Individual Irrationality and Aggregate Outcomes

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Abstract:

There is abundant evidence that many individuals violate the rationality assumptions routinely made in economics. However, powerful evidence also indicates that violations of individual rationality do not necessarily refute the aggregate predictions of standard economic models that assume full rationality of all agents. Thus, a key question is how the interactions between rational and irrational people shape the aggregate outcome in markets and other institutions. We discuss evidence indicating that strategic complementarity and strategic substitutability are decisive determinants of aggregate outcomes. Under strategic complementarity, a small amount of individual irrationality may lead to large deviations from the aggregate predictions of rational models, whereas a minority of rational agents may suffice to generate aggregate outcomes consistent with the predictions of rational models under strategic substitutability.

Keywords: Bounded rationality, Strategic interaction, Strategic complementarity

JEL-Codes: D01, D50, D84
Introduction

At least in their personal lives, many economists recognize that they are surrounded by individuals who are less than fully rational. In their professional lives, however, economists often use models which examine the interactions of fully rational agents. To reduce the cognitive dissonance of this situation, many economists believe that interactions in markets will correct or offset individually anomalous behaviors. Although the reasons for this belief are often not spelled out clearly, several assertions are often put forward.

For example, one hypothesis states that if deviations from rationality are random, they will more or less cancel out at the aggregate level. Random deviations from rationality do occur in some situations (Bossaerts, Plott and Zame 2003). However, the anomalies reported in the literature – like the biases under uncertainty reported in Kahneman, Slovic and Tversky (1982) – have a systematic pattern. Since these deviations from rationality are not random, they may plausibly affect the market equilibrium.

Another common argument is that even if individuals are irrational at times, they will learn from their mistakes. While market experience can diminish anomalous behavior in some cases (List, 2003), a number of powerful individual anomalies, like the failure to update expectations in a Bayesian manner, are very robust to individual learning in markets (Camerer 1987, 1992; Ganguly, Kagel and Moser, 2000; Kluger and Wyatt, 2003). There is no general reason to believe that markets automatically render individual decisions more rational over time.

A third powerful argument in favor of the irrelevance of individual anomalies at the aggregate level comes from the performance of competitive double auction markets in the laboratory (Smith 1982). There are conditions under which these markets converge to the competitive equilibrium allocation, even in the presence of computerized zero intelligence traders who make random bids and asks subject to a zero-profit budget constraint (Gode and Sunder 1993, Jamal and Sunder 1996). However, systematic mispricing relative to the standard prediction also characterizes important competitive double auctions (Smith, Suchanek and Williams 1998, Fehr and Falk 1999), i.e., there is no reason suggesting that these markets eliminate all individual anomalies at the aggregate level.

It is also often argued that rational agents will drive the irrational agents from the market because the former make higher profits; thus the impact of the rational agents on the aggregate
outcome will increase over time. This argument is not very convincing for labor and consumer goods markets, however. Why, for example, should a consumer or a worker, who exhibits intransitive consumption choices, be driven out of the market? In labor and consumer goods markets there are in general no mechanisms that make information about intransitive behavior readily available (Laibson and Yariv, 2004). A reduction in the quantitative weight of irrational traders is not even guaranteed in financial markets. If, for example, irrational traders take higher risks than do rational traders, the irrational traders may earn higher returns on average which may ensure their long run survival (De Long et al., 1991). In addition, the empirical evidence does not necessarily confirm the claim that professional traders are less prone to behavioral biases than are nonprofessionals. Haigh and List (2005), for example, document that professional traders from the Chicago Board of Trade are more prone to myopic loss aversion than ordinary students.

Finally, marginal buyers and marginal sellers determine the equilibrium in a market with many agents. Even if some irrational participants inhabit the far ends of the supply and demand curves, the actions of more rational individuals might determine the aggregate outcome. In fact, due to the robustness of many anomalies at the individual level, we believe that the identification of general conditions under which the rational types dominate the aggregate outcome and conditions under which the irrational types are decisive for aggregate behavior is an important task for economic research.

