfare is

$$-f\left(x^*\right)f\left(x^*\right)\left(CS^D(\mu)-CS^M\right)-f\left(x^*\right)f\left(x^*\right)\left(2\delta(\mu)\pi^M-\pi^M\right)\leq 0,$$

which is negative because $W^M-W^D(\mu)\leq 0$. We summarize these findings in the following result.

Result 8 *With a constant hazard rate, when overlapping ownership induces a shift from the stable symmetric equilibrium to a corner equilibrium, profits increase and total welfare decreases. In contrast, when overlapping ownership induces a shift from a corner equilibrium to a stable symmetric equilibrium, profits decrease and total welfare increases.*

By (18) and (19) a shift from the stable symmetric equilibrium to a corner equilibrium does not affect the total investment volume nor the probability that at least one of the firms succeeds in innovation. In this respect access to the new product is invariant to the type of investment equilibrium. Rather the effect of a shift to a corner equilibrium is to induce a monopoly in the product market also in situations where the symmetric investment equilibrium would generate a duopoly.

As shown in subsection 3.2, when overlapping ownership substantially relaxes product market competition, there can be a shift from a corner equilibrium to a stable symmetric equilibrium. By Result 8 such a shift raises total welfare at the expense of producers.

It should be emphasized that (18), (19) and Result 8 focus on an evaluation made at the degree of overlapping ownership where there is a transformation between the symmetric investment equilibrium and a corner solution. This evaluation essentially says that the shift in the nature of the investment equilibrium does not cause any discontinuity with respect to total investments or access to the new product. We remind that the effects of a higher degree of overlapping ownership within the segments that give rise to a stable symmetric equilibrium are reported by Result 1.

The probability that at least one of the firms succeeds in a corner equilibrium ($f(\hat{x})$) is always below that in a stable symmetric equilibrium $1-\left(1-f\left(x^{*}\right)\right)\left(1-f\left(x^{*}\right)\right)$. The reason is that, as investments in a symmetric equilibrium decline as a function of $\delta(\mu)$, they reach a lower bound at the fixed points where the symmetric equilibrium turns unstable ($\delta(\mu)=\mu$). Furthermore, (19) shows that that the probability that at least one of the firms succeeds in the symmetric equilibrium when $\delta(\mu)=\mu$ is equal to that in a corner equilibrium, which establishes the claim.

Finally, we illustrate the total welfare implications of overlapping ownership in a numerical example whereby overlapping ownership induces changes in the nature of the investment equilibrium. In order to enable comparisons, we focus on a combination of parameters identical to that associated with Figure 3 except that we now adjust the transportation cost so that it generates a segment with an asymmetric equilibrium. For that reason we select the transportation cost parameter $\tau=0.15$.

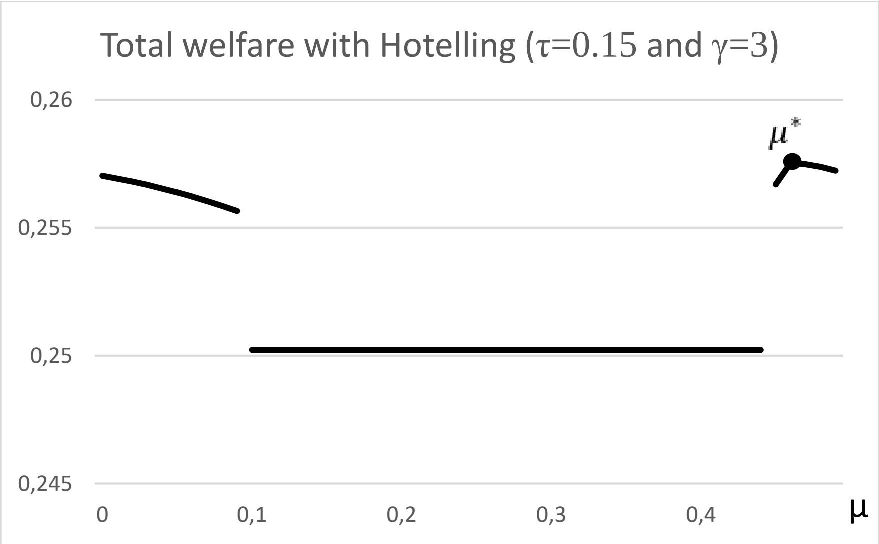


Figure 4 *Total welfare with Hotelling competition when the transportation cost is low.*

