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Social Security Design and its Political Support*

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Abstract

The design of social security systems has implications not only for households' saving and labor supply choices, but also for the political support of intergenerational transfers. We examine the effects of making pension benefits dependent on —or independent of— labor market participation, as well as the level of redistribution, on the social security tax rate, labor supply, and capital accumulation. We conduct two numerical evaluations of the model's performance. First, it can explain almost two thirds of the observed increase in pension spending following Argentina's 2005-2010 reforms aimed at universalizing coverage. Second, the model predicts that a persistent shift in work preferences following the COVID-19 pandemic in the U.S. would result in a 1.8 p.p. increase in the social security tax rate.

JEL Classification: D72, E62, H55, J46, O17

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1 Introduction

Pensions account for a large share of public sector expenditures in developed and developing countries. In 2019, the average spending among OECD countries was roughly

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8% of their GDP, while in emerging markets and middle income economies, it was close to 4%.[1] The design of pension systems holds substantial impact on the choices workers make regarding savings and labor supply, given the magnitude of transfers. The easier it is to access pensions and the more generous the benefits, the less workers tend to save and supply labor. Labor supply also varies based on the correlation between benefits and income.[2]

Positive theories of social security explain the extent of intergenerational transfers as the result of a political process that aggregates society's preferences for transfers. Retirees receive pensions, while workers finance these transfers. The design of social security affects the perceived costs and benefits of pensions, which in turn affects the size of transfers. This study examines the impact of two aspects of social security systems on its politico-economic equilibrium. First, the distinction between contributory and universal systems, where workers must meet contribution criteria to receive full pensions in the former, but automatically qualify in the latter upon reaching retirement age. Second, whether benefits are linked to earnings or flat, which we refer to as a 'Bismarckian' system and a 'Beveridgean' system, respectively.

We build on a standard overlapping generations setup with capital formation a tractable model to analyze the effects that pension system characteristics have on the equilibrium tax rate, labor supply, and capital accumulation. Households in the model, acting as economic agents, make choices regarding consumption, savings, and labor supply based on given prices, taxes, and retirement benefits. As voters, they choose among parties offering policies that consist of labor income taxes and retirement benefits. Since policy preferences vary between young and old voters, we model the resolution of this conflict through probabilistic voting. The political process lacks commitment, and elections take place every period.

Policy decisions not only impact economic outcomes, but also indirectly affect future policy decisions in the absence of commitment. Voters take into account these indirect effects, which are reflected in the equilibrium relationship between state variables and future policy choices. We focus on Markov perfect equilibria and assume that only fundamental state variables enter this equilibrium relationship.

Households decide how much time to allocate between a formal labor market and home production, or informal labor market.[3] Labor supplied in the formal market is more

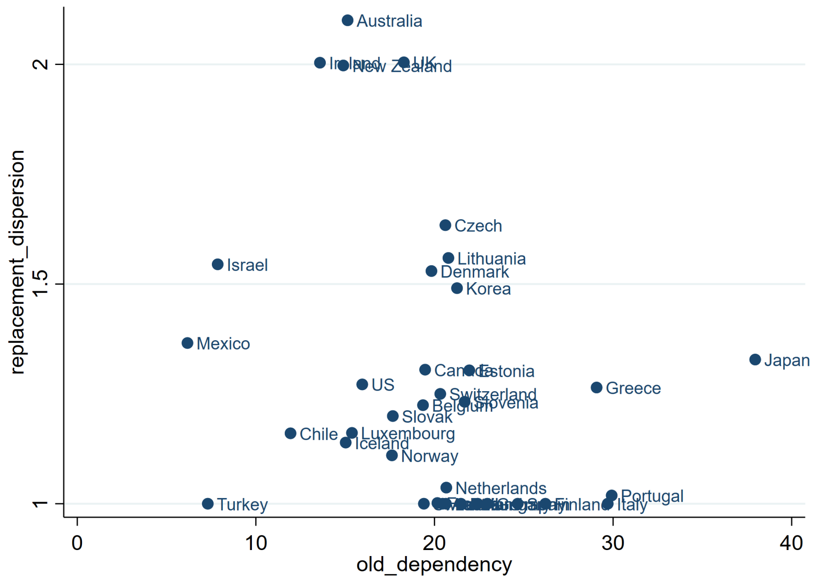
[1]There is significant heterogeneity in spending, even among OECD countries. Spending is around 2% of GDP in Ireland and Mexico, and more than 16% in Italy and Greece. See OECD (2019).

[2]See e.g. Attanasio and Rohwedder (2003) and Galiani, Gertler and Bando (2016).

[3]Based on the application, time not spent in the formal labor market work can be assumed to be allocated to either home production or participating in the underground economy through an informal

productive but incurs income taxes; home production avoids taxation. In contributive pension systems, workers must meet certain eligibility criteria to receive full pensions, otherwise they receive a minimum pension or none at all. The uncertainty of meeting these requirements by retirement age leads to old-age consumption risk. To keep the model simple and tractable, we introduce a lottery mechanism to determine whether a retiree receives full benefits under a contributive pension system. The likelihood of receiving full benefits increases with labor market participation.[4]

Figure 1: Dispersion of replacement rates and demographics



Ratio of replacement rates at half mean wages and at mean wages. Old dependency is the share of population aged 65 and above. Source: OECD (2019).

We find, as expected, that regardless of the type of social security system, transfers increase with the political power of retirees and decrease in the ratio of workers to retirees. Retirees prefer a universal system as it protects them from the risk of not receiving full pensions. They also prefer a Bismarckian system for future benefits, as it encourages current workers to work more, thereby increasing current transfers. Workers prefer a Beveridgean system as it increases their total labor income without affecting future benefits. This suggests that as the population ages and retirees gain more relative political

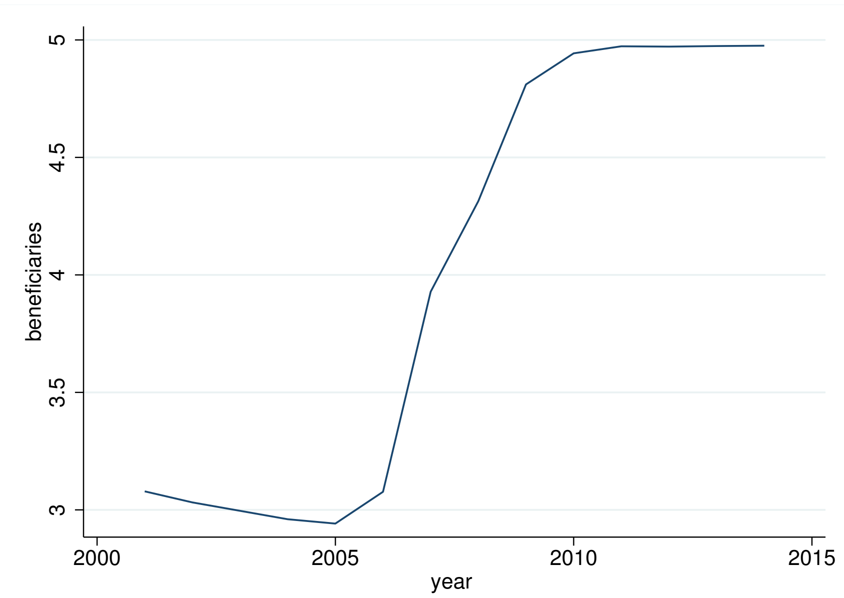
labor market.

[4]Tractability follows since the assumption renders workers homogeneous. The assumption reflects the requirement for a certain number of years of contributions to receive benefits common in social security systems. And it also captures the uncertain nature of employment that may prevent some retirees from meeting these requirements.

power, there could be a shift towards more Bismarckian systems. Data from the OECD provides some evidence of this: Figure 1 shows that countries with a higher proportion of the elderly in the population have more equal replacement rates, i.e. more Bismarckian benefits.[5]

The solution for the politico-economic equilibrium in a system with universal benefits is in closed-form and only depends on parameters and demographics. However, for a contributive system, the solution for the equilibrium tax rate can only be obtained through numerical methods. Exploiting model features, we use the backward recursive endogenous gridpoint method (EGM) algorithm to derive a series of time-dependent policy functions.[6]

Figure 2: Pension coverage in Argentina



Number of social security beneficiaries (in million). Source: Secretaría de Seguridad Social and INDEC.

To assess the model's quantitative performance we conduct two different exercises. First, we exploit recent pension reforms in Argentina. In 2005, only 68% of retirees received benefits under the country's primarily contributive system. But several reforms, including a tax amnesty in 2007 and 2008, led to an increase in coverage to 91% by 2010,

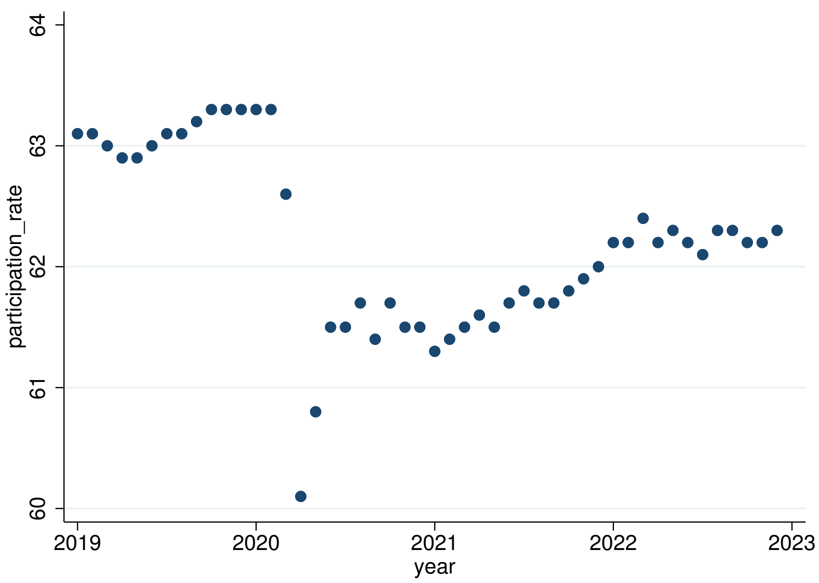
[5]The dispersion of replacement rates is given by the ratio of the replacement rate at half mean wages to that for mean wages. The correlation is significant at the 10% level, and significant at 5% when also controlling for the Gini coefficient.

[6]Compared to the standard value function iteration (VFI) approach, our approach is at least three times faster, and identifies time-dependent policy functions as opposed to the time-independent approximation that the standard VFI identifies.

see figure 2. We calibrate the model to match coverage and pension spending prior to the reform. When transitioning to universal benefits, our findings show that tax rates would increase by about 2.3 p.p., labor supply decreases by 3.1%, and savings reduced by 0.4 p.p. The rise in taxation accounts for around two-thirds of the observed increase in national government pension spending between 2005 and 2010. The decrease in private savings is mainly due to the higher taxes under universal benefits, while the change in pension incentives is responsible for about two-thirds of the decrease in labor supply.

Our second quantitative exercise examines the long-term effects of the decrease in labor force participation in the US following the COVID-19 pandemic. As shown in figure 3 the participation rate declined from an average of 63.1 in the year prior to the pandemic to 62.2 in 2022, a significant and sustained decrease.[7] As labor supply also contracted on the intensive margin, the decline in the participation rate likely indicates a change in labor supply preferences. The pandemic and the rise of remote work may have caused a re-evaluation of work-life balance in favor of the latter (Lee, Park and Shin, 2023). Our estimates suggest that if this change in preferences were to become permanent, in could lead to pressures to increase the social security tax rate by up to 1.8 p.p.

Figure 3: Recent trends in U.S. labor participation



U.S. labor participation rate. Source: Bureau of Labor Statistics.

Our work relates to the existing literature on politico-economic equilibrium for social

[7]Despite a secular decline in labor participation in the U.S., we interpret the recent drop as reflecting a shift in work preferences.

security (e.g. Cooley and Soares, 1999; Galasso, 1999; Forni, 2005; Gonzalez-Eiras and Niepelt, 2008; Song, 2011). Our approach accommodates various pension systems: Besides taking into account contributive and uniform pensions, our model allows for pension schemes to vary based on the relation between contributions and benefits. Endogenizing labor supply, by having workers allocate time between a formal labor market and home production, or informal labor market, we can estimate the impact of social security design on labor supply.

The paper closest to ours is Conde-Ruiz and Profeta (2007), which demonstrates that in certain OECD countries, greater income inequality is linked to Beveridgean systems with lower benefits. It explains these findings using a three-income-group overlapping generations model, where two types of equilibria might arise. If inequality is high, a coalition between the rich and poor will choose a small Beveridgean system, whereas if inequality is low, the middle class will favor a Bismarckian system with a higher tax rate. In our research social security systems differ along two dimensions, and our model enables a quantitative evaluation of the impact of system characteristics on the tax rate, labor supply and savings.[8]

We make a contribution to the numerical techniques used to determine politico-economic equilibrium (e.g. Krusell, Quadrini and Ríos-Rull, 1997; Song, Storesletten and Zilibotti, 2012). We develop a backward recursive EGM algorithm that takes advantage of the model's characteristics. This allows us to determine a set of time-varying policy functions based on all expected future population growth rates. We demonstrate that our method is significantly faster than the standard value function iteration approach and yields the same results. More importantly, we show that in the context of population ageing and overlapping generations models, the conventional approach in the politico-economic literature of comparing steady states (e.g. Galasso, 1999; Imrohoroğlu and Kitao, 2009) might be biased: When policy functions are calculated assuming constant population growth, the model overestimates tax rates for 2050 by between 6 and 16 p.p.

The rest of the paper is organized as follows: Section 2 presents the economic environment, and section 3 presents the economic and politico-economic equilibrium concepts. Section 4 analyzes politico-economic equilibria under different pension systems, and argues that ageing should lead to more Bismarckian systems. Section 5 outlines the numerical

[8] A shortcoming of our model is that workers are homogeneous. In our larger sample of OECD countries in 2019 we find a statistically significant relation between the share of retirees in the population and the dispersion of replacement rates, while the relation of the latter with the Gini coefficient is statistically insignificant.

solution approach for the contributive equilibrium. In section 6 we calibrate the model to Argentina to assess the quantitative effects of pension system design and to the United States to measure the effect of a permanent change in labor supply preferences. Section 7 concludes, and an appendix collects proofs, auxiliary calculation, discussion of numerical methods and other ancillary discussions.

2 The Model

2.1 Demographics and Institutions

We consider an economy populated by overlapping generations of workers and retirees. Workers allocate time to a formal labor market and home production, pay taxes, consume and save. When they reach retirement, they spend the return on their savings and any social security benefits they may receive, and die. The ratio of workers to retirees in period t follows a deterministic process, and is given by ν_t.

The government runs a pay-as-you-go (PAYG) pension system with a balanced budget. Each period, the government imposes a tax on labor supplied in the formal sector and transfers the collected amount directly to retirees. We consider social security systems that differ along two dimensions: whether workers need to qualify to receive full benefits or not, and the type of benefits provided - either tied to earnings or flat-rate. We describe the different systems in section 2.3.

Policy decisions are taken by a government that acts in the interest of voters, but lacks commitment. We consider both finite and infinite horizon economies.

2.1.1 Technology

In the formal sector, a continuum of competitive firms transforms capital and labor into output. Capital is owned by retirees and fully depreciates after a period. The economy-wide capital stock per worker, k_t, therefore corresponds to the economy-wide per-capita savings of workers in the previous period, s_{t-1}, normalized by ν_t. We assume that production technology is Cobb-Douglas with $\alpha \in (0, 1)$ denoting the income share of capital. Furthermore, for tractability we assume productive externalities as in Romer (1986) such that firm i's output is given by[9]

$$Y_t^i = A_t \left(K_t^i\right)^{\alpha} \left(H_t^i\right)^{1-\alpha}, \qquad A_t \equiv A\, p(k_t/h_t) = A\left(\frac{k_t}{h_t}\right)^{1-\alpha}. \tag{1}$$

Here K_t^i and H_t^i denotes the individual firm's use of capital and labor, while k_t and h_t are economy per-capita aggregates that the representative firm takes as given. The function $p(k_t/h_t) \equiv \left(\frac{k_t}{h_t}\right)^{1-\alpha}$ measures the strength of productive externalities (note $p' > 0$, and $p'' < 0$). In equilibrium this implies the following factor prices

$$R_t = \alpha A, \qquad w_t = (1-\alpha)A\frac{k_t}{h_t}. \tag{2}$$

Home production, or production in the informal sector, is given by the technology

$$y_t = w_t^* F(h_t) = w_t^* \frac{\xi}{1+\xi} X \left(1 - h_t^{1+\frac{1}{\xi}}\right), \tag{3}$$

where X measures baseline productivity, w_t^* denotes the formal labor sector wage rate *if* labor taxes were zero and ξ is the Frisch elasticiy of labor supply. Note that $F(1) = 0$, $F' < 0$, $F'' \leq 0$, $F''' \leq 0$. Thus, this technology only uses labor, $1-h_t$, and has decreasing returns to scale.