In this paper, we report evidence indicating that strategic complementarity and strategic substitutability are important determinants of aggregate outcomes. Under strategic complementarity, a small amount of individual irrationality may lead to large deviations from the aggregate predictions of rational models, whereas a minority of rational agents may suffice to generate aggregate outcomes consistent with the predictions of rational models under strategic substitutability. Thus, the presence of strategic substitutability or complementarity seems to be a key condition in determining when a population that is heterogeneous with regard to rationality reaches either a “rational” or an “irrational” outcome.

Strategic substitutability prevails between the actions of individual $i$ and $j$ if an increase in the action by $i$ generates an incentive for $j$ to decrease his action. Strategic complementarity exists if an increase in $i$’s action causes an incentive for $j$ to also increase his action. For example, if the fact that individual $i$ buys an asset creates incentives for individual $j$ to sell this asset, strategic substitutability prevails. If, instead, $i$’s purchase induces $j$ to buy the asset as well, strategic
complementarity prevails. Price and quantity competition in imperfect product markets represent another example. Price competition often involves strategic complementarity because if other firms lower their prices individual firms often have also an incentive to lower their price. In contrast, quantity (Cournot) competition typically involves strategic substitutability.

(Haltiwanger and Waldman, 1985, 1989) developed models showing that strategic substitutes and complements matter because they affect the impact of a given share of irrational individuals on the aggregate outcome. If, for example, the actions of the rational and irrational individuals are strategic complements, rational individuals have an incentive to partially mimic the behavior of the irrational, and the rational individuals' actions thus magnify the impact of the irrational individuals. But strategic substitutes and complements may even have deeper effects because they may change the frequency of irrational and rational behaviors. If suboptimal individual behavior responds to the cost of the mistake, then substitutability makes more people behave rationally, while complementarity renders their behavior less rational. To provide an intuition for this argument, assume that irrational individuals set a high price. Under complementarity, the rational individuals will then also be induced to opt for a high price. Thus, the distance between the actions of the rational and the irrational is relatively small. In contrast, rational individuals respond with a low price under substitutability, meaning that the distance between the actions of the rational and the irrational is relatively large. If payoff functions are smooth, this means that the cost of an irrational individual's mistake is relatively small under complementarity and relatively large under substitutability. As a consequence, complementarity is likely to generate more individual mistakes than is substitutability.

In the following we will illustrate the relevance of substitutability and complementarity and exemplify other scenarios by taking the reader through several examples. We have chosen two anomalies – money illusion and probability judgment errors – because both anomalies are based on clear data at the individual level and both have also been used in experiments that examine how the anomaly diminishes or increases at the aggregate level.
Limited rationality and strategic complementarity

Strategic complementarity can inflate a small amount of individual irrationality to such an extent that the deviation from the rational prediction at the aggregate level is much larger than at the individual level. To illustrate how this can occur, we consider the anomaly of money illusion which involves a confusion between nominal and real variables. If all wages and prices in an economy change by the same amount, a rational agent’s consumption choices will not change. If, however, the agent’s preferences depend on nominal values or if nominal values influence the perception of the consumption opportunities, then the agent will alter consumption choices and will exhibit money illusion.

Shafir, Tversky and Diamond (1997) provided questionnaire evidence for money illusion at the individual level. Their results indicate that people are prone to money illusion, and that they also expect money illusion to influence other people’s behavior. Beliefs about other people’s money illusion are illustrated by the following scenario that was given to two different groups of subjects:

Consider two individuals, Ann and Barbara, who graduated from the same college a year apart. Upon graduation, both took similar jobs with publishing firms. Ann started with a yearly salary of $30,000. During her first year on the job there was no inflation, and Ann received a 2% ($600) raise in salary in her second year. Barbara also started with a salary of $30,000. During her first year on the job there was 4% inflation, and Barbara received a 5% ($1500) raise in salary in her second year.

Respondents of group 1 were then asked the happiness question: “As Ann and Barbara entered their second year on the job, who do you think was happier?” Even though Ann obviously does better in real terms, only 36 percent thought that Ann was happier while 64 percent believed that Barbara was happier. Respondents of group 2 were asked the following question: “As they entered their second year on the job, each received a job offer from another firm. Who do you think was more likely to leave her present position for another job?” In line with the response to the happiness question, 65 percent believed that Ann, despite doing better in real terms, is more likely to leave the present job.