Figure 4 confirms Result 8 stating that a shift from the stable equilibrium to a corner equilibrium induces a drop in total welfare, whereas a shift back from a corner equilibrium to the symmetric equilibrium raises total welfare. We emphasize once more that a shift towards (away from) a corner equilibrium can only occur when the competition-softening effect of overlapping ownership is limited (substantial). On a general level, Figure 4 also confirms that the non-monotonicity of total welfare as a function of the degree of overlapping ownership is not a feature restricted to the symmetric equilibrium configuration. The non-monotonicity property can survive in configurations with shifts between symmetric and asymmetric equilibria.

5. Overlapping Ownership vs Mergers

Several recent studies[20] have analyzed the effects of mergers on investments in innovation. For example, Federico, Langus and Valletti (2018) apply simulations to demonstrate that a merger reduces innovation incentives in the absence of cost efficiencies and knowledge spillovers.[21] It could be tempting to conjecture that the investments with overlapping ownership are simply a weighted average between the investments without overlapping ownership ($\mu = 0$) and those under a merger. In this section we will investigate in greater detail whether such a conjecture holds true. We do so by characterizing the investments with a merger. We also explore whether the investments with overlapping ownership converge to those of a merged entity in the limit as $\mu \to 1/2$. This way we are able to compare the investments with overlapping ownership, which are characterized in the previous sections, against the investments without overlapping ownership and those with a merger.

To facilitate a clean comparison against a configuration with overlapping ownership, we focus on a merger which does not generate synergies at the R&D stage or in the product market. This means that there is no change in the R&D production functions of the research units (A and B), and that there is no change in marginal costs in the product market. Further, we continue to focus on the parametric specification for the R&D production function such that the hazard rate

[20] Federico, Langus and Valetti (2017, 2018) and Motta and Tarantino (2018) are examples of such studies.
[21] The model used for simulations in Federico, Langus and Valletti (2018) captures N firms which innovate to replace their old products.

is constant. In case of a single success, the merged entity earns π^M. When both innovations are successful, the profit in the product market is denoted by $\pi^M + \Delta$, whereby we define $\Delta \geq 0$ as the increase in product market profit from having two successful innovations instead of just one[22].

The merged entity decides about volumes of investment in each research unit (A and B): x_A and x_B. Formally, the optimization problem facing the merged entity is

$$\begin{aligned}&\max_{x_A,x_B}\left[f(x_A)\left(1-f(x_B)\right)+\left(1-f(x_A)\right)f(x_B)\right]\pi^M\\&+f(x_A)\,f(x_B)\left(\pi^M+\Delta\right)-\left(x_A+x_B\right)\end{aligned} \tag{20}$$

The first term captures the product market profit in case of a single successful innovation, whereas the second term denotes the profit when both innovations are successful. The final term represents the investment costs.

In the Appendix we prove the following result when there are no merger synergies and when the R&D production function is characterized by a constant hazard rate.

Result 9 *With a merger, investments are symmetric in the two research units such that* $x_A = x_B$.

We next characterize the optimal symmetric investment, which we denote by x^M, for the merged firm. With symmetric investments, the optimization problem (20) can be expressed according to

$$\max_{x^M} \quad f\left(x^M\right)\left(2-f\left(x^M\right)\right)\pi^M+\left[f\left(x^M\right)\right]^2\Delta-2x^M. \tag{21}$$

The first term equals the probability that at least one of the innovations succeeds, multiplied by the monopoly profit in the product market. The second term reflects the expected profit bonus when there are two successful innovations instead of just one.