2.2 Preferences and Household Choices

Workers and retirees in period t value consumption, $c_{1,t}^i$ and $c_{2,t}^i$ respectively. Workers discount the future at factor $\beta \in (0,1)$, and are endowed with a unit of time, supplying h_t^i in the formal sector and $1 - h_t^i$ for home production. For analytical tractability, we assume that felicity functions are logarithmic. Welfare of a worker i who chooses savings, s_t^i, and labor supply, h_t^i, is given by

$$\begin{aligned} & \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i) \\ \text{s.t.} \quad & c_{1,t}^i = w_t \left[(1-\tau_t)\, h_t^i + \frac{w_t^*}{w_t} F\left(h_t^i\right)\right] - s_t^i \equiv \mathcal{I}_t^i - s_t^i, \\ & c_{2,t+1}^i = s_t^i R_{t+1} + E_t[b_{t+1}^i]. \end{aligned} \tag{4}$$

[9]In a related context Gonzalez-Eiras and Niepelt (2012) shows that equilibrium policies are unaffected by economic growth being exogenous or endogenous.

Where τ_t is the social security tax rate, and the expectation for retirement benefits reflects that these are stochastic when the social security system is contributive. Total after tax labor income is denoted by $\mathcal{I}_t$.

Optimal savings and labor supply decisions are characterized by

$$s_t^i = s(s_{t-1}, h_{t-1}, s_t, h_t, \tau_t, \tau_{t+1}), \quad (5)$$

$$h_t^i = h(s_{t-1}, h_{t-1}, s_t, h_t, \tau_t, \tau_{t+1}), \quad (6)$$

where s_t and h_t are, respectively, aggregate saving and labor supply in period t.

2.3 Social Security System

The government runs a pay-as-you-go pension systems with a balanced budget. Every period, the government taxes labor income in the formal sector and transfers the sum directly to current retirees. The study looks at two types of systems. First, a *contributive* one such that workers have to meet a contribution requirement to receive full pension benefits. This makes the pension benefits uncertain, as workers who experience long periods of unemployment or frequent spells of unemployment may fail to meet the contribution requirements when they retire. To simplify the model, it is assumed that a worker's chance of receiving full pension benefits is based on the share of their time endowment spent in the formal sector, h_t^i.[10] The second type of system is a *universal* one, in which all workers are eligible to receive full pensions upon retirement, regardless of their contributions.

Pension schemes also differ according to the relationship between contributions and benefits. *Bismarckian* systems provide earnings-related benefits; this gives workers an additional incentive to work as they perceive this increases their future pensions. In contrast, *Beveridgean* systems offer flat payments and lack this incentive.

Since the budget is balanced, contributions collected from workers, $h_t w_t \tau_t$, are transferred to retirees. In a contributive system, taking into account the design of pensions, benefits are given by

$$b_t^{i,\epsilon,\theta,j} = \left(\theta_t \frac{h_{t-1}^i}{h_{t-1}} + (1-\theta_t)\right) \nu_t h_t w_t \tau_t \frac{1+\epsilon(2j-1)}{1+\epsilon(2h_{t-1}-1)}, \quad (7)$$

[10]By doing so we introduce the contributive principle of linking pension benefits to sufficient participation in the formal labor market while still keeping the model tractable by treating all workers as identical. It is important to note the model does not capture the heterogeneity of households within generations, meaning that workers cannot adjust their behavior after experiencing periods of unemployment, such as by saving more or working more to offset the increased risk of lower retirement income.

where superscript $\epsilon \in (0,1]$ denotes the relation between full and minimum pensions in a contributive system, and $\theta \in [0,1]$ relates to the degree of Bismarckian incentives.[11] Finally, superscript $j \in \{0,1\}$ denotes whether a worker qualifies for full pensions or not.[12]

In a universal system $\epsilon = 0$ in which case (7) reduce to

$$b_t^{i,0,\theta} = \left(\theta_t \frac{h_{t-1}^i}{h_{t-1}} + (1-\theta_t)\right) \nu_t h_t w_t \tau_t, \tag{8}$$

where superscript 0 denotes that the system is universal, and θ captures the relationship between contributions and benefits as defined for the contributive system. It is immediate that, for given tax rates, current labor supply, wage rate, and demographics, $b_t^{i,\epsilon,\theta,0} \leq b_t^{i,0,\theta} \leq b_t^{i,\epsilon,\theta,1}$.

2.4 Elections

Elections take place at the beginning of each period until T (where T may be infinite). We assume that preferences are aggregated through probabilistic voting.[13] Thus, policy maximizes a convex combination of the objective functions of all groups of voters, where the weights reflect the groups' sizes and their responsiveness to policy changes. We allow for age related variation in responsiveness, reflected in a per capita political influence weight of unity for young voters and a per capita weight of $\omega \geq 0$ for retired voters. Furthermore, we assume that the political weight of a retiree is independent of the amount of benefits she receives.

3 Equilibrium

3.1 Competitive Equilibrium

The state is given by z_t, which includes exogenous demographics as well as savings per capita, s_{t-1}, and past labor supply, h_{t-1}. Conditional on z_t, the production function as well as competition among firms determine factor prices, w_t and R_t. Given the type of pension system, τ_t then determines capital accumulation, s_t, labor supply, h_t, and thus

[11]Extremes $\theta = 0$ (1) reflect pure Beveridgean (Bismarckian) cases.

[12]Since $\epsilon > 0$, $b_t^{i,\epsilon,\theta,1} > b_t^{i,\epsilon,\theta,0}$ and workers face retirement income risk. Note that the ratio of minimum to full pensions is given by $\frac{1-\epsilon}{1+\epsilon}$.

[13]See Lindbeck and Weibull (1987).

z_{t+1}. Thus, conditional on z_t, a policy sequence $\{\tau_s\}_{t\leq s\leq T}$ fully determines an allocation and price system.

Definition 1. A *competitive equilibrium* conditional on z_0 and a policy sequence $\{\tau_t\}_{0\leq t\leq T}$ is given by an allocation and price system such that

i. households optimize: (5) and (6) hold for all i, t;

ii. capital evolves according to $k_t = s_{t-1}/\nu_t$, markets clear, and factor prices are determined according to (2) for all t; and

iii. the government budget constraints (7) or (8) are satisfied for all t.

3.2 Politico-Economic Equilibrium

In politico-economic equilibrium political decision makers optimally choose tax rates, taking all implications of their actions into account and forming rational expectations about future policy choices. We assume that these choices are Markov, i.e. they are functions of the fundamental state variables. The decision maker at date t takes s_{t-1} and h_{t-1} as well as $\tau^{t+1}(\cdot)$ as given. Furthermore, given the continuation tax function the policymaker takes as given the following law of motion for the state variables as a function of current policy choices[14]

$$z_{t+1} = \zeta_t(z_t, \tau_t, \tau^{t+1}(\cdot)). \tag{9}$$

The policymaker chooses τ_t to maximize

$$\begin{aligned} &\mathcal{W}_t(z_t, \tau_t; \tau^{t+1}(z_{t+1})) \equiv \omega\mathcal{O}(z_t, \tau_t) + \nu_t\mathcal{Y}(z_t, \tau_t; \tau^{t+1}(z_{t+1})) \\ &\text{s.t.} \;\; (2), (7) \text{ (or } (8)), (9), \end{aligned} \tag{10}$$

[14]These laws of motion formally follow from (5) and (6). These in competitive equilibrium imply the following economic laws of motion:

$$\begin{aligned} s_t &= S(s_{t-1}, h_{t-1}, \tau_t, \tau_{t+1}), \\ h_t &= H(s_{t-1}, h_{t-1}, \tau_t, \tau_{t+1}). \end{aligned}$$

Equation (9) follows when we replace future policy by the continuation policy function,

$$\begin{aligned} s_t &= \tilde{S}(s_{t-1}, h_{t-1}, \tau_t, \tau^{t+1}(\cdot)), \\ h_t &= \tilde{H}(s_{t-1}, h_{t-1}, \tau_t, \tau^{t+1}(\cdot)). \end{aligned}$$

where the objective function is the weighted sum of the indirect utility functions of workers, $\mathcal{Y}(z_t, \tau_t; \tau^{t+1}(z_{t+1}))$, and retirees, $\mathcal{O}(z_t, \tau_t)$.

Definition 2. A *politico-economic equilibrium* as of period t conditional on z_t consists of a sequence of tax functions, $\{\tau_\iota(\cdot)\}_{t\leq\iota\leq T}$; a sequence of continuation tax functions, $\{\tau^{\iota+1}(\cdot)\}_{t\leq\iota<T}$; a sequence of laws of motion for the state variables, $\{\zeta_\iota(\cdot)\}_{t\leq\iota<T}$; policy choices, $\{\tau^\star_\iota\}_{t\leq\iota\leq T}$; and a competitive equilibrium allocation such that

i. tax functions are optimal subject to continuation tax functions:

$$\tau_\iota(z_\iota) \in \arg\max_{\tau_\iota} \mathcal{W}_\iota(z_\iota; \tau^{\iota+1}(\cdot)) \text{ for all } z_\iota, t \leq \iota \leq T;$$

ii. continuation tax functions are consistent with tax functions:[15]

$$\tau^\iota(z_\iota) = (\tau_\iota(z_\iota), \tau^{\iota+1}(z_{\iota+1}(\cdot))) \text{ for all } z_\iota, t \leq \iota \leq T;$$

iii. laws of motion are consistent with the policy and continuation policy functions according to (9);

iv. equilibrium tax choices are generated by the continuation tax function,

$$\{\tau^\star_\iota\}_{t\leq\iota\leq T} = \tau^t(z_t),$$

and $\{\tau^\star_\iota\}_{t\leq\iota\leq T}$ implements a competitive equilibrium allocation.

Note that for infinite horizon and a recursive time-autonomous structure, the policy and continuation policy functions are time-autonomous functions of the state as well, and conditions i. and ii. above are combined in a fixed point requirement.

[15]With a finite horizon, $\tau^{T+1}(z_T) = \emptyset$.

4 Analysis

4.1 The Economic Equilibrium

In appendix A we show that, given policy, equilibrium savings and consumption functions are given by:

$$s_t^{\epsilon,\theta} = \delta_t^{\epsilon,\theta}\mathcal{I}_t \tag{11}$$

$$c_{1,t}^{\epsilon,\theta} = \left(1-\delta_t^{\epsilon,\theta}\right)\mathcal{I}_t \tag{12}$$

$$c_{2,t+1}^{\epsilon,\theta,j} = \alpha A\delta_t^{\epsilon,\theta}\gamma_t^{\epsilon,\theta,j}\mathcal{I}_t, \tag{13}$$

where $\delta_t^{\epsilon,\theta} \equiv \delta^{\epsilon,\theta}(h_t,\tau_{t+1})$ is the propensity to save out of disposable labor income and $\gamma_t^{\epsilon,\theta,j} \geq 1$ are auxiliary functions that capture the return inclusive of the pension system (see appendix A). The after tax labor income function, $\mathcal{I}_t$, is given by

$$\mathcal{I}_t = (1-\alpha)A\frac{s_{t-1}}{\nu_t}\left[1-\tau_t + X^{\xi}F\left(h_t\right)\right] \tag{14}$$

which is independent of direct effects from future policy choices and log-separable in state variables s_{t-1} and ν_t.[16]

Equilibrium in the labor market is characterized implicitly by the fixed-point requirement

$$h_t = \left\{\frac{1-\tau_t}{X^{1+\xi}} + \beta\left[\frac{1-\tau_t}{X^{1+\xi}} + \frac{\xi}{1+\xi}\left(1-h_t^{1+1/\xi}\right)\right]\Gamma\left(h_t,\tau_{t+1}\right)\right\}^{\frac{\xi}{1+\xi}}, \tag{15}$$

where $\Gamma(\cdot)$ is detailed in appendix A.

We impose assumptions to guarantee a unique interior solution, and that taxation reduces labor income.

Assumption 1.

i. Baseline productivity in home production, X, is sufficiently large:

$$X > \left(1+\beta\frac{\left(1+\frac{1-\alpha}{\alpha}\right)\ln\left(1+\frac{1-\alpha}{\alpha}\right)+\frac{1-\alpha}{\alpha}}{1+\beta+\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1+\xi}}.$$

[16]Note that labor income depends on the social security system characteristics as this affects savings. Likewise, labor supply also depends on ϵ, and θ. We make this dependence explicit when necessary.

ii. For all tax rates, aggregate labor supply satisfies

$$-X^{1+1/\xi} h_t^{1/\xi} \frac{\partial h_t}{\partial \tau_t} < 1.$$

We remark that condition ii. is always satisfied in universal systems.[17] Under Assumption 1 we can compare the economic equilibria under various pension systems.

Proposition 1. Consider the economic equilibria under Assumption 1.

i. Given policy there exists a unique equilibrium, characterized by $h_t \in [0, 1)$.

ii. Labor supply increases with θ and ϵ. The semi-elasticity of labor supply with respect to the tax rate is decreasing in θ.

iii. The propensity to save is always higher under contributive than universal pension benefits. It does not depend on θ in universal systems, and is decreasing in θ when $\epsilon \approx 1$.

iv. Labor supply is decreasing in the current tax rate, and increasing in future taxes.

Proof. See appendix A.2. □

Proposition 1 states that when policies are given, transitioning to universal benefits results in a reduction of private savings and labor supply. This occurs because workers self-insure against the risk of not receiving full pension benefits in a contributive system by increasing their labor supply and private savings. Furthermore, in Bismarckian systems we find, as expected, that labor supply is higher given that workers perceive their benefits to be tied to their labor supply. This holds both for contributive and universal systems. Labor supply is chosen to maximize after tax labor income only when $\theta = \epsilon = 0$. Thus, given policy, an increase in θ reduces workers' welfare. The effect of labor incentives on the savings rate is in general ambiguous. But when $\epsilon = 1$ an increase in labor supply reduces saving. Thus, given the effect of θ on labor supply, the savings rate will be higher with Beveridgean incentives in this polar case.

4.2 Politico-economic equilibrium with universal pensions

When the system is universal every retired household receives the same pension benefit, characterized by (8). Proposition 1 states that, given policy, equilibrium in the universal

[17] In appendix A we show labor supply is a unique solution of parameters and taxes in this case. We also verified numerically that for our calibrations the condition is satisfied when $\epsilon > 0$.

case does not depend on θ and is given by (11)-(15) with (see appendix A)

$$\gamma_t^{0,\theta} = \gamma_t^0 = 1 + \frac{1-\alpha}{\alpha}\tau_{t+1}, \qquad \delta_t^{0,\theta} = \delta_t^0 = \frac{\beta}{\beta + \gamma_t^0}.$$

When pensions are universal, the political aggregator function is given by

$$\mathcal{W}^{0,\theta}(z_t) = \omega \ln\left(c_{2,t}^{0,\theta}\right) + \nu_t \left[\ln\left(c_{1,t}^{0,\theta}\right) + \beta \ln\left(c_{2,t+1}^{0,\theta}\right)\right]. \tag{16}$$

The political process maximizes (16), subject to the constraints that the economy is in a competitive equilibrium, and taking the future policy function, $\tau_{t+1}(z_t)$, as given. Since the equilibrium allocation —in particular labor income— and the objective function do not depend on h_{t-1}, then only savings per capita might be a relevant endogenous state variable with universal pensions. The following proposition characterizes politico-economic equilibrium in this case.

Proposition 2. Consider a universal pensions system. There is a unique Markov perfect equilibrium in the limit of the finite horizon. For any θ, the equilibrium policy function is given by:

$$\tau^{0,\theta}(\nu_t) = \tau^0(\nu_t) = \min\left\{1,\ \max\left\{0,\ \frac{1}{\omega + \nu_t(1+\beta)}\left[\omega\left(1 + \xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_t(1+\beta)\right]\right\}\right\}.$$

Proof. See appendix B.1. □

We note here that as the policy function does not depend on any endogenous states, future taxes τ_{t+1} become independent of τ_t as well. Notwithstanding this orthogonality, the trade-offs underlying the equilibrium tax rates are dynamic in nature as they relate contemporaneous tax revenue and benefits to future wages and tax revenue. The tractability of the model comes from specifying functional forms that render the factor price elasticities and the derivatives of the indirect utility functions orthogonal to the capital stock.[18] As will be shown next, when pensions are contributive we lose the ability to generate closed form solutions since, although the capital stock remains orthogonal to the derivatives of the indirect utility functions, past labor supply affects political trade-offs.