Beliefs about the money illusion of others also even exist among managers who are routinely involved in wage setting. Agell and Bennmarker (2002) conducted a representative survey of Swedish human resource managers and asked them the following question:
Assume hypothetically, that your enterprise is making a small surplus. There is no inflation and unemployment is high. There are many job seekers applying for a job at your unit. Under these circumstances you decide to propose a wage cut of 5%. How do you think that your employees would find this proposal?

A full 95 percent of the managers believed that this proposal was unacceptable to the workers. However, if the scenario was changed to a situation with 10 percent inflation and a nominal wage increase of 5 percent, only 50 percent of the managers thought that workers would find the proposal unacceptable.

Behavioral effects of money illusion also have been documented in other contexts. Genesove and Mayer (2001) show that condominium owners behave in ways indicating a strong aversion to nominal losses when selling their condominiums. Kooreman, Faber and Hofmans (2004) provide evidence suggesting that money illusion influences charitable donations. Specifically, donations to a large Dutch charity increased by roughly 11 percent in the year after the introduction of the euro (in 2002) for no apparent reason other than the currency change. The exchange rate between the Dutch guilder and the euro was 2.20371. Thus, if the donors wanted to keep their donations at the previous level but applied the rule of thumb that 1 Euro equals roughly 2 guilders, donations would have increased by a bit more than 10 percent. Cohen, Polk and Vuolteenaho (2005) recently provided evidence indicating that the stock market tends to discount cash flows at nominal discount rates, i.e., investors seem to be driven by money illusion. Their results suggest that when inflation was high, the stock market was undervalued because cash flows were discounted with a high nominal discount rate while if inflation was low or negative the stock market was overvalued.

Taken together, these studies suggest that a nonnegligible share of people exhibit money illusion and/or believe that others exhibit money illusion. This raises the question when strategic interactions diminish or inflate the impact of individual money illusion. Fehr and Tyran (2001) studied this question in an experimental price setting game with strategic complementarity and a unique equilibrium in which they ensured that individual-level money illusion was small. However, a small amount of individual level money illusion had large aggregate effects in this setting. In particular, money illusion caused sticky nominal prices and a very slow adjustment to the new equilibrium after a fully anticipated monetary shock. This result contrasts sharply with the standard rational expectations approach which predicts instantaneous price adjustment to a fully anticipated money shock in this setting.
The price setting game in Fehr and Tyran was inspired by the macroeconomic models of monopolistic competition by Akerlof and Yellen (1985) or Blanchard and Kiyotaki (1987). In these models, the reduced form real profit function for firms can be written as \( \pi_i = \pi_i(P_i/\bar{P}, M/\bar{P}) \) where \( \pi_i \) is firm \( i \)'s real profit, \( P_i \) is the nominal price set by firm \( i \), \( \bar{P} \) is the aggregate price level and \( M \) denotes the nominal supply of money. Thus each firm’s real profit is a function of the relative price \( P_i/\bar{P} \) and the real money supply \( M/\bar{P} \). The real money supply \( M/\bar{P} \) is proportional to real aggregate demand in these models. Strategic complementarity is a natural feature of monopolistic competition because a rise in the nominal prices of other firms (i.e., a rise in \( \bar{P} \)) provides, in general, an incentive for each individual firm to also raise its nominal price.

Groups of four subjects play the price setting game in Fehr and Tyran (2001). Each subject is in the role of a firm that is selling a product, and each subject must simultaneously choose a nominal selling price \( P_i \) between 1 and 30. Each subject’s real payoff depends both on the own nominal price \( P_i \) and on the nominal average price set by the other three players in the group (which, for convenience, we also denote by \( \bar{P} \)). The experimenter sets the money supply \( M \), which is exogenous for the subjects. To study the impact of money illusion on price adjustment, the subjects faced a high money supply in the first half of experiment (the pre-shock phase); the money supply was reduced by 2/3 at the beginning of the second half of the experiment (the post-shock phase). Since the subjects choose \( P_i \) simultaneously, they do not yet know the average price the other players set when they make their choices; thus they have to form expectations about \( \bar{P} \). In fact, the main task of the subjects in this experiment is to predict \( \bar{P} \) correctly because – as we show below – it was very easy to choose the best reply to a given expectation about \( \bar{P} \).