The first-order condition for optimization is obtained by differentiating (21) and it can be formulated according to

[22] Our Hotelling model, for example, has the feature that $\Delta > 0$ because the two products innovations are horizontally differentiated.

$$f'\left(x^M\right)\left(1-f\left(x^M\right)\right)\pi^M + f'\left(x^M\right)f\left(x^M\right)\Delta - 1 = 0. \tag{22}$$

Condition (22) essentially requires that it is optimal to invest x^M in one of the product innovations, taking as given that the amount invested in the other innovation also equals x^M. Specifically, the first term in (22) represents the situation where the other innovation is unsuccessful, which happens with probability $1-f\left(x^M\right)$. Investing a higher amount then raises the probability that one of the innovations succeeds, in which case the merged entity earns π^M in the product market. The second term captures the situation where the other innovation succeeds, which happens with probability $f\left(x^M\right)$. In this case a second successful innovation makes it possible to earn an extra profit in the product market equal to Δ.

We are now ready to compare the investments of the merged entity to those in a configuration with overlapping ownership. The comparison is conducted with reference to our models with Cournot competition and Hotelling competition presented section 3.3.

(1) Cournot competition in a homogeneous industry

In the model of Cournot competition in a homogenous industry we have that $\Delta = 0$. The optimal investment of the merged entity, given by (22), can therefore be simplified as

$$f'\left(x^M\right)\left(1-f\left(x^M\right)\right)\pi^M - 1 = 0. \tag{23}$$

From the perspective of the merged entity, achieving success with one of the product innovations is only valuable when the other innovation is unsuccessful (which happens with probability $1-f\left(x^M\right)$). When instead the other innovation is successful, there is no point in achieving a second success, as the two product innovations are identical to each other.

With Cournot competition, the symmetric investment volumes decrease as a function of the degree of overlapping ownership, as we showed in Result 4(a). Further, we can also see that these investment volumes converge to those associated with a merger. Formally, from (11) we obtain that $\delta\left(\frac{1}{2}\right) = \frac{1}{2}$, from which we can conclude that (4) simplifies to (23). These observations imply that with Cournot competition the investments in product innovation are decreasing in the degree of overlapping ownership and reach a minimum when firms engage in a merger.

(2) Hotelling competition in a differentiated industry

With Hotelling competition, the product market profit when there is a single successful innovation equals $\pi^M = 1-\tau$. When there are two successful innovations, the profit of the merged entity in the product market equals $1-\frac{\tau}{2}$. This means that $\Delta = \tau/2$. The merged entity's optimal investment (22) can thus be written as

$$f'\left(x^M\right)\left(1-f\left(x^M\right)\right)\pi^M + f'\left(x^M\right) f\left(x^M\right)\tau/2 - 1 = 0. \qquad (24)$$

From (12), in this configuration we have that $\delta\left(\frac{1}{2}\right) = \frac{1-\frac{\tau}{2}}{2(1-\tau)}$. Substitution of $\mu = 1/2$ into (4) shows that the symmetric investments with overlapping ownership given by (4) can be written as

$$f'\left(x^*\right)\pi^M\left(\frac{1}{2} - f\left(x^*\right)\left(1-\frac{1-\frac{\tau}{2}}{2(1-\tau)}\right)\right) - \frac{1}{2} = 0\,. \qquad (25)$$

We can further simplify (25), using $\pi^M = 1-\tau$, to conclude that the symmetric investments with overlapping ownership converge to those associated with a merger, characterized by (24).

Finally, we observe from (4) and (12) that with Hotelling competition the investment levels under a merger, given by (24), are identical to those without overlapping ownership ($\mu = 0$). This means that there is no change in product innovation when there is a switch from a configuration without overlapping ownership to a merger. This finding is interesting in light of our analysis of overlapping ownership in sections 3 and 4. Specifically, as we have shown in Result 4(b), the symmetric equilibrium involves investment volumes that are nonmonotonic as a function of the degree of overlapping ownership. In addition, in section 4 we have also characterized circumstances such that overlapping ownership generates asymmetric investment equilibria – a feature which does not emerge with a merger (Result 9).