The equilibrium policy is increasing in ω and decreasing in ν_t, as retirees prefer higher transfers and workers lower taxes. For both groups the direct effect of taxation dominates over the indirect effects, working through a reduction in labor supply and an increase in

[18] As shown elsewhere, these functional form restrictions tend to be of minor importance for the quantitative predictions of the model. See Gonzalez-Eiras and Niepelt (2005) for an analysis in a related context.

wages.[19] Thus, the model with universal pension benefits confirms the findings in related literature that an ageing population and a larger relative political power of retirees result in higher equilibrium taxes. Note also that an increase in the productivity of home production, X, or the elasticity of labor supply, ξ, increases the equilibrium tax rate.

The outcome that the equilibrium tax rate is unaffected by θ is due to the fact that although pension incentives influence labor supply and labor income, the semi-elasticity of labor income with respect to taxes, and thus the perceived cost of taxation by workers, is independent of θ. As the benefit of taxation for retirees is also unaffected by θ, equivalence of equilibrium tax rates follows. However, this does not mean that Bismarckian and Beveridgean uniform systems are politico-economic equivalent as the equilibrium labor supply, and thus allocations, still differ (Gonzalez-Eiras and Niepelt, 2015).

4.3 Politico-economic equilibrium with contributive pensions

With contributive pensions some retirees do not receive full pensions, as they presumably fail to meet eligibility criteria for receiving them due to long or repeated spells of unemployment. We assume that with probability h_t^i retired household i receives full pension benefits $b_{t+1}^{\epsilon,\theta,1}$, and with probability $(1-h_t^i)$ she only receives $b_{t+1}^{\epsilon,\theta,0} < b_{t+1}^{\epsilon,\theta,1}$, see (7). The probability of receiving pension benefits is tied to *individual* household's labor participation. Our model incorporates this contributive principle, where benefits are linked to adequate involvement in the labor market, while still maintaining tractability by assuming young households are all alike.

The equilibrium labor supply depends on θ, and is implicitly determined by (15):

$$h_t^{\epsilon,\theta} = \left\{ \frac{1-\tau_t}{X^{1+\xi}} + \beta \left[\frac{1-\tau_t}{X^{1+\xi}} + \frac{\xi}{1+\xi} \left(1 - (h_t^{\epsilon,\theta})^{1+1/\xi} \right) \right] \Gamma^{\epsilon,\theta} \left(h_t^{\epsilon,\theta}, \tau_{t+1} \right) \right\}^{\frac{\xi}{1+\xi}} \tag{17a}$$

with auxiliary function

$$\Gamma^{\epsilon,\theta} = \left(1 - \delta_t^{\epsilon,\theta}\right) \left(h_t^{\epsilon,\theta} \ln \left(\frac{\gamma_t^{\epsilon,\theta,1}}{\gamma_t^{\epsilon,\theta,0}} \right) + \theta \left(\frac{\gamma_t^{\epsilon,\theta,1} - 1}{\gamma_t^{\epsilon,\theta,1}} h_t^{\epsilon,\theta} + \frac{\gamma_t^{\epsilon,\theta,0} - 1}{\gamma^{\epsilon,\theta,0}} (1 - h_t^{\epsilon,\theta}) \right) \right), \tag{17b}$$

where we note that $(\delta_t^{\epsilon,\theta}, \gamma_t^{\epsilon,\theta,j})$ are functions of $(h_t^{\epsilon,\theta}, \tau_{t+1})$. When pension benefits are

[19] For workers, consumption is proportional to after tax labor income and thus political support can be summed up by how $\mathcal{I}_t$ is affected.

contributive, the political aggregator function is given by

$$\mathcal{W}^{\epsilon,\theta}(z_t) = \omega \left[h^{\epsilon,\theta}_{t-1} \ln\left(c^{\epsilon,\theta,1}_{2,t}\right) + \left(1 - h^{\epsilon,\theta}_{t-1}\right) \ln\left(c^{\epsilon,\theta,0}_{2,t}\right)\right] \\ + \nu_t \left[\ln\left(c^{\epsilon,\theta}_{1,t}\right) + \beta\left(h^{\epsilon,\theta}_{t} \ln\left(c^{\epsilon,\theta,1}_{2,t+1}\right) + \left(1 - h^{\epsilon,\theta}_{t}\right) \ln\left(c^{\epsilon,\theta,0}_{2,t+1}\right)\right)\right], \tag{18}$$

and the political process seeks taxes to maximize it, subject to the constraints that the economy is in a competitive equilibrium, and taking as given $\tau^{\epsilon,\theta}_{t+1}(\cdot)$, the policy function that determines future policy as a function of state variables. Compared to the case of universal benefits, $h^{\epsilon,\theta}_{t-1}$ becomes a potentially relevant state variable.

The main results of the contributive politico-economic equilibrium are summarized in the following proposition.

Proposition 3. Consider a contributive pensions system. In the finite horizon economy ending at time T, the terminal equilibrium policy function, $\tau^{\epsilon,\theta}(h^{\epsilon,\theta}_{T-1}, \nu_T)$, is unique, and implicitly determined by:

$$\frac{\tau^{\epsilon,\theta}_{T}(h^{\epsilon,\theta}_{T-1}, \nu_T)}{1 - \tau^{\epsilon,\theta}_{T}(h^{\epsilon,\theta}_{T-1}, \nu_T) + \xi X^{1+\xi}} = \frac{\omega}{\nu_T(1+\beta)} \left[h^{\epsilon,\theta}_{T-1} \frac{\gamma^{\epsilon,\theta,1}_{T-1} - 1}{\gamma^{\epsilon,\theta,1}_{T-1}} + (1 - h^{\epsilon,\theta}_{T-1}) \frac{\gamma^{\epsilon,\theta,0}_{T-1} - 1}{\gamma^{\epsilon,\theta,0}_{T-1}}\right] \tag{19}$$

For $t < T$ the politico-economic equilibrium has to be solved numerically. The policy function is increasing in ω and decreasing in ν_t, the policy function is increasing in $h^{\epsilon,\theta}_{t-1}$, and retirees' support for taxes is unambiguously lower compared to the universal system.

Proof. See Appendix B.2. □

The reasoning behing the equilibrium tax being increasing in ω and decreasing in ν_t is is similar to that in the case of a universal system: Retirees (workers) prefer higher (lower) taxes. Shifts in ω and ν_t move the relative weights the political process gives to the two groups of voters and thus the equilibrium tax. Compared to the universal case however, there are a number of additional effects.

The benefit of taxes for retirees is given by

$$\left[h^{\epsilon,\theta}_{t-1} \ln\left(c^{\epsilon,\theta,1}_{2,t}\right) + \left(1 - h^{\epsilon,\theta}_{t-1}\right) \ln\left(c^{\epsilon,\theta,0}_{2,t}\right)\right] \leq \ln\left(h^{\epsilon,\theta}_{t-1} c^{\epsilon,\theta,1}_{2,t} + (1 - h^{\epsilon,\theta}_{t-1}) c^{\epsilon,\theta,0}_{2,t}\right) \tag{20}$$

Since $\ln\left(h^{\epsilon,\theta}_{t-1} c^{\epsilon,\theta,1}_{2,t} + (1 - h^{\epsilon,\theta}_{t-1}) c^{\epsilon,\theta,0}_{2,t}\right)$ is the benefit of (same level of) taxation for retirees in a universal system,[20] there will be lower support for taxation from the retired, relative

[20]This follows since, for given taxes, $h^{\epsilon,\theta}_{t-1}\gamma^{\epsilon,\theta,1}_{t-1} + (1 - h^{\epsilon,\theta}_{t-1})\gamma^{\epsilon,\theta,0}_{t-1} = \gamma^{0}_{t-1}$.

to the universal pension system. Given that $c_{2,t}^{\epsilon,\theta,1} > c_{2,t}^{\epsilon,\theta,0}$, an increase in $h_{t-1}^{\epsilon,\theta}$ increases equilibrium taxes.

The equilibrium relationship between incentives and political support for taxation is indeterminate. Similarly, the impact of converting pension benefits to a contributive system on political support from workers is ambiguous. When benefits are determined by contributions, a tax increase results in two impacts on worker well-being: A *labor income* effect and a *consumption smoothing* effect. Conversely, if benefits are universal, taxes only affect workers' welfare through labor income.

As seen in (11)-(15), equilibrium consumption functions are proportional to labor income $\mathcal{I}_t$. As in the case of universal benefits, taxation produces a direct loss of income from working in the formal sector, and a positive effect on wages from capital deepening. Under assumption 1 ii. the direct effect always prevails. It is ambiguous whether the marginal effect of taxation is higher or lower than when pension benefits are universal.[21]

When the pension system is contributive, it creates old-age consumption risk. To counteract this risk, workers increase their private savings and work more, as stated in proposition 1. This leads to a higher marginal utility of current consumption, given their labor income, and magnifies the impact of taxation on the indirect utility of workers, thereby reducing the political support for taxes.

4.4 Political choice of system characteristics

Our goal is to understand what pension characteristics would be chosen by workers and retirees, and how these characteristics might change in response to demographic shifts. We conjecture that future decisions regarding the social security system will not be influenced by the current choice. To explore this, we first examine the choice between a contributive or uniform system, i.e. on ϵ_t, while keeping incentives constant. The political objectives in each case, given the current and future taxes, are expressed as follows:

$$\begin{aligned}
\mathcal{W}_t^{\epsilon,\theta} &= \omega \left[h_{t-1}^{\epsilon,\theta} \ln \left(\gamma_{t-1}^{\epsilon,\theta,1} \right) + (1 - h_{t-1}^{\epsilon,\theta}) \ln \left(\gamma_{t-1}^{\epsilon,\theta,0} \right) \right] + \nu_t \mathcal{Y}_t(\tau_t, \tau_{t+1}), \\
\mathcal{W}_t^{0,\theta} &= \omega \ln \left(\gamma_{t-1}^{0} \right) + \nu_t \mathcal{Y}_t(\tau_t, \tau_{t+1}),
\end{aligned}$$

where $\mathcal{Y}_t$ is the indirect utility of workers. Crucially, for given tax rates, $\mathcal{Y}_t$ is the *same* regardless of whether benefits are contributive or universal today: When making their

[21] Indeterminacy arises because the size of the effect depends on the equilibrium response of labor supply to taxes, and in the contributive system this depends on the endogenous response of future taxes, which has to be solved for numerically.

labor supply and saving decisions, workers only care about the benefits they will receive when they retire. Thus, workers are indifferent on the type of benefits.

As shown in (20), $h_{t-1}^{\epsilon,\theta} \ln\left(\gamma_{t-1}^{\epsilon,\theta,1}\right) + (1 - h_{t-1}^{\epsilon,\theta}) \ln\left(\gamma_{t-1}^{\epsilon,\theta,0}\right) \leq \ln(\gamma_t^0)$. Thus, retirees are always better off with universal pensions as they benefit from the risk sharing this provides. Therefore,

$$\mathcal{W}_t^{0,\theta}(\tau_t^{0,\theta*}) \geq \mathcal{W}_t^{0,\theta}(\tau_t^{\epsilon,\theta*}) \geq \mathcal{W}_t^{\epsilon,\theta}(\tau_t^{\epsilon,\theta*}) \tag{21}$$

and a universal system would always be preferred.[22]

When considering labor supply incentives, with benefits type fixed, retirees prefer more Bismarckian incentives, i.e. a higher θ_{t+1}, as this boosts labor supply and the tax base for transfers.[23] This reduces workers' welfare because in equilibrium their future benefits are unaffected by incentives and it is straightforward to verify that, for given tax rates, $\mathcal{I}_t^{\epsilon,0} \geq \mathcal{I}_t^{\epsilon,\theta}$, see (23b) in the appendix. Hence, workers would opt for a more Beveridgean system if given a choice, i.e. a lower θ_{t+1}. The model suggests that an aging population, which gives more weight to retirees in the political process, should lead to a shift towards more Bismarckian systems; see figure 1.[24] Our conjecture that the current choice of pension characteristics would not affect future choices is confirmed.

5 Numerical Solution with Contributive Pensions

In order to determine the politico-economic equilibrium for contributive pension benefits, a numerical solution is required. Our modeling methodology, however, enables us to efficiently find the policy function for each period through numerical means. Specifically, as the labor supply function in (17) can be expressed as a closed-form relation between τ_t, h_t, and τ_{t+1} for both Beveridgean and Bismarkian incentives, we use an EGM following Carroll (2006) to solve for the policy function.

In the terminal period we construct a grid of the endogenous state h_{T-1}. The bounds are found by evaluating the labor supply function at $(\tau_{T-1}, \tau_T) = (0, 1)$ and $(\tau_{T-1}, \tau_T) = (1, 0)$ respectively. For each node on the grid we solve for the equilibrium terminal policy

[22] Most countries have contributive systems. A potential rationale for this is that private savings, and thus capital accumulation, is lower under a universal system. If workers, at least partially, internalize these effects on their future wages, they might prefer a contributive system. We explore this in appendix C where we show that for the calibration used in subsection 6.1 it is the case that a contributive system has higher steady state political welfare than a universal system.

[23] As we care about incentive effects on labor supply, the relevant choice in period t would be on θ_{t+1}.

[24] Recall that the negative correlation is statistically significant at the 10% level and at 5% when controlling for the Gini coefficient.

as given by (19). We approximate this policy function by interpolation over the solution grid. For each pair (h_{T-1}, τ_T) we use the labor supply function to define the implied tax rate τ_{T-1}. With the terminal policy given we solve for all policies $t < T$ recursively. For this we exploit the fact that we can express period t taxes as a closed-form expression of current labor supply and period $t+1$ taxes.

The policy function at time t, $\tau_t(z_t)$, may depend on the entire future path of the exogenous state $\{\nu_\iota\}_{t \leq \iota \leq T}$, in a way that is parsimoniously captured by the continuation policy function $\tau^{t+1}(z_{t+1})$. Given parameter values and an initial value h_0 we can simulate a model realization using the identified policy functions. For the numerical analysis carried out in section 6, we identify the policy functions using several numerical methods besides the finite horizon EGM-type approach outlined above. These includes an infinite horizon version of the model with time-dependent policy functions and a steady state approximation with time-autonomous structure.

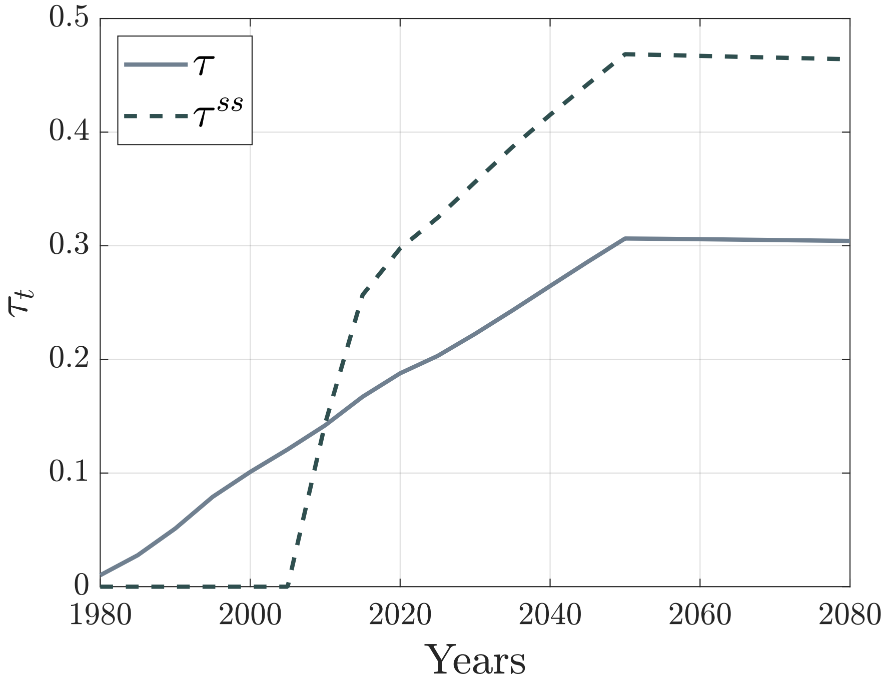
The numerical simulations yield two key insights: First, in terms of computational speed, the finite horizon approach is only three times faster than the infinite horizon version with time-dependent policy functions, but more than two orders of magnitude faster than the steady state approximation.