Since a subject’s payoff for a given level of the money supply \( M \) only depends on \( P_i \) and \( \bar{P} \), payoffs can be represented in matrix form. Table 1 shows parts of a typical payoff matrix that subjects faced in this game. The two payoff matrices in Table 1 show the nominal payoff a subject earns for different combinations of \( P_i \) and \( \bar{P} \). A subject’s nominal payoff in the experiment is simply given by the multiplication of the real payoff \( \pi_i \) with the average price \( \bar{P} \). The first payoff matrix is based on the pre-shock (high) money supply; the second matrix on the post-shock (low) money supply. Table 1 has several noteworthy features. First, it is very easy to choose a best reply

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1 The full version of Fehr and Tyran (2001), including what appeared in the American Economic Review and also a set of appendices that provide full details of the experiment, including the instructions and payoff matrices given to subjects, is available at [http://www.iew.unizh.ch/wp/iewwp045.pdf](http://www.iew.unizh.ch/wp/iewwp045.pdf).
for a given expectation about $\bar{P}$. The subject simply has to choose the highest number in the column that corresponds to the expected average price. For example, if the subject expects $\bar{P}$ to be equal to 15, the best selling price is given by $P_i = 27$ and the associated nominal payoff is 600.

Second, it is very easy to compute the real payoff for any given level of $\bar{P}$. For example, if $\bar{P} = 15$ and $P_i = 27$ the subject’s real payoff is simply given by the nominal payoff divided by $\bar{P} = 15$ ($600/15 = 40$). Third, the best reply to a given expectation about $\bar{P}$ is the same regardless of whether the payoff matrix shows the nominal payoffs (as in Table 1) or the real payoffs. Since the nominal payoff is just given by $\bar{P} \pi_i$, the highest nominal payoff at a given level of $\bar{P}$ is also always the highest real payoff at that level of $\bar{P}$. Thus, it is equally easy to play a best reply when the payoff matrix shows the nominal payoffs as when it shows the real payoffs; subjects merely have to choose the highest nominal or real number in a given column. Fourth, the payoff matrix exhibits strategic complementarity, i.e., if $\bar{P}$ rises it is generally in the individual subjects' interest to raise their prices as well. This is indicated by the shaded cells which shows a subject’s best reply to $\bar{P}$. Fifth, the price setting game has a unique but asymmetric equilibrium because there are two types of players in each group. Only half the subjects in a group were paid according to the payoff functions displayed in Table 1. The other half faced a different payoff function with the same qualitative features (i.e., strategic complementarity). Heterogeneity was introduced to rule out the “symmetry heuristic” as an equilibrium detection device. Thus, in equilibrium (see circled payoffs in Table 1) $P_i$ and $\bar{P}$ do not coincide. The average equilibrium price across all group members is 18 in the pre-shock phase and 6 in the post-shock phase.

In every period of the game, subjects first simultaneously chose their prices and privately announced an expectation about the average price $\bar{P}$; they received feedback about the actual level of $\bar{P}$ immediately afterwards. They then proceeded to the next period. The money supply was

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2 The subjects’ payoff matrices did not have shaded cells but most subjects marked the best reply with a color marker. Table 1 also shows that the subjects always earn the same real payoff (i.e. 40) when they play a best reply to their expectation. We deviated from monopolistic competition in this regard because we wanted to rule out collusion incentives. Pilot experiments had shown that collusion incentives slow down convergence to equilibrium and we wanted to examine the pure effect of money illusion on price convergence after a fully anticipated money shock.
reduced by 2/3 at the beginning of the second half of the experiment. The shock was implemented by giving subjects new payoff tables based on the reduced money supply.\(^3\) Since the payoff matrices were based on a money-neutral equilibrium, the monetary shock did not affect real payoffs, i.e. all subjects earned the same real payoffs in the pre-shock and the post-shock equilibrium. However, the reduction in the money supply required lowering nominal prices in the post-shock equilibrium. This is illustrated in the displayed parts of the pre and post-shock payoff matrices in Table 1. In the pre-shock phase, the equilibrium choice for the subjects who faced these payoff matrices was \(P_i = 27\) with an associated nominal (real) payoff of 600 (40), while the equilibrium choice is \(P_i = 9\) with a nominal (real) payoff of 200 (40) in the post-shock phase. Thus, the monetary shock shifted subjects’ best reply functions to lower nominal prices but left their real equilibrium payoffs unaffected.