6. Concluding Comments

In this study we have analysed the effect of overlapping ownership on investments in product innovation. We have shown that this effect is the outcome of an interplay between two opposing forces: (1) Overlapping ownership induces firms to internalize that success on their own behalf erodes the rival's business, reducing investment incentives, and (2) overlapping ownership, by softening competition in the product market, enhances investment incentives. These two forces determine the effect of overlapping ownership in the stable symmetric investment equilibrium: overlapping ownership gives rise to an investment expansion if and only if the competition-softening effect is sufficiently strong. Furthermore, these two forces also determine potential shifts between the stable symmetric investment equilibrium and asymmetric equilibria whereby only one of the firms has a positive investment volume, whereas its rival invests zero. We have found that a strong competition-softening effect of overlapping ownership makes the investment stage less prone to having such asymmetric equilibria and enhances the possibility that the stability condition of the symmetric equilibrium is satisfied. Finally, we compared the investments in a configuration of overlapping ownership with those under a merger. We demonstrated that overlapping ownership generates a rich pattern of equilibrium investments that can differ significantly from a weighted average of the investments with no overlapping ownership and those with a merger.

Overall, we have characterized the effect of overlapping ownership on investments for a given number of firms actively pursuing product innovation (intensive margin) as well as the effect on this number of firms actively investing (extensive margin). Along both of these dimensions our welfare analysis has revealed that the competition-softening effect can give scope for overlapping ownership to raise total welfare. This is an interesting conclusion because it means that spillovers associated with process innovation, which are the source of the potential welfare gains from overlapping ownership in López and Vives (2019), do not constitute a necessary condition for overlapping ownership to yield welfare benefits. We have exemplified our general analysis with Cournot and Hotelling competition as explicit modes of competition.

A promising direction for future research is to generalize our model to capture R&D-spillovers. Since the success of product innovation is uncertain, the character of the spillovers plays a central role for evaluating the effects of overlapping ownership on R&D investments and welfare. For example, whereas "R&D input spillovers" in the innovation production process enhance the probability that at least one firms succeeds, “R&D output spillovers”, which make it possible for firms to exploit a successful innovation of the rival, can only induce a monopoly market structure to turn into a duopoly.[23] Our model with a general mode of competition in the product market can serve as a basis for investigating the role of these different types of spillovers.

From the perspective of innovation policy, it would be valuable to systematically evaluate how overlapping ownership compares with various forms of R&D collaboration. R&D collaboration is encouraged as a central element of industrial policy consistent with competition laws in jurisdictions such as the European Union and the United States. However, there are concerns that R&D collaboration could facilitate collusion also in the product market (see for example Duso, Röller and Seldeslachts (2014)). It would be interesting for future research to study whether overlapping ownership is less prone than R&D collaboration to induce product market collusion.

References

Antón, Miguel, Florian Ederer, Mireia Giné, and Martin C. Schmalz. 2021a. “Innovation: The Bright Side of Common Ownership?” Available at SSRN 3099578.

Antón, Miguel, Florian Ederer, Mireia Giné, and Martin C. Schmalz. 2021b. “Common Ownership, Competition, and Top Management Incentives”. Available at SSRN 2802332.

Azar, José. 2016. “Portfolio Diversification, Market Power, and the Theory of the Firm.” Available at SSRN 2811221.

Azar, José, Sahil Raina, and Martin Schmalz. 2021. “Ultimate Ownership and Bank Competition.” *Financial Management:* doi/epdf/10.1111/fima.12368.

Azar, José, Martin Schmalz, and Isabel Tecu. 2018. “Anti-competitive Effects of Common Ownership.” *Journal of Finance* 73: 1513-1565.

[23] See Hauenschild (2003) for a presentation of the distinction between R&D input spillovers and R&D output spillovers.

Backus, Matthew, Christopher Conlon, and Michael Sinkinson. 2021. "Common ownership in America: 1980–2017." *American Economic Journal: Microeconomics 13.3*: 273-308.

Banal-Estañol, Albert, Jo Seldeslachts, and Xavier Vives. 2020. “Diversification, common ownership, and strategic incentives.” *AEA Papers and Proceedings* 110: 561-64.