Second, in terms of accuracy, we find that the steady state approach produces significantly different predicted future policies, see figure 4 which corresponds to the case of Argentina analyzed in 6.1. The predicted policy for 2050 is around 16 p.p. higher under the assumption that demographics in each period correspond to a steady state. In appendix D we perform a robustness analysis by recalibrating the model with the assumption of an initial tax rate at roughly double its original level. Simulations still show that the steady state approximation predicts a future tax rate that is around 6 p.p. higher than the rate predicted by time-dependent policies. This suggests that results obtained from the steady state approximation should be reevaluated as they might overestimate the effect of demographic changes on policy (e.g Galasso, 1999; Imrohoroğlu and Kitao, 2009).

6 Quantitative Analysis

To evaluate the model's quantitative performance we carry out two evaluations. One involves analyzing the impact of a set of reforms in Argentina from 2005 to 2010 that introduced near-universal pensions. We calculate the effect these reforms had on the tax rate, labor participation, and savings rate. The other evaluation involves examining

Figure 4: Comparison of time-dependent policies and steady state approximation



Calibration to Argentina before social security reforms of 2005-2010. See subsection 6.1.

the recent decline in labor participation in the United States following the COVID-19 pandemic and estimating the effects of a permanent decrease in the preference for work on the equilibrium social security tax rate.

6.1 Pension System Reform in Argentina

Argentina is a middle-income country that in 2005 faced high levels of labor informality. The social security system had strict contribution requirements, which resulted in about a third of individuals approaching retirement age not being eligible for full benefits[25] Since 2005 a number of reforms were introduced to increase pension coverage.[26] In particular, a tax amnesty resulted in 2.7 million additional beneficiaries and raised the coverage rate of the elderly from 68% to 91%, see figure 2. This was achieved by allowing workers with less

[25]A pension reform in 1994 introduced an optional fully-funded pillar to its social security system. Workers were required to have 30 years of contribution to receive full benefits.

[26]In 2007 workers under the fully-funded system were given the option to return to a pay-as-you go system. Then in December 2008 the fully-funded pension funds were nationalized, and its beneficiaries were transferred to the pay-as-you-go system.

than the required 30 years of contributions to receive a pension.[27] Thus, in a few years, Argentina experienced a significant increase in non-contributive pensions: Between 2005 and 2010 social security spending increased by 1.9% of GDP with most of this increase, 1.8% of GDP, due to the tax amnesty (Cetrángolo and Grushka, 2016).

Before the tax amnesty, retirees over the age of 70 who had made contributions for a minimum of 10 years were eligible for basic pension coverage. This coverage consisted of 70% of a basic pension and a proportional amount from other contributions.[28] We calculated the ratio of basic benefits to full pensions before the reform, $(1-\epsilon)/(1+\epsilon)$, by considering the retirement age for men (65) and women (60), and their estimated life expectancy from retirement age which was 14.45 years for men and 22.55 years for women.[29] The estimate of the contributive nature of the pension system prior to the reforms is calculated by taking the average of the estimates for men and women, resulting in $\epsilon = 0.560$, see table 1. To identify the Bismarckian incentives, we use OECD data on the ratio of replacement rates at half mean and mean wages for males and females such that $\theta = 0.815$.[30]

After the reforms, we set $\epsilon = 0$ since there are virtually no contribution requirements to receive pensions. To gauge the impact of transitioning to more universal benefits on taxes, informality, and savings, we calibrate the model to Argentina in 2005-2010 and take a period to be 30 years. The capital income share, $\alpha = 0.50$, comes from Frankema (2010) and Restrepo-Echevarría (2017). In the baseline we take the elasticity of labor supply, $\xi = 0.35$, and explore values between 0.25 and 0.45 as robustness.[31] The average savings rate between 1994 and 2007 is 20.7% (from World Bank national accounts data)

[27]See Cetrángolo and Grushka (2016) and Rofman and Oliveri (2012) for details on recent pension reforms in Argentina, and Gonzalez-Rozada and Ruffo (2015) for an estimate of the effect of the tax amnesty on savings.

[28]The basic pensions was set by law 24347 of year 1995 at 70% of the first pillar, denoted “prestación básica universal”, plus the corresponding fully-funded or pay-as-you-go benefit. For simplicity we assume the latter on average was also 70% of the benefits received by workers that had at least 30 years of contributions.

[29]To estimate life expectancy at retirement age for Argentina we conduct a regression analysis using data for men and women life expectancy at birth and at age 65 for 40 OECD countries for the year 2000. We then use the fitted regression and data from the World Bank on male and female life expectancy at birth in Argentina in 2000. Our estimate of $(1-\epsilon)/(1+\epsilon)$ is $0.7 * \beta^{5/30} * 9.45/14.45$ for men and $0.7 * \beta^{10/30} * 12.55/22.55$ for women, where β is the estimated 30-years discount factor.

[30]In a pure Bismarkian system the replacement rate would be the same at all wage levels, while in a pure Beveridgean it would double if we halve wages. In Argentina the gross replacement rate for men at mean wages is 0.712 (0.644 for women) while it climbs to 0.837 (0.769) at half mean wages (OECD, 2019). Then θ is 2 minus the averages for men and women of the ratio of replacement rates at half mean and mean wages.

[31]Changes in this parameter affect the calibration but have minor effects on the implications of pension reform. This is consistent with Imrohoroğlu and Kitao (2009) that finds aggregate labor supply to be insensitive to this elasticity.

and is used to calibrate β. We take the pre-reform informality rate in 2005-2010 to be 37.4%, calculated from the proportion of beneficiaries before and after the 2005-2010 reforms and considering non-contributive pensions prior to the reform (Cetrángolo and Grushka, 2016); this pins down parameter X.[32] To construct the time series for ν_t we follow Gonzalez-Eiras and Niepelt (2008) and use 30-year gross population growth rate and projections from census data. Finally, we use social security spending before the 2001 crisis of 7.1% of GDP,[33] which gives an equivalent social security tax rate of $\tau_{2010} = 0.142$, to calibrate ω.[34]

Table 1: Calibration

Parameter	Value	Calibration target
ϵ (*pre*-reform)	0.560	Minimum-to-full pension coverage
θ	0.815	Replacement rate dispersion
α	0.50	Factor income shares
ν_t	$[1.65, 0.99]$	30-year gross population growth rates
ξ	0.35	Elasticity of labor supply
β	$0.297 = 0.96^{30}$	Private savings rate of 20.7%
X	4.07	Informality rate of 37.4%
ω	0.58	Social security tax of 14.2%

The model predicts higher tax rates under a universal system, with the difference increasing with ageing. Under universal benefits we also have higher informality and lower savings, see table 2. In 2010 the model predicts that a shift to a universal pension system increases tax rates by about 2.3 p.p. In 2010, after the reform, the savings rate drops 0.4 p.p., while labor supply drops by roughly 3.1%.

To study the impact of pension characteristics, we run various simulation scenarios and estimate the politico-economic equilibrium under extreme regimes, i.e. considering $(\epsilon, \theta) \in \{(0,0), (0,1), (1,0), (1,1)\}$. We also consider the response of savings and labor supply if system characteristics are changed but labor taxes are kept constant. Results

[32] It should be noted that International Labor Organizaton's ILOSTAT reports a higher rate of labor informality for this time period at 48%. However, using this rate would overestimate the probability of not receiving pensions, as workers who are informal in a given year may still have enough formal years over their working life to claim a social security benefit upon retirement.

[33] Data from ANSES. Using the 2005 spending of 5.8% of GDP or the actual social security tax rate of 27.1% affect the model's quantitative predictions. We prefer to use pre-crisis spending as it was relatively stable during that time. Following the 2001 crisis, pension spending was reduced because only minimum pensions were initially adjusted for inflation. It's important to also note that after 2010, contributive pensions have returned to their pre-crisis levels (Cetrángolo and Grushka, 2016).

[34] We do this by following the general outline of the nested fixed point algorithm, cf. Rust (1987): Given parameter values an inner loop solves the model and calculates the difference from target values τ_t. An outer hill-climbing algorithm searches the parameter space to minimize this difference.

are shown in table 3. First, changes in the level of contribution-based benefits have a much greater impact on equilibrium taxes and savings compared to changes in the relationship between benefits and earnings.[35] Second, the impact on labor supply is roughly double when the system transitions from purely contributive to universal, compared to when benefits switch from fully Bismarckian to Beveridgean. Third, we observe that while maintaining constant taxes, the majority of changes in savings stem from the rise in taxation, while most of the changes in labor supply stem from changes in the system's features.

Table 2: Pension system reform, year 2010.

Variable:	**Tax rate**	**Savings rate**	**Labor supply**
Initial (B) (ϵ=0.56, θ=0.82)	14.2%	20.70%	0.616
Universal (U) (ϵ=0, θ=0.82)	16.5%	20.31%	0.597

Politico-economic equilibrium for social security under the initial regime and after making pension benefits universal.

Table 3: Effect of polar changes in system characteristics, 2010

	$\boldsymbol{\Delta(\epsilon,\theta)}$			
	$\mathbf{(1,1)\rightarrow(0,1)}$	$\mathbf{(1,0)\rightarrow(0,0)}$	$\mathbf{(1,1)\rightarrow(1,0)}$	$\mathbf{(0,1)\rightarrow(0,0)}$
$\boldsymbol{\Delta\tau}$	5.58 p.p.	6.10 p.p.	-0.52 p.p.	0.00 p.p.
$\boldsymbol{\Delta\delta}$	-0.91 p.p.	-0.98 p.p.	0.08 p.p.	0.00 p.p.
$\boldsymbol{\Delta h}$	-3.84 %	-4.36 %	-2.08 %	-2.60 %
$\boldsymbol{\Delta\delta}$ (τ constant)	-0.10 p.p.	-0.10 p.p.	0.01 p.p.	0.00 p.p.
$\boldsymbol{\Delta h}$ (τ constant)	-3.04 %	-2.60 %	-3.01 %	-2.60 %

6.2 Permanent Shift in Leisure Preferences

The purpose of our second quantitative examination is to evaluate the potential consequences for social security in the United States of the change in labor supply preferences

[35] That taxes are weakly increasing with Bismarckian incentives is in accordance with the corollary in Conde-Ruiz and Profeta (2007). Our work suggests this effect is quantitatively minor.

induced by the COVID-19 pandemic. The labor participation rate was 63.1% for the 12 months before the pandemic and after a sharp drop and recovery, it stabilized at an average of 62.2% for 2022, see figure 3.

The labor force participation rate in the United States has undergone significant fluctuations in the past. From the mid-1960s to 2000, it rose steadily, reaching a peak of 67.3% in 2000. Since then participation declined, first smoothly and then sharply after the Great Recession, hitting a low of 62.4% in 2015 before gradually recovering to 63.3% in February 2020. These fluctuations were largely influenced by social, technological, and demographic factors such as increased female participation, technology favoring skilled workers, and an aging population.[36] The recent drop in labor force participation following the pandemic, however, appears to be due to changes in workers' labor supply preferences.

Table 4: Calibration, US

Parameter	Value	Calibration target
ϵ	0.516	Minimum-to-full pension coverage
θ	0.728	Replacement rate dispersion
α	0.281	Factor income shares
ν_t	$[1.5, 1.07]$	30-year gross population growth rates
ξ	0.35	Elasticity of labor supply
β	$0.2839 = 0.959^{30}$	Private savings rate of 17.9%
X (*pre* Covid)	3.91	Labor participation rate of 63.1%
X (*post* Covid)	4.09	Labor participation rate of 62.2%
ω	0.31	Social security tax of 12.4%

Lee et al. (2023) supports the notion that the COVID-19 pandemic has altered workers' labor supply preferences. The paper studies labor market trends following the pandemic and finds that about half of the decrease in hours worked between 2019 and 2022 is due to a contraction on the intensive margin. It argues that the tight labor market in 2022 suggests workers were opting to reduce their labor supply voluntarily, rather than due to pandemic-related factors like illness, fear of infection, or school closures. Therefore, the authors of this study conjecture that the pandemic prompted a reconsideration of life priorities, causing workers to "stop idolizing work and seek more work-life balance".

It's premature to determine if the reduction in labor supply will be permanent, but we can estimate the impact of a permanent shift in workers' leisure preferences on the equilibrium social security tax rate. To do so, we calibrate the model to the U.S. in 2020

[36]See Juhn and Potter (1987) for a description of labor force participation trends in the second half of the twentieth century.

using the capital income share from Piketty and Saez (2003), the elasticity of labor supply from Chetty, Guren, Manoli, and Weber (2013), the average savings rate from World Bank data and labor participation from the Bureau of Labor Statistics (average between March 2019 and February 2020) as h_{2020}.[37] From Census Bureau data and projections we take the time series of ν_t and take the social security tax $\tau_{2020} = 0.124$. To calibrate ϵ we used the online Social Security Quick Calculator and took the ratio of benefits when contributions provided the required minimum of 40 credits to benefits when contributions were made throughout the entire worklife.[38] Finally, θ is taken from OECD data as the ratio of replacement rates at half mean and mean wages. The calibrated parameters are in table 4.

Table 5: COVID-19 induced change in preferences for leisure

Variable:	**Tax rate**	**Savings rate**	**Labor supply**
Pre pandemic ($X = 3.94$)	12.4%	17.9%	0.631
Post pandemic ($X = 4.13$)	14.2%	17.3%	0.622

The change in preferences is assumed to have been unexpected. Taking the path estimated for (h_t, δ_t, τ_t) as given under the calibration for $t < 2020$ we estimate what new value of X would result in the lower participation rate observed in 2022, estimating at the same time what are the implications for the equilibrium tax rate and the savings rate, see table 5. The predicted increase in tax rates of 1.8 p.p. suggests that a modest permanent increase in the preference for leisure of 4.4% can have a significant impact on social security.[39]

We leverage this quantitative analysis to address the difference in hours worked between the U.S. and Europe since the 1970s. Prescott (2004) attributes the divergence to taxes and pensions using an infinitely lived representative household real business cycle model. Our study asserts that the underlying drivers of taxes and intergenerational transfers must first be examined, and it suggests Blanchard's (2004) alternative explanation

[37] Given the life cycle structure of our model we take the estimate of the Hicksian aggregate hours elasticity of 0.7 in Chetty et al. (2013) for the model presented in Rogerson and Wallenius (2009). Results are robust to minor changes around our chosen value of 0.35 for the extensive margin elasticity.

[38] The calculator is available at `https://www.ssa.gov/OACT/quickcalc/index.html`.

[39] By construction the model is silent on whether changes would entirely fall on the size of intergenerational transfers or lead to a redesign of the pension system.

giving a significant role to preferences for leisure is at least quantitatively plausible. A more comprehensive quantitative investigation of this issue will be the subject of future research.

7 Conclusions

Our politico-economic equilibrium model showcases the impact of various pension system features on the size of transfers and, directly and indirectly, savings and labor supply. In particular, the model takes into account two key aspects of heterogeneity. First, we considered systems that either have or don't have a requirement for years of contributions to receive full benefits, and used the ratio between minimum and full pensions as a measure of the degree by which a system is contributive. Second, we considered whether benefits are earnings-related or flat, thus allowing for both Bismarckian and Beveridgean systems.

Retirees prefer universal pensions for the security they offer against the risk of not getting full benefits when retiring, and Bismarckian systems as this increases labor supply leading to greater pension benefits. Conversely, younger generations prefer Beveridgean systems as this increases their labor income without affecting their pensions. Thus, the model implies that as the population ages and retirees become more influential in the political process, there is a trend towards adopting more Bismarckian pension systems.

The model was calibrated based on the experience of Argentina, which transitioned from mostly contributive pensions to universal pensions between 2005 and 2010. The model accounts for roughly two thirds of the observed increase in pension spending spending and predicts a 0.4 percentage point decrease in private savings and a 3.1% decrease in labor supply. Taxation was found to be the main factor behind the reduction in private savings, while the change in pension system characteristics was the primary reason for the decrease in labor supply. Simulations suggest that changes in the degree to which benefits are Bismarckian have a much smaller effect on equilibrium taxes compared to changes in the degree of contributiveness of the system.