The basic experiment had two treatment conditions. In one, the payoff matrices were expressed in nominal terms; in the second, they were expressed in real terms. If there were no money illusion, subjects’ price choices should be identical across the real and the nominal treatments. However, if subjects choose different nominal prices across treatments, beliefs about money illusion must be present, either because some subjects confuse nominal and real values or because they believe that others will do so.

Subjects' actual behavior is displayed in Figure 1a. The figure shows that in the pre-shock phase (periods -20 to -1) the average price across groups quickly converged to the equilibrium in both treatments. Prices also converged relatively quickly to the equilibrium in the post-shock phase in the real treatment. However, it took a long time for prices to settle close to the post-shock equilibrium in the nominal treatment. The large difference in post-shock price adjustment indicates that money illusion causes strong nominal inertia in the nominal treatment. Some subjects indicated in the post-experimental questionnaire that they took high nominal payoffs as a proxy for high real payoffs or that they expected that other subjects would behave in this way. Note that if subjects behave in this way, they perceive a collusion incentive for remaining at high nominal prices because if all players choose high nominal prices, the nominal payoffs will also be high (see Table 1). For this reason, subjects may have been hesitant in cutting their prices after the shock or they believed that others were hesitant. In fact, the post-shock expectations about the average price \(\bar{P}\)

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\(^3\) Every subject knew the payoff matrices of all players in a group.
are much higher in the nominal compared to the real treatment, suggesting that money illusion affected subjects’ expectations either directly or indirectly because they believed that others are prone to money illusion. Such expectations, in turn, will also induce rational subjects to keep their post-shock prices at the high pre-shock level because of strategic complementarity. Thus, even if very few subjects are prone to money illusion themselves, money illusion can have large aggregate effects on price adjustment because a rational subject’s mere belief that other players keep their post-shock prices high because of money illusion will also induce the rational subject to set a high post-shock price.

Insert Figure 1a and 1b about here

To check whether individual-level money illusion has been amplified because rational subjects expected other group members to choose high post-shock prices, we introduced two other treatments that enabled us to rule out the impact of expectations on post-shock price adjustment. In these treatments a group of four players consisted of one human subject and three computer players that were programmed to respond optimally to the subject’s price choices. The subjects received exactly the same payoff matrices as in the previous treatments and the computers’ optimal actions were also based on these matrices. Subjects knew that they were playing against rational computers in these sessions; in fact, they were even told the computers’ aggregate best replies against their own choices. If the subjects solve this individual optimization problem differently in the real and the nominal treatment, when they know that the computers respond rationally, we have evidence of individual-level money illusion. In addition, it is not possible for expectations about others’ money illusion to affect prices in these treatments. Since we expected little individual-level money illusion and quick price adjustment in these experiments, the game with the computerized opponents was only played for 10 post-shock periods.

The line with the diamond in Figure 1b shows price adjustment in the nominal treatment with computer opponents, the plain line shows adjustment in the corresponding real treatment. Post-shock price adjustment is very quick in Figure 1b, even in the nominal treatment, and the difference between the nominal and real treatments is very small, indicating only a small amount of individual level money illusion. This pattern contrasts sharply with the price differences when human players interact with each other. It suggests that while many players do not suffer from money illusion themselves, they believe that other players have money illusion, creating large aggregate effects. In
fact, the aggregate effects of money illusion caused a decrease in subjects’ income in the treatments with the human players during the periods of disequilibrium of roughly 50 percent.

These results show how beliefs about other players’ irrationality or beliefs about other players’ beliefs about others’ irrationality can lead to large aggregate effects, even though the existing amount of individual-level irrationality is actually quite small. However, the experiment also shows that players do eventually converge to the new equilibrium.