Bayona, Anna and Ángel López. 2018. “Silent financial interests and product innovation.” *Economics Letters* 170: 109-112.

Bindal, Shradha. 2019. “When Does Common Ownership Matter?” University of Kansas, Working Paper.

Branstetter, Lee, Chirantan Chatterjee, and Matthew J. Higgins. 2016. “Regulation and welfare: evidence from paragraph IV generic entry in the pharmaceutical industry.” *The RAND Journal of Economics* 47: 857-890.

d’Aspremont, Claude and Alexis Jacquemin. 1988. “Cooperative and Noncooperative R&D in Duopoly with Spillovers.” *American Economic Review* 78: 1133-1137.

Dietzenbacher, Erik, Bert Smid, and Bjørn Volkerink. 2000. “Horizontal integration in the Dutch financial sector.” *International Journal of Industrial Organization* 18: 1223-1242.

Duso, Tomaso, Lars-Hendrik Röller, and Jo Seldeslachts. 2014. "Collusion through joint R&D: An empirical assessment." *Review of Economics and Statistics 96.2*: 349-370.

Farrell, Joseph and Carl Shapiro. 1990. “Asset ownership and market structure in oligopoly.” *The RAND Journal of Economics* 21: 275-292.

Federico, Giulio, Gregor Langus, and Tommaso Valletti. 2017. “A Simple Model of Mergers and Innovation.” *Economics Letters* 157: 136-140.

Federico, Giulio, Gregor Langus, and Tommaso Valletti. 2018. “Reprint of: Horizontal Mergers and Product Innovation.” *International Journal of Industrial Organization* 61: 590-612.

Gutiérrez, Germán and Thomas Philippon. 2017a. “Investmentless Growth: An Empirical Investigation.” *Brookings Papers on Economic Activity Fall* 2017: 89-169.

Gutiérrez, Germán and Thomas Philippon. 2017b. “Declining Competition and Investment in the U.S..” *NBER Working Paper* No. 23583.

Hauenschild, Nils. 2003. “On the Role of Input and Output Spillovers when R&D Projects are Risky.” *International Journal of Industrial Organization* 21: 1065-1089.

He, Jie and Jiekun Huang. 2017. “Product Market Competition in a World of Cross-Ownership: Evidence from Institutional Blockholdings.” *Review of Financial Studies* 30: 2674-2718.

Heim, Sven, Kai Hüschelrath, Ulrich Laitenberger, and Yossi Spiegel. 2019. “The Anticompetitive Effect of Minority Share Acquisitions: Evidence from the Introduction of National Leniency Programs.” *American Economic Journal: Microeconomics 14:* 366-410.

Kamien, Morton, Eitan Muller, and Israel Zang. 1992. “Research Joint Ventures and R&D Cartels.” *American Economic Review* 82: 1293-1306.

Li, Sanxi, Hongkun Ma and Chenhang Zeng. 2015. “Passive Cross Holding as a Strategic Entry Deterrence.” *Economics Letters* 134: 37-40.

López, Ángel and Xavier Vives. 2019. “Overlapping Ownership, R&D Spillovers, and Antitrust Policy.” *Journal of Political Economy* 127: 2394-2437.

Motta, Massimo and Emanuele Tarantino. 2021. “The effect of horizontal mergers, when firms compete in prices and investments.” *International Journal of Industrial Organization* 78*:* 102774.

Nain, Amrita, and Yan Wang. 2016. “The Product Market Impact of Minority Stake Acquisitions.” *Management Science* 64.2: 825-844.

Newham, Melissa, Jo Seldeslachts, and Albert Banal-Estañol. 2018. “Common Ownership and Market Entry: Evidence from the Pharmaceutical Industry.” University of Pompeu Fabra, Department of Economics Working Paper No. 1612.

O’Brien, Daniel and Steven Salop. 2000. “Competitive Effects of Partial Ownership: Financial Interest and Corporate Control.” *Antitrust Law Journal* 67: 559-614.