In addition, the model was calibrated to the United States in 2020 to evaluate the potential impact of the reduction in labor supply preferences caused by the COVID-19 pandemic on social security. Our results, which showed that a modest rise in leisure preferences could lead to a 1.8 p.p. increase in the tax rate if the preference change is permanent, highlight the significance of leisure preferences in explaining disparities in labor supply between the United States and Europe.

We made a contribution to the field of numerical methods for finding politico-economic equilibrium. By using a backward recursive EGM algorithm, we identified a series of time-dependent policy functions that depend on future policies. Importantly we showed that the conventional approach, in the context of population ageing in overlapping generations models, of approximating time-dependent policies with time-independent steady state policies might overestimate future changes in tax rates.

Our model could be valuable in exploring the challenges posed by the rise of new types of platform work, such as zero-hour contracts with no guaranteed hours, which make formal labor less attractive. This raises concerns about the pension coverage for workers engaged in these activities, as these workers face low job security under existing legislation. Governments also face difficulties in offering pension protection, as it is difficult to determine the level of self-employment or dependency of workers under these types of contracts.

It is important to consider the limitations of our results. By assuming that young households are ex-ante identical, we cannot account for involuntary unemployment and income inequality. Workers who face a higher risk of losing pension benefits due to past unemployment would increase their labor supply and engage in higher precautionary savings, and would generally not behave like workers who are not affected by unemployment and have a lower risk of losing benefits. Based on Song (2011), we speculate that these distributional concerns could make universal benefits even more politically desirable.

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A The Economic Equilibria

A.1 Deriving the economic equilibria

The representative young household i solves

$$\max_{c^i_{1,t},c^{i,1}_{2,t+1},c^{i,0}_{2,t+1},h^i_t,s^i_t} \ln(c^i_{1,t}) + \beta\left[h^i_t \ln(c^{i,1}_{2,t+1}) + \left(1-h^i_t\right)\ln\left(c^{i,0}_{2,t+1}\right)\right] \tag{22a}$$

$$\text{s.t. } c^i_{1,t} = w_t\left[(1-\tau_t)h^i_t + \frac{w^*_t}{w_t}F(h^i_t)\right] - s^i_t \tag{22b}$$

$$c^{i,1}_{2,t+1} = s^i_t R_{t+1} + b^{i,1}_{t+1} \tag{22c}$$

$$c^{i,0}_{2,t+1} = s^i_t R_{t+1} + b^{i,0}_{t+1}. \tag{22d}$$

where benefits follow

$$b^{i,1}_{t+1} = \left(\theta\frac{h^i_t}{h_t} + (1-\theta)\right) g(h_t)\nu_{t+1}w_{t+1}h_{t+1}\tau_{t+1}(1+\epsilon), \qquad g(h_t) = \frac{1}{1+\epsilon(2h_t-1)},$$

$$b^{i,0}_{t+1} = \left(\theta\frac{h^i_t}{h_t} + (1-\theta)\right) g(h_t)\nu_{t+1}w_{t+1}h_{t+1}\tau_{t+1}\left(1-\epsilon\right).$$

In the universal case $\epsilon = 0$, and in the contributive case $\epsilon \in (0,1]$. The first order conditions are in general given by

$$\frac{1}{c^i_{1,t}} = \beta\left[\frac{h^i_t R_{t+1}}{c^{i,1}_{2,t+1}} + \frac{(1-h^i_t)R_{t+1}}{c^{i,0}_{2,t+1}}\right] \tag{23a}$$

$$\frac{w_t\left[1-\tau_t+\frac{w^*_t}{w_t}F'(h^i_t)\right]}{c^i_{1,t}} + \beta\left[\ln\left(\frac{c^{i,1}_{2,t+1}}{c^{i,0}_{2,t+1}}\right) + \frac{h^i_t}{c^{i,1}_{2,t+1}}\frac{\partial b^{i,1}_{t+1}}{\partial h^i_t} + \frac{1-h^i_t}{c^{i,0}_{2,t+1}}\frac{\partial b^{i,0}_{t+1}}{\partial h^i_t}\right] = 0 \tag{23b}$$

We note that (23a) is the same in the Bismarckian and the Beveridgean cases. Furthermore, with ex-ante identical households, $h^i_t = h_t$. Thus, given tax rates and labor supply, the pension benefits are identical as well. Using the equilibrium wage and interest rates, the budget constraints, along with the assumption of ex-ante identical households, second period consumption, $c^{i,1}_{2,t+1}, c^{i,0}_{2,t+1}$, can be written as:

$$c^{i,1}_{2,t+1} = s_t\alpha A\left[1 + \frac{1-\alpha}{\alpha}\tau_{t+1}g(h_t)\left(1+\epsilon\right)\right]$$

$$c^{i,0}_{2,t+1} = s_t\alpha A\left[1 + \frac{1-\alpha}{\alpha}\tau_{t+1}g(h_t)\left(1-\epsilon\right)\right].$$

To simplify notation we introduce the following auxiliary parameters

$$\gamma^{\epsilon,\theta,0} = 1 + \frac{1-\alpha}{\alpha}\tau_{t+1}g\left(h_t^{\epsilon,\theta}\right)(1-\epsilon)$$
$$\gamma^{\epsilon,\theta,1} = 1 + \frac{1-\alpha}{\alpha}\tau_{t+1}g\left(h_t^{\epsilon,\theta}\right)(1+\epsilon).$$

Note that the γ's only depend on θ through its potential impact on h_t and τ_{t+1}. Using the first order condition for savings and imposing $s_t^i = s_t$ and $h_t^i = h_t$, we get:

$$s_t^{\epsilon,\theta} = \frac{\beta\left(\gamma^{\epsilon,\theta,1}(1-h_t^{\epsilon,\theta}) + \gamma^{\epsilon,\theta,0}h_t^{\epsilon,\theta}\right)}{\beta\left(\gamma^{\epsilon,\theta,1}(1-h_t^{\epsilon,\theta}) + \gamma^{\epsilon,\theta,0}h_t^{\epsilon,\theta}\right) + \gamma^{\epsilon,\theta,0}\gamma^{\epsilon,\theta,1}} w_t\left[(1-\tau_t)h_t^{\epsilon,\theta} + \frac{w_t^*}{w_t}F(h_t^{\epsilon,\theta})\right]$$
$$\equiv \delta^{\epsilon,\theta}\mathcal{I}_t \tag{24a}$$

where $\delta^{\epsilon,\theta}$ is the propensity to save out of labor income ($\mathcal{I}_t$). We note that θ does not directly affect δ, but might still be relevant in equilibrium through the effects on h_t and τ_{t+1}. From budget constraints we then have

$$c_{1,t}^{\epsilon,\theta} = \left(1-\delta^{\epsilon,\theta}\right)\mathcal{I}_t \tag{24b}$$
$$c_{2,t+1}^{\epsilon,\theta,0} = \alpha A\delta^{\epsilon,\theta}\gamma^{\epsilon,\theta,0}\mathcal{I}_t \tag{24c}$$
$$c_{2,t+1}^{\epsilon,\theta,1} = \alpha A\delta^{\epsilon,\theta}\gamma^{\epsilon,\theta,1}\mathcal{I}_t. \tag{24d}$$

In the general case, the labor supply function is implicitly determined by the h_t that solves:

$$h_t = \left\{\frac{1-\tau_t}{X^{1+\xi}} + \beta\left[\frac{1-\tau_t}{X^{1+\xi}} + \frac{\xi}{1+\xi}\left(1-h_t^{1+1/\xi}\right)\right]\Gamma\left(h_t,\tau_{t+1}\right)\right\}^{\frac{\xi}{1+\xi}}, \tag{24e}$$

where the function Γ is defined as

$$\Gamma^{\epsilon,\theta}\left(h_t,\tau_{t+1}\right) \equiv \left(1-\delta^{\epsilon,\theta}\right)\left[h_t\ln\left(\frac{\gamma^{\epsilon,\theta,1}}{\gamma^{\epsilon,\theta,0}}\right) + \theta\left(\frac{\gamma^{\epsilon,\theta,1}-1}{\gamma^{\epsilon,\theta,1}}h_t + \frac{\gamma^{\epsilon,\theta,0}-1}{\gamma^{\epsilon,\theta,0}}(1-h_t)\right)\right]. \tag{24f}$$

We note that if the right-hand side of (24e)>1 for $h_t = 1$ the solution would be the corner with $h_t = 1$. Imposing assumption 1, however, guarantees an interior solution.

From (24e) and (24f) it follows that when $\epsilon = \theta = 0$ labor supply is chosen to maximize after tax labor income $\mathcal{I}_t$. Thus, $\mathcal{I}_t$ is decreasing in both ϵ and θ.

Furthermore, if we differentiate the individual choice from the aggregate choice in (23b) it follows that whenever $\epsilon = 0$, an increase in h_t will give an individual household an incentive to reduce h_t^i. When $\epsilon > 0$ and $\theta = 0$ this FOC can be rewritten as

$$\frac{w_t^* h^{i^{1/\xi}} - w_t(1-\tau_t)}{c_{1,t}^i \ln\left(\frac{c_{2,t+1}^{i,1}}{c_{2,t+1}^{i,0}}\right)} = \beta,$$

where $w_t^* = (1-\alpha)AX^{1/\xi}k_t$. An increase in h_t reduces w_t, $c_{1,t}^i$, and $\frac{c_{2,t+1}^{i,1}}{c_{2,t+1}^{i,0}}$. Thus, unambiguously it must be met with a reduction in h_t^i. When $\theta > 0$ there is a new term in the denominator, see (23b). But this term is also decreasing in h_t.[40] Thus, we conclude that aggregate employment and individidual employment are always strategic substitutes.

A.1.1 Contributive system

A contributive social security system is described by (24) with $\epsilon \in (0, 1]$, such that households are exposed to pension risk. We note that for a given level of labor supply and future taxes (h_t, τ_{t+1}), the consumption and savings functions, γ, $c_{1,t}$, and $c_{2,t+1}$, are all independent of θ. The Γ function, however, does rely explicitly on both ϵ and θ, and thus, so will the resulting laborS1mon2012 supply $h_t^{\epsilon,\theta}$.

A.1.2 Universal system

In a universal social security system $\epsilon = 0$. Savings and consumption functions still do not depend on θ. In this case the savings rate is $\delta^0 = \dfrac{\beta}{\beta+\gamma^0}$, and auxiliary parameters and the Γ function simplify to

$$\begin{aligned}\gamma^{0,\theta,0} &= \gamma^{0,\theta,1} = \gamma^0 \equiv 1 + \frac{1-\alpha}{\alpha}\tau_{t+1}.\\ \Gamma^{0,\theta} &= \theta\frac{\gamma^0 - 1}{\beta+\gamma^0}\end{aligned}$$

Using this, the labor supply function (24e) simplifies to

$$h_t^{0,\theta} = \left\{\frac{1-\tau_t}{X^{1+\xi}} + \beta\theta\frac{\gamma^0-1}{\gamma^0+\beta}\left[\frac{1-\tau_t}{X^{1+\xi}} + \frac{\xi}{1+\xi}\left(1-\left(h_t^{0,\theta}\right)^{1+1/\xi}\right)\right]\right\}^{\frac{\xi}{1+\xi}}.$$

[40]This follows since $\frac{g(h_t)}{c_{2,t+1}^{i,j}}$ is decreasing in h_t for $j = 0, 1$.

We note that as γ^0 does not depend on h_t, we can represent $h_t^{0,\theta}$ in closed form in this case; after some algebraic steps we get:

$$h_t^{0,\theta} = \left(\frac{\left(\frac{1-\tau_t}{X^{1+\xi}}\right)\left(1+\theta\beta\frac{\gamma^0-1}{\gamma^0+\beta}\right)+\theta\beta\frac{\gamma^0-1}{\gamma^0+\beta}\frac{\xi}{1+\xi}}{1+\theta\beta\frac{\gamma^0-1}{\gamma^0+\beta}\frac{\xi}{1+\xi}} \right)^{\frac{\xi}{1+\xi}}. \tag{25}$$

A.1.3 Some derivations for the economic labor supply function

Consider the labor supply function in the general case. Define the auxiliary function $\chi(h_t;\tau_t,\tau_{t+1},\theta,\epsilon)$ as the right-hand side of (24e). It is straightforward that:

$$\frac{\partial\chi}{\partial\tau_t}\leq 0, \qquad \frac{\partial\chi}{\partial\tau_{t+1}}\geq 0, \qquad \frac{\partial\chi}{\partial\epsilon}\geq 0, \qquad \frac{\partial\chi}{\partial\theta}\geq 0, \qquad \chi\geq 0, \tag{26}$$

for all values $(h_t,\tau_t,\tau_{t+1},\epsilon,\theta)\in[0,1]^5$. For $h_t=0$ this simplifies to

$$\chi(0)=\left\{\frac{1-\tau_t}{X^{1+\xi}}\left(1+\theta\frac{\beta(\gamma^{\epsilon,\theta,0}-1)}{\beta+\gamma^{\epsilon,\theta,0}}\right)+\theta\frac{\xi}{1+\xi}\frac{\beta(\gamma^{\epsilon,\theta,0}-1)}{\beta+\gamma^{\epsilon,\theta,0}}\right\}^{\frac{\xi}{1+\xi}},$$

Similarly around $h_t=1$ we have

$$\chi(1)=\left(\frac{1-\tau_t}{X^{1+\xi}}\right)^{\frac{\xi}{1+\xi}}\left(1+\frac{\beta\gamma^{\epsilon,\theta,1}}{\beta+\gamma^{\epsilon,\theta,1}}\ln\left(\frac{\gamma^{\epsilon,\theta,1}}{\gamma^{\epsilon,\theta,0}}\right)+\theta\frac{\beta(\gamma^{\epsilon,\theta,1}-1)}{\beta+\gamma^{\epsilon,\theta,1}}\right)^{\frac{\xi}{1+\xi}},$$

To guarantee an interior solution, consider the requirement that

$$\max_{\tau_t,\tau_{t+1},\epsilon,\theta\in[0,1]}\left(\chi(1;\tau_t,\tau_{t+1},\epsilon,\theta)\right)<1.$$

This is equivalent to ensuring that

$$X>\left(1+\beta\frac{\left(1+\frac{1-\alpha}{\alpha}\right)\ln\left(1+\frac{1-\alpha}{\alpha}\right)+\frac{1-\alpha}{\alpha}}{1+\beta+\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1+\xi}}. \tag{27}$$

The labor response to a tax change (in economic equilibrium) is defined as:[41]

$$\frac{\partial h_t}{\partial \tau_t} = -\frac{1+\beta\Gamma}{X^{1+\xi}\left\{\beta\Gamma h_t^{\frac{1}{\xi}} + \frac{1+\xi}{\xi}h_t^{\frac{1}{\xi}} - \beta\left[\frac{1-\tau_t}{X^{1+\xi}} + \frac{\xi}{1+\xi}(1-h_t^{1+1/\xi})\right]\frac{\partial\Gamma}{\partial h_t}\right\}}.$$

A.2 Proof of proposition 1

First, recall that given labor supply and tax rates, the optimal consumption and savings functions, $\delta^{\epsilon,\theta}, \gamma^{\epsilon,\theta,0}, \gamma^{\epsilon,\theta,1}$, are not affected by θ. Parameter θ only affects the equilibrium level of labor supply and, in politico-economic equilibrium, taxes.

Consider labor in equilibrium, given policy. Note that $\chi(h_t;\cdot)$ is continuously differentiable and $\chi(h_t;\cdot) \geq 0$ for all $h_t \in [0,1]$ (with equality only for $\tau_t = 1$), and under Assumption 1, $\chi(1;\cdot) < 1$. Thus, there exists at least one $0 \leq h_t < 1$ such that $h_t = \chi(h_t;\cdot)$. Uniqueness follows since we have proved that individual and aggregate labor supply are strategic substitutes for all ϵ and θ.

Consider two pension systems (x^1, x^2):

$$\text{If } \forall h_t \in [0,1]: \ \chi^1(h_t;\cdot) \geq \chi^2(h_t;\cdot), \qquad \text{then } h_t^1 \geq h_t^2, \tag{28}$$

Using this it is straightforward to show that for two pension systems where $\epsilon^1 > \epsilon^2$ we have that $h_t^1 \geq h_t^2$. Similarly, for two pension systems where $\theta^1 > \theta^2$ we know that $h_t^1 \geq h_t^2$. In other words: Given taxes, labor supply is higher when incentives are Bismarckian compared to Beveridgean, and when benefits are contributive compared to universal.