**Limited Rationality and Coordination Failure**

Even if almost all people eventually learn to play the equilibrium, as shown in the previous example, limited rationality may have permanent effects due to its impact on equilibrium selection. A good and a bad equilibrium can exist in certain settings, where the good equilibrium is Pareto-superior to the bad one. However, once individuals are in a bad equilibrium, unilateral deviations are costly so that they can be locked in that equilibrium.

Fehr and Tyran (2004) illustrate this possibility in a price setting game with strategic complementarity that was very similar to that described above. The subjects in this game also simultaneously chose a price in every period and privately announced an expectation about the average price of the other group members. Their payoffs depended only on their own price $P_i$ and the average price of the other players $\bar{P}$; the subjects received feedback about the actual level of $\bar{P}$ at the end of each period. The money supply remained constant in the new game. However, the payoffs were designed so that multiple equilibria prevailed. Equilibrium A is Pareto efficient because every subject earns the highest real payoff in this equilibrium, but nominal payoffs and the average price level are both low in A. At equilibrium C, subjects earn a high nominal payoff, but a low real payoff due to a high average price level. There is also an unstable equilibrium B between these two, with intermediate levels of own price and average price level. Thus, if subjects have to play this game with nominal payoff matrices, seeking high nominal returns and money illusion may cause the players to end up at the Pareto inferior equilibrium C.

Figure 2 shows subjects’ average price choices and their expectations about others’ average prices in two conditions: in the “real” treatment, where subjects are given real payoff information, and in the “nominal” treatment, where they receive nominal payoff information. The figure shows a
striking divergence in behavior and expectations across treatments. Subjects’ choices and expectations quickly converge to the efficient equilibrium A in the real treatment, whereas they never come close to playing this equilibrium in the nominal treatment. Instead they converge slowly but steadily to the inefficient equilibrium C.

This drive towards the inefficient equilibrium is due to the fact that many subjects do exhibit money illusion, at least at the beginning of the game. However, almost all subjects eventually learn to play the efficient equilibrium in a different version of this game where they faced computers playing rationally, although many subjects initially start with money illusion and seek high nominal payoffs. But when playing against humans in a setting of strategic complementarity, the rational players face high costs if they do not follow the irrational players' choices. Several rational players tried to push their groups towards the efficient equilibrium for several periods by choosing very low prices that were not a best reply to the expected price, but setting an exceptionally low price in an environment where the price level is high is a costly strategy, and they eventually gave up. However, the rational players’ attempts to induce the irrational ones to play the efficient equilibrium explain the slower convergence towards equilibrium C in the nominal treatment than toward equilibrium A in the real treatment.

In fact, the irrational players in the nominal treatment may have never even learned that their equilibrium is inefficient. While we know that most players learn this when they have the chance to play against rational computers, it is an open question whether they learn it while playing against other human players who act roughly as they do. Strategic complementarity may well inhibit individual learning processes because there is little behavioral difference between rational and irrational players.

**Aggregate rationality and strategic substitutability**

Clearly, a small amount of individual irrationality can have large aggregate effects through its effect on equilibrium selection. Now we ask the opposite question: is it possible for a small number of rational people to generate a rational aggregate outcome? If so, what are the conditions under which this happens? Haltiwanger and Waldman (1985, 1989) show that a given amount of irrationality causes radically different aggregate patterns depending on whether players’ actions are strategic
complements or strategic substitutes. In particular, the presence of a small number of rational players has a large effect on the speed of adjustment towards equilibrium if substitutability prevails; this is not the case if complementarity prevails.

This finding can be illustrated with a price setting game implemented in Fehr and Tyran (2002) which has similar qualitative features as that described in Fehr and Tyran (2001). In the former, we conducted a *ceteris paribus comparison* of the impact of strategic complementarity and substitutability on aggregate behavior. For this purpose, we created a price setting game with completely identical real payoffs in the substitutability and the complementarity condition, except that the best reply $P_i$ is negatively related to $\bar{P}$ under substitutability, while it is positively related to $\bar{P}$ under complementarity.