OECD. 2017. “Common Ownership by Institutional Investors and its Impact on Competition.” Available from https://www.oecd.org/daf/competition/common-ownership-and-its-impact-on-competition.htm

Reynolds, Robert J. and Bruce R. Snapp. 1986. "The Competitive Effects of Partial Equity Interests and Joint Ventures." *International Journal of Industrial Organization* 4: 141-153.

Schmalz, Martin. 2018. “Common-Ownership Concentration and Corporate Conduct.” *Annual Review of Financial Economics* 10: 413-448.

Seldeslachts, Jo, Melissa Newham, and Albert Banal-Estanol. 2017. “Changes in Common Ownership of German Companies.” *DIW Economic Bulletin* 7: 303-311.

Shelegia, Sandro and Yossi Spiegel. 2015. “Cost-Reducing Investments under Partial Cross-Ownership.” Working Paper, Tel Aviv University.

Vives, Xavier. 2020. “Common Ownership, Market Power, and Innovation.” *International Journal of Industrial Organization* 70: 102528.

Zormpas, Dimitrios and Richard Ruble. 2021. “The Dynamics of Preemptive and Follower Investments with Overlapping Ownership.” *Journal of Economic Dynamics & Control* 129: 104175.

Appendix

Proof of property (6)

We introduce the following notation to denote second-order derivatives evaluated at equilibrium investments:

$$\partial_{x_i,x_i}\Gamma_i \equiv \left.\frac{\partial^2\Gamma_i}{\partial x_i^2}\right|_{\substack{x_i=x_i^* \\ x_j=x_j^*}},\ \partial_{x_i,x_j}\Gamma_i \equiv \left.\frac{\partial^2\Gamma_i}{\partial x_i\partial x_j}\right|_{\substack{x_i=x_i^* \\ x_j=x_j^*}},\ \partial_{x_i,\mu}\Gamma_i \equiv \left.\frac{\partial^2\Gamma_i}{\partial x_i\partial \mu}\right|_{\substack{x_i=x_i^* \\ x_j=x_j^*}}.$$

The effect of overlapping ownership (μ) on investment is obtained by totally differentiating the first-order conditions (3) for firms A and B. The system of equations becomes

$$\partial_{x_A,\mu}\Gamma_A + \frac{dx_A^*}{d\mu}\partial_{x_A,x_A}\Gamma_A + \frac{dx_B^*}{d\mu}\partial_{x_A,x_B}\Gamma_A = 0$$
$$\partial_{x_B,\mu}\Gamma_B + \frac{dx_B^*}{d\mu}\partial_{x_B,x_B}\Gamma_B + \frac{dx_A^*}{d\mu}\partial_{x_B,x_A}\Gamma_B = 0.$$

Solving for $\frac{dx_A^*}{d\mu}$ and $\frac{dx_B^*}{d\mu}$ we obtain that

$$\frac{dx_A^*}{d\mu} = \frac{\partial_{x_A,x_B}\Gamma_A \partial_{x_B,\mu}\Gamma_B - \partial_{x_B,x_B}\Gamma_B \partial_{x_A,\mu}\Gamma_A}{\partial_{x_A,x_A}\Gamma_A \partial_{x_B,x_B}\Gamma_B - \partial_{x_A,x_B}\Gamma_A \partial_{x_B,x_A}\Gamma_B}$$

and

$$\frac{dx_B^*}{d\mu} = \frac{\partial_{x_B,x_A}\Gamma_B \partial_{x_A,\mu}\Gamma_A - \partial_{x_A,x_A}\Gamma_A \partial_{x_B,\mu}\Gamma_B}{\partial_{x_B,x_B}\Gamma_B \partial_{x_A,x_A}\Gamma_A - \partial_{x_B,x_A}\Gamma_B \partial_{x_A,x_B}\Gamma_A}.$$