Next, consider the way labor supply changes with tax rates in the universal case. The semi-elasticity is generally defined as:

$$-\frac{\partial ln(h_t^{0,\theta})}{\partial \tau_t} = \frac{\xi}{X^{1+\xi}(1+\xi)}\left(h_t^{0,\theta}\right)^{-\frac{1+\xi}{\xi}} \frac{1+\beta\theta\frac{\gamma^0-1}{\gamma^0+\beta}}{1+\beta\theta\frac{\gamma^0-1}{\gamma^0+\beta}\frac{\xi}{1+\xi}}.$$

Substituting $h_t^{0,\theta}$ from (25) it follows that the semi-elasticity is decreasing in θ.

[41] This follows from the derivative of the equilibrium function of labor supply given policy, where the auxiliary derivative is defined as

$$\frac{\partial\Gamma}{\partial h_t} = -\frac{\partial\delta}{\partial h_t}\frac{\Gamma}{1-\delta} + (1-\delta)\left[\ln\left(\frac{\gamma^1}{\gamma^0}\right) + h_t\left(\frac{\partial\ln(\gamma^1)}{\partial\ln(h_t)} - \frac{\partial\ln(\gamma^0)}{\partial\ln(h_t)}\right)\right.$$
$$\left. +\theta\left(\frac{1}{\gamma^1}\frac{\partial\ln(\gamma^1)}{\partial\ln(h_t)} + \frac{1-h_t}{h_t\gamma^0}\frac{\partial\ln(\gamma^0)}{\partial\ln(h_t)} + \frac{\gamma^1-\gamma^0}{\gamma^1\gamma^0}\right)\right],$$

where we have used that the functions $\gamma^0, \gamma^1, \delta$ all only depend on τ_t through h_t.

We now turn to the effect of system characteristics on savings rates. Note that for a universal system, the savings rate does not depend on h_t:

$$\delta^{0,\theta}(\tau_{t+1}) = \frac{\beta}{1+\beta+\frac{1-\alpha}{\alpha}\tau_{t+1}} = \frac{\beta}{\beta+\gamma^0}$$

whereas in a contributive system the propensity to save in general depends on h_t directly, and through the γ functions:

$$\delta^{\epsilon,\theta}(h_t,\tau_{t+1}) = \frac{\beta(\gamma^{\epsilon,\theta,1}(1-h_t^{\epsilon,\theta})+\gamma^{\epsilon,\theta,0}h_t^{\epsilon,\theta})}{\beta(\gamma^{\epsilon,\theta,1}(1-h_t^{\epsilon,\theta})+\gamma^{\epsilon,\theta,0}h_t^{\epsilon,\theta})+\gamma^{\epsilon,\theta,0}\gamma^{\epsilon,\theta,1}}.$$

In general, the derivative

$$\begin{aligned}\frac{\partial\delta}{\partial h_t} =& \frac{\beta(\gamma^{\epsilon,\theta,0}-\gamma^{\epsilon,\theta,1})}{\left[\beta(\gamma^{\epsilon,\theta,1}(1-h_t^{\epsilon,\theta})+\gamma^{\epsilon,\theta,0}h_t^{\epsilon,\theta})+\gamma^{\epsilon,\theta,0}\gamma^{\epsilon,\theta,1}\right]^2}\\ &\times\left(\gamma^{\epsilon,\theta,0}\gamma^{\epsilon,\theta,1}-\frac{h_t^{\epsilon,\theta}(\gamma^{\epsilon,\theta,0})^2(1+\epsilon)+(1-h_t^{\epsilon,\theta})(\gamma^{\epsilon,\theta,1})^2(1-\epsilon)}{1+\epsilon(2h_t^{\epsilon,\theta}-1)}\right),\end{aligned}$$

can be either positive or negative. Thus, the full effect of θ on the propensity to save is analytically ambiguous. We note, however, that in the case of purely contributive pensions with $\epsilon=1$ the sign is unambiguously negative $\partial\delta/\partial h_t<0$. With h_t being an increasing function of θ, an increase in θ thus reduces savings in the special case of $\epsilon=1$.

Furthermore, we can show that given tax rates, the savings rate is higher under contributive systems. First, note that for $h_t=0$, $\delta^{0,\theta}\leq\delta^{\epsilon,\theta}$ (strict inequality if $\epsilon>0$) and that for $h_t=0$, $\delta^{0,\theta}=\delta^{\epsilon,\theta}$. From the equation above it is straightforward to show that $\partial\delta/\partial h_t>0$ in a neighborhood of $h_t=0$ and $\partial\delta/\partial h_t=0$ only once for $h_t\in[0,1]$ (and $\epsilon\in(0,1]$). Thus, $\delta^{\epsilon,\theta}\geq\delta^{0,\theta}$.

Finally, it is straightforward to show that the labor supply in all scenarios is decreasing in τ_t, and increasing in τ_{t+1}. By (28), a parameter that increases $\chi(h_t;\cdot)$ for all $h_t\in[0,1]$, increases the equilibrium labor supply. It follows that in general $\partial\chi/\partial\tau_t\leq0$ and $\partial\chi/\partial\tau_{t+1}\geq0$.

B The Politico-Economic Equilibrium

B.1 A note on the equilibrium labor income function

With our assumptions, the equilibrium labor income function is given by:

$$\begin{aligned}\mathcal{I}_t^{\epsilon,\theta} &= (1-\alpha)A\frac{k_t^{\epsilon,\theta}}{h_t^{\epsilon,\theta}}(1-\tau_t)h_t^{\epsilon,\theta} + w_t^* F(h_t^{\epsilon,\theta}) \\ &= (1-\alpha)Ak_t^{\epsilon,\theta}(1-\tau_t) + w_t^* F(h_t^{\epsilon,\theta}),\end{aligned}$$

Note that in this case the direct effect of a tax change is given by

$$\frac{\partial \mathcal{I}_t^{\epsilon,\theta}}{\partial \tau_t} = -(1-\alpha)Ak_{t-1}^{\epsilon,\theta} + w_t^* F'(h_t^{\epsilon,\theta})\frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t}.$$

Note that there are opposing effects on $\mathcal{I}_t^{\epsilon,\theta}$ from a tax change. The reason is that in the economic equilibrium we have a capital thinning effect that lowers wages when h is raised. Thus reducing aggregate formal labor supply increases the per-unit wage rate. To ensure that the presence of home production does not make it optimal for young households to impose a large labor tax on themselves we need to ensure that

$$\begin{aligned} w_t^* F'(h_t^{\epsilon,\theta})\frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t} &< (1-\alpha)Ak_t^{\epsilon,\theta}, \\ -X^{1+1/\xi} h_t^{\epsilon,\theta\,\frac{1}{\xi}} \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t} &< 1. \end{aligned} \tag{29}$$

In the last expression we used that $w_t^* = (1-\alpha)Ak_tX^{\xi}$ and $F'(h_t) = Xh_t^{1/\xi}$. For the universal system condition (29) is satisfied. Given (25),

$$-X^{1+1/\xi} h_t^{\epsilon,\theta\,\frac{1}{\xi}} \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t} = \frac{\xi}{1+\xi}\frac{1+\theta\beta\frac{\gamma^0-1}{\gamma^0+\beta}}{1+\theta\beta\frac{\gamma^0-1}{\gamma^0+\beta}\frac{\xi}{1+\xi}} < 1.$$

For contributive systems, when $\epsilon \approx 0$ the condition will be satisfied by continuity. We impose assumption 1 ii. to guarantee it holds for all $0 < \epsilon \leq 1$. In our numerical simulations it is always the case that (29) is satisfied.

B.2 Proof of Proposition 2

We start by conjecturing that the policy function $\tau^{0,\theta}(\cdot)$ is independent of endogenous states, $s_{t-1}^{0,\theta}$ and $h_{t-1}^{0,\theta}$ (note that at the time benefits are received $(\theta h^i/h + (1-\theta)) = 1$ independently of past incentives). Under this conjecture, we can substitute the economic equilibrium consumption functions into the political problem and rewrite it as follows:

$$\tau^{0,\theta}(\nu_t) = \arg\max_{\tau_t \in [0,1]} \omega \ln\left(1 + \frac{1-\alpha}{\alpha}\tau_t\right) + \nu_t(1+\beta)\ln(\mathcal{I}_t) + e_t,$$

where e_t contains all the terms that under our conjecture are independent of the choice of τ_t. This yields the FOC for equilibrium tax

$$\frac{\omega(1-\alpha)}{\alpha + (1-\alpha)\tau_t} + \nu_t(1+\beta)\frac{\partial \mathcal{I}_t / \partial \tau_t}{\mathcal{I}_t} = 0, \tag{30}$$

The labor income function in the universal case can be written as

$$\mathcal{I}_t^{0,\theta} = A(1-\alpha)k_t \frac{(1-\tau_t)\frac{1}{1+\xi} + \frac{\xi}{1+\xi}X^{1+\xi}}{1 + \beta\theta\frac{\gamma^0 - 1}{\gamma^0+\beta}\frac{\xi}{1+\xi}}$$

Recall that γ^0 is independent of θ in the universal case. While it depends on τ_{t+1}, this is independent of τ_t under our conjecture that the policy function $\tau^{0,\theta}$ is independent of endogenous states. The implication is that $\mathcal{I}_t^{0,\theta}$ is log-separable in all terms related to θ and τ_{t+1}.

Thus, independently of θ, we can derive the equilibrium tax rate as:

$$\tau_t^{0,\theta} = \tau_t^0 = \frac{1}{\omega + \nu_t(1+\beta)}\left[\omega\left(1 + \xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_t(1+\beta)\right]. \tag{31}$$

Note that if $\tau_t^0 > 1$ then the solution is in the corner of $\tau_t^0 = 1$ and if $\tau_t^0 < 0$ then we have the corner solution $\tau_t^0 = 0$. Note furthermore that this confirms our conjecture that the policy function is indeed independent of endogenous states $s_{t-1}^{0,\theta}$ and $h_{t-1}^{0,\theta}$. It is straightforward to verify that this function is increasing in ω and decreasing in ν_t. If there is a terminal date, say T, the political objective at such date is derived in a similar manner, but with workers living only one period. This is given by:

$$\tau^0(\nu_T) = \max_{\tau_T \in [0,1]} \omega \ln\left(c_{2,T}^0\right) + \nu_T \ln\left(c_{1,T}^0\right),$$

where $c_{2,T}^0$ follows the usual household solution formula and $c_{1,T}^0 = \mathcal{I}_T$ from imposing

$s_T = 0$ in the budget. This yields the same problem as for $t < T$ where $\beta = 0$ is imposed. It is immediate that the policy function $\tau^0(\cdot)$ is the unique Markov perfect equilibrium in the limit of the finite horizon economy.

B.3 Proof of Proposition 3

Substituting the economic equilibrium constraints in the political program and omitting all terms independent of the choice of policy, we can write an equivalent objective function for the policy choice at time t, for contributive system:

$$\begin{aligned} W_t^{\epsilon,\theta} =& \omega \left[h_{t-1}^{\epsilon,\theta} \ln\left(\gamma_{t-1}^{\epsilon,\theta,1}\right) + (1 - h_{t-1}^{\epsilon,\theta}) \ln\left(\gamma_{t-1}^{\epsilon,\theta,0}\right) \right] \\ &+ \nu_t \left[\ln\left(1 - \delta_t^{\epsilon,\theta}\right) + \ln\left(\mathcal{I}_t\right)(1+\beta) + \beta\left(\ln\left(\delta_t^{\epsilon,\theta}\right) + h_t^{\epsilon,\theta} \ln\left(\gamma_t^{\epsilon,\theta,1}\right) + (1 - h_t^{\epsilon,\theta}) \ln\left(\gamma_t^{\epsilon,\theta,0}\right) \right) \right] \end{aligned}$$

where the labor income function ($\mathcal{I}$), labor supply function (h), and parameter functions γ, δ are defined in appendix A. The policymaker maximizes $W_t^{\epsilon,\theta}$ subject to the constraint that future policies are defined by continuation policies, i.e. $\tau_{t+1} = \tau_{t+1}^{\epsilon,\theta}(z_{t+1})$. Before we proceed to the characterization of the equilibrium policy, we define the *politico-economic equilibrium labor function* as the labor function that internalizes the effect through the continuation tax function, i.e. the h_t implicitly determined from

$$h_t \equiv h\left(\tau_t, \tau_{t+1}^{\epsilon,\theta}(z_{t+1})\right), \quad h_t \in z_{t+1} \tag{32}$$

The total response of labor to taxes from this function is defined as

$$\frac{dh_t^{\epsilon,\theta}}{d\tau_t} = \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t} + \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}^{\epsilon,\theta}}{\partial \tau_t} < 0, \qquad \frac{\partial \tau_{t+1}^{\epsilon,\theta}}{\partial \tau_t} \equiv \frac{\partial \tau_{t+1}^{\epsilon,\theta}}{\partial h_t} \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t} < 0,$$

where the partial derivatives of $h_t^{\epsilon,\theta}$ are defined from the labor equilibrium condition, i.e. $\partial h_t^{\epsilon,\theta}/\partial \tau_t < 0$, $\partial h_t^{\epsilon,\theta}/\partial \tau_{t+1} > 0$, and as we will show shortly, an increase in h_t increases in the following period the benefit of taxation for retirees, and $\partial \tau_{t+1}^{\epsilon,\theta}/\partial h_t > 0$. Note that we can rewrite this as

$$\frac{dh_t^{\epsilon,\theta}}{d\tau_t} = \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t} \left[1 + \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}^{\epsilon,\theta}}{\partial h_t^{\epsilon,\theta}} \right] \leq \frac{\partial h_t^{\epsilon,\theta}}{\partial \tau_t}. \tag{33}$$

In other words, we can confirm that the politico-economic labor response to an increase in τ_t is numerically larger (more negative) than the equilibrium response given policies. The reason is that an increase in the current tax lowers h_t on impact; in the following period $t+1$ this implies that a smaller share of retirees receive benefits. As a smaller share of retirees prefers higher taxes, this reduces the equilibrium tax rate in $t+1$. In other words, increasing the current tax rate leads to the expectation that future taxes will be lowered; with the prospect of lower future taxes, the expected future pension transfers drops as well, further reducing the incentives to work in the formal sector.

To investigate the effects of τ_t on the political objective $W_t^{\epsilon,\theta}$ we start looking at three separate terms.

First, the marginal effect on retirees' political support from a marginal tax increase ($\mathcal{E}_{2,t}^{\epsilon,\theta}$):

$$\begin{aligned}\mathcal{E}_{2,t}^{\epsilon,\theta} =& \omega\left[h_{t-1}^{\epsilon,\theta}\frac{d\ln\left(\gamma_{t-1}^{\epsilon,\theta,1}\right)}{d\tau_t} + (1-h_{t-1}^{\epsilon,\theta})\frac{d\ln\left(\gamma_{t-1}^{\epsilon,\theta,0}\right)}{d\tau_t}\right] \\ =& \frac{\omega}{\tau_t}\left[h_{t-1}^{\epsilon,\theta}\frac{\gamma_{t-1}^{\epsilon,\theta,1}-1}{\gamma_{t-1}^{\epsilon,\theta,1}} + (1-h_{t-1}^{\epsilon,\theta})\frac{\gamma_{t-1}^{\epsilon,\theta,0}-1}{\gamma_{t-1}^{\epsilon,\theta,0}}\right] \geq 0\end{aligned}$$

We want to show that $\mathcal{E}_{2,t}^{\epsilon,\theta} \leq \mathcal{E}_{2,t}^{0,\theta}$. We start by noting that, in the special case where $\epsilon = 1$, $\mathcal{E}_{2,t}^{\epsilon,\theta}$ simplifies to

$$\mathcal{E}_{2,t}^{1,\theta} = \frac{\omega(1-\alpha)}{\alpha + (1-\alpha)\tau_t/h_{t-1}^{\epsilon,\theta}} \leq \mathcal{E}_{2,t}^{0,\theta}.$$

This holds with equality when $h_{t-1} = 1$. Note that it is also the case that $\mathcal{E}_{2,t}^{\epsilon,\theta} = \mathcal{E}_{2,t}^{0,\theta}$ when $\epsilon \to 0$. Thus, all we need to prove is that $\frac{d\mathcal{E}_{2,t}^{\epsilon,\theta}}{d\epsilon} \leq 0$ for all $0 \leq \epsilon \leq 1$. For this we take derivative of $\mathcal{E}_{2,t}^{\epsilon,\theta}$ (taking $h_{t-1}^{\epsilon,\theta}$ as given):

$$\begin{aligned}\frac{d\mathcal{E}_{2,t}^{\epsilon,\theta}}{d\epsilon} &= \frac{\omega}{\tau_t}\left[h_{t-1}^{\epsilon,\theta}\frac{d\gamma_{t-1}^{\epsilon,\theta,1}}{(\gamma_{t-1}^{\epsilon,\theta,1})^2 d\epsilon} + (1-h_{t-1}^{\epsilon,\theta})\frac{d\gamma_{t-1}^{\epsilon,\theta,0}}{(\gamma_{t-1}^{\epsilon,\theta,0})^2 d\epsilon}\right] \\ &= \frac{\omega}{\tau_t}\left[(1-h_{t-1}^{\epsilon,\theta})\left(\frac{1}{(\gamma_{t-1}^{\epsilon,\theta,0})^2} - \frac{1}{(\gamma_{t-1}^{\epsilon,\theta,1})^2}\right)\frac{d\gamma_{t-1}^{\epsilon,\theta,0}}{d\epsilon}\right] \leq 0,\end{aligned}$$

where in the last equality we have used that $h_{t-1}^{\epsilon,\theta}\gamma_{t-1}^{\epsilon,\theta,1} + (1-h_{t-1}^{\epsilon,\theta})\gamma_{t-1}^{\epsilon,\theta,0}$ is independent of ϵ.

Second, the marginal effect on workers' political support from labor-income changes induced by a marginal tax increase ($\mathcal{E}^I_{1,t}$):

$$\mathcal{E}^{\epsilon,\theta,I}_{1,t} = \nu_t \left(1+\beta\right) \frac{X^\xi F'(h^{\epsilon,\theta}_t)\frac{dh^{\epsilon,\theta}_t}{d\tau_t} - 1}{1-\tau_t + X^\xi F(h^{\epsilon,\theta}_t)}.$$

By assumption 1 ii., $\mathcal{E}^{\epsilon,\theta,I}_{1,t} \leq 0$, guaranteeing that higher taxes on current labor income reduce total income. Given that the labor response to a tax rate can only be solved numerically, it is not possible to identify how ϵ, θ affects the magnitude of this effect.

Finally, there are a number of reallocation effects on workers from a tax increase. We collect these in the following:

$$\begin{aligned}\mathcal{E}^{\epsilon,\theta,r}_{1,t} =& \nu_t \frac{d\delta^{\epsilon,\theta}}{d\tau_t}\frac{\beta - \delta^{\epsilon,\theta}(1+\beta)}{(1-\delta^{\epsilon,\theta})\delta^{\epsilon,\theta}} \\ &+ \nu_t\beta\left[\frac{dh^{\epsilon,\theta}_t}{d\tau_t}\ln\left(\frac{\gamma^{\epsilon,\theta,1}_t}{\gamma^{\epsilon,\theta,0}_t}\right) + \frac{\theta}{h^c_t}\left(h^{\epsilon,\theta}_t \frac{d\ln\left(\gamma^{\epsilon,\theta,1}_t\right)}{d\tau_t} + (1-h^{\epsilon,\theta}_t)\frac{d\ln\left(\gamma^{\epsilon,\theta,0}\right)}{d\tau_t}\right]\right)\end{aligned}$$

Total derivatives include partial derivatives of the economic equilibrium functions and strategic effects through the continuation policy ($\partial\tau^{\epsilon,\theta}_{t+1}/\partial\tau_t$). Again, it is not possible to determine how $\mathcal{E}^{\epsilon,\theta,r}_{1,t}$ is affected by θ or ϵ.

Consider now the terminal policy function in (19). To show this we proceed as for the universal system and set up the terminal period political objective function:

$$\tau^{\epsilon,\theta}_T\left(h^{\epsilon,\theta}_{T-1}, \nu_T\right) = \max_{\tau_T\in[0,1]} \omega\Big\{h^{\epsilon,\theta}_{T-1}\ln\left(c^{\epsilon,\theta,1}_{2,T}\right) + \left(1-h^{\epsilon,\theta}_{T-1}\right)\ln\left(c^{\epsilon,\theta,0}_{2,T}\right)\Big\} + \nu_T \ln\left(c^{\epsilon,\theta}_{1,T}\right),$$

where $c^{\epsilon,\theta}_{1,T}$ is defined from the budget of the young. Note that the objective is the same for Bismarkian and Beveridgean systems as labor decisions are static in the last period. Plugging in the economic equilibrium functions and omitting terms not relevant for the choice of τ_T we can equivalently present the political objective function as:

$$\tau^{\epsilon,\theta}_T\left(h^{\epsilon,\theta}_{T-1}, \nu_T\right) = \max_{\tau_T\in[0,1]} \omega\left[h_{T-1}\ln\left(\gamma^{\epsilon,\theta,1}_{T-1}\right) + (1-h^{\epsilon,\theta}_{T-1})\ln\left(\gamma^{\epsilon,\theta,0}_{T-1}\right)\right] + \nu_T\ln\left(\mathcal{I}_T\right),$$

where the terminal period's choice of labor supply is equivalent to that of the universal pension system's such that $\mathcal{I}_T$ is independent of θ, ϵ. The marginal effect of τ_T is given by

$$\mathcal{E}_T = \frac{\omega}{\tau_T}\left[h^{\epsilon,\theta}_{T-1}\frac{\gamma^{\epsilon,\theta,1}_{T-1}-1}{\gamma^{\epsilon,\theta,1}_{T-1}} + (1-h^{\epsilon,\theta}_{T-1})\frac{\gamma^{\epsilon,\theta,0}_{T-1}-1}{\gamma^{\epsilon,\theta,0}_{T-1}}\right] - \frac{\nu_T(1+\beta)}{1-\tau_T+\xi X^{1+\xi}} \tag{34}$$

We note that this is independent of θ. Furthermore, note that $\partial \mathcal{E}_T / \partial \tau_T \geq 0$ for all $\tau_T \in [0,1]$. Thus, the first order condition is sufficient if there exists a $\tau_T \in [0,1]$, where this holds. Conversely, if an interior solution does not exist, the optimal tax is 0 (1) if $\mathcal{E}_T < 0$ (> 0) for any $\tau_T \in [0,1]$.

For $t < T$ we can characterize the equilibrium policy by use of the marginal effect terms in $\mathcal{E}$. Firstly, note that if the equilibrium policy is not in a corner, it must be the case that

$$\mathcal{E}_{2,t}^{\epsilon,\theta} + \mathcal{E}_{1,t}^{\epsilon,\theta,I} + \mathcal{E}_{1,t}^{\epsilon,\theta,r} = 0.$$

First note that for us to have an interior solution at τ_t^*, we know that at least in a neighborhood of τ_t^*, $\sum \mathcal{E}$ must be positive and decreasing for $\tau_t < \tau_t^*$, and negative decreasing for $\tau_t^* > \tau_t^*$. Note furthermore that $\mathcal{E}_{2,t}^{c,\theta}$ is always positive, implying that in an interior solution we have that $\mathcal{E}_{1,t}^{\epsilon,\theta,I} + \mathcal{E}_{1,t}^{\epsilon,\theta,r} \leq 0$. Finally, note that a marginal change in $h_{t-1}^{\epsilon,\theta}$ or ω simply shifts $\mathcal{E}_{2,t}^{\epsilon,\theta}$ upwards implying an increase in the equilibrium tax rate. Conversely, ν_t simply enters as a proportional factor in $\mathcal{E}_{1,t}^{\epsilon,\theta,I} + \mathcal{E}_{1,t}^{\epsilon,\theta,r} \leq 0$. Thus an increase in ν_t reduces $\mathcal{E}_{1,t}^{\epsilon,\theta,I} + \mathcal{E}_{1,t}^{\epsilon,\theta,r}$ implying a lower equilibrium tax rate.

C Contributive Versus Universal Steady States

As shown in 4.4, given a choice among social security systems, society would always choose a universal one. Since most countries have contributive benefits, we conjecture that there are welfare loses from, presumably, higher taxes in the universal system that depress capital accumulation over longer horizons. To gauge this we compare steady state political welfare across systems. We note that with productive externalities the model features endogenous growth at rate

$$\left(\frac{k_{t+1}}{k_t}\right)^{\epsilon,\theta,BGP} = \frac{(1-\alpha)A}{\nu}\delta^{\epsilon,\theta}\left[1 - \tau^{\epsilon,\theta} + X^{\xi}F(h^{\epsilon,\theta})\right].$$

Given different levels of saving rates, tax levels and labor supply with the two pension systems the balanced growth paths diverge. To abstract from these long run effects, we now eliminate productive externalities.[42]

[42]Recall they were only introduced for analytical tractability.

C.1 Economic Equilibrium without productive externalities

The results are similar to those with externalities. In general, the labor supply functions are not changed. In the universal, Beveridgean case we further have:

$$\begin{aligned}
s_t^{0,0} =& \delta^0 \mathcal{I}_t, \\
c_{1,t}^{0,0} =& (1-\delta^0)\mathcal{I}_t, \\
c_{2,t+1}^{0,0,j} =& \alpha A \gamma^0 \left[s_t^{0,0}\right]^\alpha \left(\nu_{t+1} h_{t+1}^{0,0}\right)^{1-\alpha}, \\
\mathcal{I}_t =& (1-\alpha) A \left(\frac{s_{t-1}^{0,0}}{\nu_t}\right)^\alpha \left[(h_t^{0,0})^{1-\alpha}(1-\tau_t) + X^{\alpha\xi} F(h_t^{0,0})\right],
\end{aligned}$$

For the general case $\mathcal{I}_t$ is similarly defined and we have

$$\begin{aligned}
s_t^{\epsilon,\theta} &= \delta^{\epsilon,\theta} \mathcal{I}_t \\
c_{1,t}^{\epsilon,\theta} &= (1-\delta^{\epsilon,\theta})\mathcal{I}_t \\
c_{2,t+1}^{\epsilon,\theta,0} &= \alpha A \gamma^{\epsilon,\theta,0} \left[\delta^{\epsilon,\theta}\mathcal{I}_t\right]^\alpha \left(\nu_{t+1} h_{t+1}\right)^{1-\alpha} \\
c_{2,t+1}^{\epsilon,\theta,1} &= \alpha A \gamma^{\epsilon,\theta,1} \left[\delta^{\epsilon,\theta}\mathcal{I}_t\right]^\alpha \left(\nu_{t+1} h_{t+1}\right)^{1-\alpha}.
\end{aligned}$$

Imposing steady state implies the savings rate, for $k \in \{u, c\}$,

$$s^{\epsilon,\theta} = \left(\delta^{\epsilon,\theta}(1-\alpha) A \nu^{-\alpha} \left[h^{1-\alpha}(1-\tau) + X^{\alpha\xi} F(h)\right]\right)^{\frac{1}{1-\alpha}}.$$

C.2 Steady State welfare without productive externalities

The steady state political welfares are a useful proxy for the outcome of a political process that has more commitment than what we allow in our model.[43] In the universal and contributive cases these are given by

$$\begin{aligned}
\mathcal{U}^{0,\theta} =& \omega \ln\left(c_2^{0,\theta}\right) + \nu\left[\ln\left(c_1^{0,\theta}\right) + \beta \ln\left(c_2^{0,\theta}\right)\right] \\
\mathcal{U}^{\epsilon,\theta} =& \omega\left[h \ln\left(c_2^{\epsilon,\theta,1}\right) + (1-h)\ln\left(c_2^{\epsilon,\theta,0}\right)\right] + \nu\left[\ln\left(c_1^{\epsilon,\theta}\right) + \beta\left[h\ln\left(c_2^{\epsilon,\theta,1}\right) + (1-h)\ln\left(c_2^{\epsilon,\theta,0}\right)\right]\right]
\end{aligned}$$

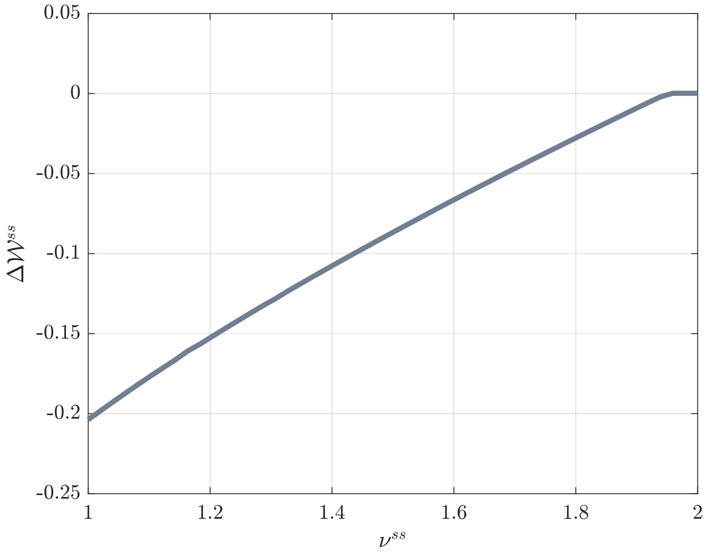
[43] In our calibration a period lasts for 30 years, so policies are in effect long lasting.

The steady state difference can be reduced to

$$\Delta\mathcal{W} = \nu\left\{\Delta\ln(1-\delta) + \frac{\alpha}{1-\alpha}\Delta\ln(\delta) + \frac{1}{1-\alpha}\Delta\ln\left[h^{\epsilon,\theta 1-\alpha}(1-\tau) + X^{\alpha\xi}F(h)\right]\right\}$$
$$+ (\omega+\beta\nu)\left\{(1-\alpha)\Delta\ln(h^{\epsilon,\theta 1-\alpha}) + \ln(\gamma^0) - h^{\epsilon,\theta}\ln(\gamma^{\epsilon,\theta,1}) - (1-h^{\epsilon,\theta})\ln(\gamma^{\epsilon,\theta,0})\right.$$
$$\left.+\frac{\alpha}{1-\alpha}\left[\Delta\ln(\delta) + \Delta\ln\left[h^{\epsilon,\theta 1-\alpha}(1-\tau) + X^{\alpha\xi}F(h)\right]\right]\right\},$$

where we have used the notation $\Delta f(x) = f(x^{0,\theta}) - f(x^{\epsilon,\theta})$. For our calibration, detailed in the next section, we show that for all $\nu \in [1,2]$ the contributive system offers higher welfare in steady state. Figure 5 shows that $\Delta\mathcal{W} \leq 0$ and increasing in ν.

Figure 5: Difference in steady state (political) welfare



D Numerical Methods

This appendix deals with the numerical methods that can be applied when solving for the PEE with contributive pensions. The appendix discusses the difference between the *infeasible general problem*, the steady state infinite horizon assumption and the finite horizon version. Specifically, we compare three models: The time-dependent model with a finite horizon (FH), the time-dependent model with an infinite horizon (IH) assumption, and the time-independent, steady state approximation (SS). The various methods are simulated to illustrate efficiency as well as differences in outcomes.

There are three main takeaways from this numerical exercise: First, in terms of accuracy, we find that for certain parameter values the SS approach produces significant errors, overestimating PEE tax rates by as much as 16.2 p.p. by 2050. To verify the robustness of this result we recalibrated the model for the case of Argentina imposing an initial tax of 27.1%, about double its value. We find that under the SS approach the predicted tax rates are still more than 6 p.p. Second, the FH and IH solution approaches produce virtually identical solutions (with the exception of the terminal state). Third, in terms of efficiency, we find that the FH solution is roughly 3 times faster than IH and more than two orders of magnitude faster than the SS (≈ 179 times). Table 6 and figures 6 and 7 illustrate the main results.

Table 6: Comparison of methods identifying the PEE

Solution method:	Time
Finite Horizon (FH)	11 seconds
Infinite Horizon (IH)	31 seconds
Steady State approximation (SS)	32 minutes

The following subsections elaborate on the specific algorithms applied for each of the three methods (FH, IH, SS) and how they differ.