Since firms invest symmetrically, we can substitute in that

$$\begin{aligned}
&\partial_{x_A,\mu}\Gamma_A = \partial_{x_B,\mu}\Gamma_B = \partial_{x_i,\mu}\Gamma_i \\
&\partial_{x_A,x_B}\Gamma_A = \partial_{x_B,x_A}\Gamma_B = \partial_{x_i,x_j}\Gamma_i \\
&\partial_{x_A,x_A}\Gamma_A = \partial_{x_B,x_B}\Gamma_B = \partial_{x_i,x_i}\Gamma_i \\
&\frac{dx_A^*}{d\mu} = \frac{dx_B^*}{d\mu} = \frac{dx^*}{d\mu}
\end{aligned}$$

to obtain

$$\frac{dx^*}{d\mu} = \frac{\partial_{x_i,\mu}\Gamma_i \left(\partial_{x_i,x_j}\Gamma_i - \partial_{x_i,x_i}\Gamma_i\right)}{\partial_{x_i,x_i}\Gamma_i \partial_{x_i,x_i}\Gamma_i - \partial_{x_i,x_j}\Gamma_i \partial_{x_i,x_j}\Gamma_i},$$

or

$$\frac{dx^*}{d\mu} = \frac{\partial_{x_i,\mu}\Gamma_i \left(\partial_{x_i,x_j}\Gamma_i - \partial_{x_i,x_i}\Gamma_i\right)}{\left(\partial_{x_i,x_j}\Gamma_i - \partial_{x_i,x_i}\Gamma_i\right)\left(-\partial_{x_i,x_j}\Gamma_i - \partial_{x_i,x_i}\Gamma_i\right)} = \frac{\partial_{x_i,\mu}\Gamma_i}{-\partial_{x_i,x_j}\Gamma_i - \partial_{x_i,x_i}\Gamma_i},$$

which can be written as (6). ■

Proof of Result 4

(1) Cournot competition in a homogeneous industry

Straightforward differentiation of (11) shows that

$$\delta'(\mu) = \frac{4-8\mu}{(3-2\mu)^3}.$$

This means that condition (7) is equivalent to

$$\frac{4-8\mu}{(3-2\mu)\left[(3-2\mu)^2-4(1-\mu)\right]}<\frac{1}{1-\mu}$$

for Cournot competition. This condition, in its turn, is equivalent to the inequality

$$8\mu^3-20\mu^2+22\mu-11<0\,.$$

This inequality always holds, because it can be decomposed as follows:

$$8\underbrace{\left(\mu^3-\mu^2\right)}_{<0}-12\mu^2+11\underbrace{(2\mu-1)}_{<0}<0\,.$$

(2) Hotelling competition in a differentiated industry

In light of (7) we investigate the condition $\frac{\delta'(\mu)}{1-\delta(\mu)}<\frac{1}{1-\mu}$. Given the analytical expression for $\delta(\mu)$ given by (12), we separate the case $\mu<\mu^*$ from that of $\mu\geq\mu^*$.

First, for $\mu<\mu^*$, in light of (12) the condition $\frac{\delta'(\mu)}{1-\delta(\mu)}<\frac{1}{1-\mu}$ is equivalent to

$$\frac{\tau}{(1-2\mu)\left[(2-3\tau)-(4-5\tau)\mu\right]} \quad < \quad \frac{1}{1-\mu}\,.$$

The left-hand side is positive when $(2-3\tau)-(4-5\tau)\mu>0$, a condition we can rewrite as $\mu<\frac{2-3\tau}{4-5\tau}$. Since $\mu<\mu^*$ this condition is always met. Consequently, we can rewrite $\frac{\tau}{(1-2\mu)\left[(2-3\tau)-(4-5\tau)\mu\right]} \quad < \quad \frac{1}{1-\mu}$ as the quadratic inequality

$$\left(8-10\tau\right)\mu^2-\left(8-12\tau\right)\mu+2(1-2\tau)>0\,. \qquad \text{(A1)}$$

The associated equation $\left(8-10\tau\right)\mu^2-\left(8-12\tau\right)\mu+2(1-2\tau)=0$ has two solutions: $\mu=\tilde{\mu}\equiv\frac{(2-3\tau)-\sqrt{\tau(1-\tau)}}{4-5\tau}$ and $\mu=\tilde{\tilde{\mu}}\equiv\frac{(2-3\tau)+\sqrt{\tau(1-\tau)}}{4-5\tau}$. It can be shown algebraically, using $0<\tau<1/2$, that $\tilde{\mu}<\mu^*<\tilde{\tilde{\mu}}$. This means that the feasible solution to the inequality (A1) is $\mu<\tilde{\mu}$. This, in turn, means that the symmetric investment is decreasing for $\mu\in\left[0,\tilde{\mu}\right[$, whereas it is increasing for $\mu\in\left]\tilde{\mu},\mu^*\right[$.

Second, for $\mu \geq \mu^*$ we see that $\delta'(\mu) = 0$, from which we can directly conclude that (7) holds, meaning that the investment is decreasing in μ. ■

Proof of Result 9

We denote the average investment level by $x^M = \frac{x_A + x_B}{2}$. We can therefore express x_B as $2x^M - x_A$. For a given value of x^M, the profit of the merged entity equals

$$\begin{aligned}&\left(1 - f(x_A)\right) f\left(2x^M - x_A\right)\pi^M + f(x_A)\left(1 - f\left(2x^M - x_A\right)\right)\pi^M \\ &+ f(x_A)\, f\left(2x^M - x_A\right)\left(\pi^M + \Delta\right) - 2x^M \quad ,\end{aligned} \tag{A2}$$

which can simplified according to

$$\begin{aligned}&\left[f\left(2x^M - x_A\right) + f(x_A) - f(x_A)\, f\left(2x^M - x_A\right)\right]\pi^M \\ &+ f(x_A)\, f\left(2x^M - x_A\right)\Delta - 2x^M .\end{aligned} \tag{A3}$$

Treating the average investment volume x^M as given, we can analyze the incentive for the merged entity to redistribute investments away from x_B and towards x_A by differentiating (A3) with respect to x_A. The first-order derivative equals

$$\begin{aligned}&\left[-f'\left(2x^M - x_A\right) + f'(x_A) - f'(x_A)\, f\left(2x^M - x_A\right) + f(x_A)\, f'\left(2x^M - x_A\right)\right]\pi^M \\ &+ \left[f'(x_A)\, f\left(2x^M - x_A\right) - f(x_A)\, f'\left(2x^M - x_A\right)\right]\Delta.\end{aligned} \tag{A4}$$

Since the R&D production function is characterized by a constant hazard rate, we know that $f'(x_A) = \gamma\left[1 - f(x_A)\right]$ and $f'\left(2x^M - x_A\right) = \gamma\left[1 - f\left(2x^M - x_A\right)\right]$. Substituting these expressions into (A4) and simplifying yields:

$$\gamma\left[\left[1 - f(x_A)\right] f\left(2x^M - x_A\right) - f(x_A)\left[1 - f\left(2x^M - x_A\right)\right]\right]\Delta . \tag{A5}$$

When $\Delta = 0$, the product market profit of the merged entity is invariant to whether there is one successful innovation or two successful innovations. This means that the product innovations replicate each other in the sense that each of them generates the ability to produce the same homogenous product. Our explicit mode of competition à la Cournot exemplifies this feature.

By (A5), when $\Delta = 0$, the profit of the merged entity does not depend on how a given total investment is distributed between x_A and x_B. To simplify our exposition, we assume that in this case the merged entity breaks the tie by investing symmetrically such that $x_A = x_B$.

We next demonstrate that, when $\Delta > 0$, it is strictly optimal for the merged entity to invest symmetrically. First, notice that the corner investment whereby $x_A = 0$ cannot be optimal, as then (A5) would be positive, meaning that there would be an incentive to redistribute investments away from x_B and towards x_A. Second, an analogous observation rules out the corner investment whereby $x_A = 2x^M$. These features imply that the optimal distribution of investments must be interior and that at the optimum the first-order derivative (A5) equals zero. Formally, we can rewrite this condition as

$$\frac{f\left(2x^M - x_A\right)}{1 - f\left(2x^M - x_A\right)} = \frac{f\left(x_A\right)}{1 - f\left(x_A\right)} \ , \tag{A6}$$

which enables us to conclude that the merged entity invests symmetrically. ■