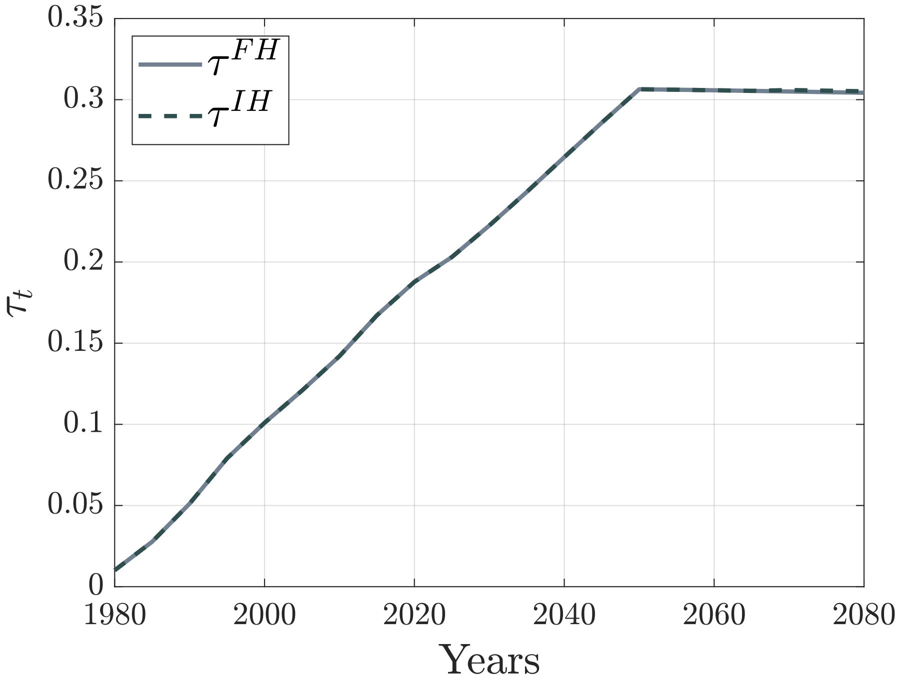
D.1 The General Case vs. Steady State Assumption

To start with, we will argue that the standard infinite horizon version is only an approximation of the true time-dependent structure. Consider the infinite horizon version of the model. As outlined in section 3 the policymaker maximizes the *political aggregator* function, here repeated for convenience:

$$\mathcal{W}(z_t, \tau_t; \tau^{t+1}(z_{t+1})) = \omega \mathcal{O}(z_t, \tau_t) + \nu_t \mathcal{Y}\left(z_t, \tau_t; \tau^{t+1}(z_{t+1})\right). \tag{35}$$

Here z_t denotes the vector of relevant states for the political decision at time t. In the general infinite horizon case this consists of a single *endogenous* state variable (h_{t-1}), as

Figure 6: Comparison of finite and infinite horizon models



well as the *entire* future path of population weights, that is:[44]

$$z_t = \Big(h_{t-1},\ \ \{\nu_\iota\}_{\iota\geq t}\Big).$$

With an infinite-dimensional state space this general infinite horizon problem is infeasible to solve without additional assumptions. The conventional way to get around this obstacle is to assume that the model converges to a steady state after T^{ss} periods. In this case the model has a recursive time-autonomous structure (for $t \geq T^{ss}$), in which case the identification of the policy and continuation policy functions is done by a fixed-point requirement (see definition 2).

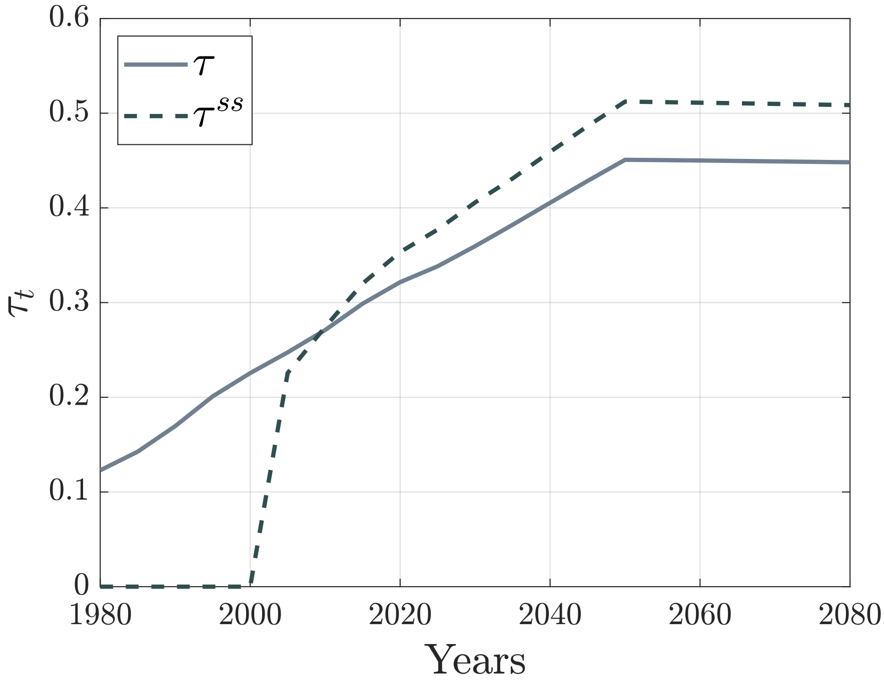
One of the traditional ways of solving for the politico-economic equilibrium in infinite horizon models is to identify the steady state time-autonomous policy function as a

[44]To see why this is the case, conjecture that the state space consists of (h_{t-1}, ν_t) and denote the corresponding continuation policy $\tilde{\tau}^t(\cdot)$. This entails that the relevant state space for the continuation policy is given by (h_t, ν_{t+1}). In this case ν_{t+1} becomes a relevant state for the political decision at time t **unless** the political decision defined by

$$\tau = \arg\max_{\tau'} \big(\omega\mathcal{O}(h_{t-1}, \nu_t, \tau') + \nu_t\mathcal{Y}\left(h_{t-1}, \tau'; \tilde{\tau}^{t+1}(h_t, \nu_{t+1})\right)\big) \tag{36}$$

is independent of ν_{t+1} (which only happens when $\epsilon = 0$). If ν_{t+1} is a relevant state at time t, then ν_{t+2} is a relevant state at time $t+1$. Following this argument all ν_ι for $\iota \geq t$ become relevant states.

Figure 7: Comparison of time-dependent policies and steady state approximation



Calibration to Argentina before social security reforms of 2005-2010 assuming initial tax of 27.1%.

function of the simplified 2-dimensional steady state state space $z = (h_{-1},\ \nu)$, (Krusell et al., 1997). We note that using the steady state policy function outside of steady state should however only be considered an approximation of the true *time-dependent* structure.

D.2 The Steady State Solution

We suggest solving the infinite horizon version of the model in one of two ways: A standard iteration scheme and an endogenous gridpoint-like method.

D.2.1 The Steady State Solution with VFI

Consider the maximization problem in (35). The complication for the numerical solution is that there is no closed form representation of $\mathcal{W}$; thus the problem faced by the policy-maker is for our purposes here more accurately presented by the constrained maximization problem

$$\tau = \arg\max_{\tau'} \left\{ W^c\left(h_{-1}, \nu, \tau', h, \tau_{+1}\right) \quad \text{s.t. } \tau_{+1} = \tau^{+1}(h, \nu) \quad \text{and } h = h(\tau', \tau_{+1}) \right\}. \tag{37}$$

The mapping $W^c(\cdot)$ is given by equation (18) and the mapping $h(\cdot)$ is the economic equilibrium condition defined by equation (17). Importantly, the economic equilibrium condition only *implicitly* defines h as a function of τ, τ_{+1}, whereas the condition can be rewritten to yield an *analytical* solution for τ:

$$\tau = 1 - \frac{X^{1+\xi}}{1 + \beta h \Gamma_{+1}} \left[h^{1+1/\xi} - \beta h \Gamma_{+1} \frac{\xi}{1+\xi} \left(1 - h^{1+1/\xi}\right) \right]. \tag{38}$$

With this in place define the ordered grids over the relevant state variables:

$$\begin{aligned} \mathcal{G}_h &\equiv \left\{ h_{-1}^1, h_{-1}^2, \ \dots \ , h_{-1}^{N_h} \right\}, \qquad & h_{-1}^1 = h(1,0), \qquad & h_{-1}^{N_h} = h(0,1). \\ \mathcal{G}_\nu &\equiv \left\{ \nu^1, \nu^2, \ \dots \ , \nu^{N_\nu} \right\}. \end{aligned}$$

The VFI approach now proceeds as follows:

i. Let $n = 0$ and define an initial guess of a policy function $\tilde{\tau}^0(h_{-1}, \nu)$.

ii. Given $\tilde{\tau}^n$, for each pair of states (i, j) on the grids $(\mathcal{G}_h, \mathcal{G}_\nu)$, solve the maximization problem

$$\tau^{i,j} = \arg\max_{\tau'} \left\{ W^c\left(h_{-1}^i, \nu^j, \tau', h, \tau_{+1}\right) \quad \text{s.t. } \tau_{+1} = \tilde{\tau}^n(h, \nu^j) \quad \text{and } h = h(\tau', \tau_{+1}) \right\}.$$

Let $\mathcal{G}_\tau^n$ define the grid of solutions from the policy guess $\tilde{\tau}^n$ on the grids $(\mathcal{G}_h, \mathcal{G}_\nu)$.

iii. Update the policy guess $\tilde{\tau}^{n+1}(h_{-1}, \nu)$ using some functional approximation approach, e.g. interpolation.

iv. Define a tolerance level $\Delta > 0$. If $\sup_{i,j} \left| \tilde{\tau}^{n+1}(h_{-1}^i, \nu^j) - \tilde{\tau}^n(h_{-1}^i, \nu^j) \right| > \Delta$ then $n \rightsquigarrow n+1$ and repeat steps ii.-iv.

The computational cost of the VFI approach is the constrained maximization problem that is carried out in the innermost loop (in step ii.) that is repeated for all (n, i, j).

D.2.2 The Steady State Solution With EGM

The idea of the EGM-type approach is to replace the constrained maximization problem with an unconstrained one that circumvents the root-finding operation of the labor equilibrium. To do this in the infinite horizon model, we have to perform two steps. First, we pre-approximate the labor equilibrium function $h(\tau, \tau_{+1})$ on a grid of (τ, τ_{+1}). Secondly, we define the policy function on an endogenous grid of τ_{-1} instead of the exogenous grid of h_{-1}. This allows us to exploit the analytical mapping (38) from h, τ_{t+1} to τ, and thus circumvent a root-finding operation for each (n, i, j).

The steady state EGM-type approach now proceeds as follows:

i. Approximate the labor equilibrium function $h = h(\tau, \tau_{+1})$ on grids of (τ, τ_{+1}). Denote the approximate function $h^a(\tau, \tau_{+1})$.

ii. Let $n = 0$ and define an initial policy guess $\tilde{\tau}^n(\tau_{-1}, \nu)$.

iii. Given $\tilde{\tau}^n$, for each pair of states (i, j) on the (exogenous) grids $(\mathcal{G}_h, \mathcal{G}_\nu)$, solve the *unconstrained* maximization problem

$$\tau^{i,j} = \arg\max_{\tau'} W^c\left(h^i_{-1}, \nu^j, \tau', \underbrace{h^a\big(\tau', \tilde{\tau}^n(\tau', \nu^j)\big)}_{\approx h}, \underbrace{\tilde{\tau}^n(\tau', \nu^j)}_{=\tau_{+1}}\right).$$

Note that in place of h we use the approximate function h^a that through $\tilde{\tau}^n$ only depends on the current tax rate (τ') and the exogenous state ν^j.

iv. From the exogenous grid of h_{-1} back out the corresponding value of τ_{-1} using (38) to obtain the *endogenous* state grid $\mathcal{G}^n_{\tau_{-1}} \equiv \left\{\tau^{1,n}_{-1}, \tau^{2,n}_{-1}, ..., \tau^{N^n_\tau,n}_{-1}\right\}$. Let $\mathcal{G}^n_\tau$ define the grid of solutions.

v. Update the policy function $\tilde{\tau}^{n+1}(\tau_{-1}, \nu)$ using some functional approximation approach, e.g. interpolation.

vi. Define a tolerance level $\Delta > 0$. If $\sup_{i,j}\left|\tilde{\tau}^{n+1}(h^i_{-1}, \nu^j) - \tilde{\tau}^n(h^i_{-1}, \nu^j)\right| > \Delta$ then $n \rightsquigarrow n + 1$ and repeat steps iii.-vi.

The EGM-type approach has the added computational cost of having to compute an approximation for the labor function in step i., whereas the gain comes from a simpler unconstrained maximization problem in step iii. In the table 6 we refer to the EGM-like approach which is roughly 2-3 times faster than the standard VFI approach.

D.2.3 Computational speed

As highlighted in the main comparison in table 6, the steady state approximation is orders of magnitude slower than the finite horizon solution. To better understand why, we gradually increase the number of gridpoints in the grid for ν ($\mathcal{G}_\nu$).

Table 7 shows the effect of using 10, 20, and 30 nodes on the grid respectively. The grids are linearly spaced from the minimum to the maximum observed ν in the Argentina data. We report the number of iterations needed to obtain convergence (# iterations), the time needed to solve the model, and a measure of convergence.[45]

Table 7: Steady State Approximation Solution Time

# nodes	# iterations	Solution time	$\min_n \sup_{i,j} \lvert\tilde{\tau}_{i,j}^{n+1} - \tilde{\tau}_{i,j}^{n}\rvert$
10	28	8.14 minutes	9.8e-06
20	19	10.93 minutes	9.0e-06
30	50	43.27 minutes	8.0e-05

Table 7 illustrates that there is a highly nonlinear relationship between the number of gridpoints and the solution time; this is because of the vastly different number of iterations required to reach convergence. As it turns out, for relatively large and small values of ν, the algorithm converges within 5-10 iterations; in these ranges of ν, the algorithm takes roughly 7-15 seconds per value of ν. The slow convergence is due to the kink in the policy function that can be seen in figures 4 and 7.

D.3 The Finite Horizon Version

Consider the finite horizon version of the model. In this case the terminal period policy (T) is defined analytically, here repeated for convenience:

$$\tau_T\left(h_{T-1}, \nu_T\right) = \min\left\{1,\ \max\left\{0,\ \frac{1}{\omega + \nu_T/h_{T-1}}\left[\omega\left(1+\xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_T\right]\right\}\right\}.$$

In this case we can solve the model as follows:

[45] In the computational literature a standard way of assessing the accuracy of approximation methods is to derive the so-called Euler-errors (Judd, 1992; Barillas and Fernández-Villaverde, 2007). As we do not have an analytical first order condition for the optimal choice of the tax rate, there is not a straightforward way of doing this in our setup. An alternative measure could be to compute deviations from the labor equilibrium condition; however as this constraint enters directly in the numerical problem, this level of error can be controlled by an option in the solver (tolerance level).

i. Create an exogenous grid of the state $\mathcal{G}_h \equiv \{h_1, ..., h_{N_h}\}$. Given the exogenous value of ν_T this solves for the equilibrium τ_T on each node on the grid using (19). Denote the corresponding grid of solutions $\mathcal{G}_\tau^T$.

ii. Given the solutions for (τ_T, h_{T-1}, ν_T) back out an endogenous grid of τ_{T-1} using the economic labor equilibrium in equation (38). Denote the endogenous grid $\mathcal{G}_{\tau_{-1}}^T$.

iii. Define the terminal period policy function (τ_T) as the interpolation approximation over the grid of τ_{T-1}. Define the PEE labor equilibrium function $(\tilde{h}_{T-1})$ as the interpolation approximation over the grid of τ_{T-1}.

For $t < T$:

iv. On the exogenous grid of the state $(\mathcal{G}_h)$ the political objective function is on node i given by

$$\tau_t^i = \arg\max_{\tau_t \in [0,1]} \mathcal{W}\left(h_{t-1}^i, \nu_t, \tau_t, \tilde{h}(\tau_t), \tau_{t+1}(\tau_t)\right), \tag{39}$$

where $h_t = \tilde{h}(\tau_t)$ and $\tau_{t+1} = \tau_{t+1}(\tau_t)$ are the approximate solution functions from $t+1$.

v. Given the solution for (τ_t, h_{t-1}, ν_t) back out an endogenous grid of τ_{t-1} using the economic equilibrium in equation (38). Denote the endogenous grid $\mathcal{G}_{\tau_{-1}}^t$.

vi. Define the period t policy function (τ_t) as the interpolation approximation over the grids $(\mathcal{G}_\tau^t, \mathcal{G}_{\tau_{-1}}^t)$. Define the PEE labor equilibrium function $(\tilde{h}_{t-1})$ as the interpolation approximation over over the grids $(\mathcal{G}_h, \mathcal{G}_{\tau_{-1}}^t)$.

D.4 The Infinite Horizon Version with time-dependent Functions

The infinite horizon version with time-dependent policy functions out of steady state is essentially a combination of the previous two subsections: We apply the steady state solution approach from section D.2 to determine the policy for the terminal value of population growth ν_T. Given this terminal policy, we use the method from section D.3 to iterate back for $t < T$.