As we already know, money illusion may generate sticky expectations and sticky price adjustment after an anticipated money supply shock in such a price setting game. Figure 3 shows the *predicted* price adjustment after the shock under strategic complementarity and substitutability if three of the four players have adaptive price expectations while the other player is fully rational, and all four players choose a best reply to their price expectations. Here, adaptive expectations mean that the last period’s average price is also assumed to be this period’s average price; rationality means that a player fully anticipates the actions of the adaptive players.

If all players were fully rational, they should immediately jump from the pre-shock to the post-shock equilibrium because the money shock is fully anticipated and there is a unique post-shock equilibrium. However, as Figure 3 shows, the economy is predicted to exhibit long-lasting disequilibrium for 11 periods under strategic complementarity in the presence of 75 percent adaptive players, whereas the economy is predicted to return to equilibrium in 2 periods under substitutability. These differences in adjustment patterns are due to the rational players’ different incentives in the two conditions. Rational players have an incentive for overshooting relative to the post-shock equilibrium under substitutability, causing a large decrease in average prices which then induces the adaptive players to play much closer to the new equilibrium in the next period. In contrast, the rational players choose prices similar to the adaptive players under complementarity, resulting in only a gradual price decrease relative to the pre-shock equilibrium. Fehr and Tyran (2002) also show that if all players had adaptive expectations, the differences in the adjustment speed towards the new equilibrium would vanish: full adjustment would then take 12 periods both
under complementarity and under substitutability. Thus, the heterogeneity of players in terms of rationality is key for the differences in adjustment speed.

The experimental results of Fehr and Tyran (2002) show that substitutability indeed causes a much quicker adjustment towards the post-shock equilibrium. In fact, equilibrium adjustment is almost instantaneous under substitutability, implying that the rational expectations hypothesis is a very good predictor of aggregate adjustment behavior in this setting. But if there is heterogeneity among players, no player should play the equilibrium right after the shock: after all, the adaptive players will choose the pre-shock price and the rational players will, therefore, overshoot relative to the new equilibrium.

However, roughly 70 percent of the subjects jump instantaneously to the post-shock equilibrium in the experiments, which suggests that substitutability not only diminishes the impact of a given amount of individual irrationality on aggregate outcomes but also reduces individual irrationality directly. Roughly twice as many subjects under substitutability already exhibit equilibrium expectations in the first post-shock period, suggesting that they know the new equilibrium and believe that the other players will play it. This pattern contrasts sharply with the complementarity condition, where many subjects believe that the other players choose prices close to the pre-shock equilibrium. Thus, the players also seem to attribute more rationality to the other players under substitutability.

A plausible reason for a direct effect of substitutability on individual irrationality can be given in terms of the cost of the expectation error. A rational player has an incentive for following the irrational crowd under complementarity, meaning that the rational players choose actions similar to those of the adaptive players. In contrast, a rational player has an incentive for choosing an action that is very different (“far away”) from that of an adaptive player under substitutability, implying that the payoff gain from playing rationally is relatively large. Therefore, strategic substitutability provides stronger incentives for forming rational expectations. These patterns suggest that the degree of rationality should not be taken as exogenously given. Instead, whether individuals will act rationally should be viewed as a variable that can respond to economic conditions.

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4 The cognitive hierarchy model by Camerer, Ho and Chong (2004) explains the large difference in price adjustment in the first post-shock period across the substitutability and the complementarity condition remarkably well. One reason for this success is that the model explicitly captures the strategic response of more rational players to the predicted behavior of the less rational players.
Strategic Substitutability and Individual Choice Anomalies in Competitive Markets

Do anomalies cause deviations from the predictions of competitive price theory in markets that are known to be very competitive or do they only exert minimum effect on trading prices? To study this question, it is necessary to measure the anomaly at the individual level and to examine how it is aggregated in markets. This has been done in detail with two choice anomalies: the base rate fallacy and the Monty Hall problem (Camerer, 1987, 1992; Ganguly, Kagel and Moser, 2000; Kluger and Wyatt, 2003). Both of these anomalies involve the Bayesian updating of prior probabilities.5

The base rate fallacy involves a neglect of base rates when assessing the probability of a particular event. It can be explained with the help of the following scenario invented by Kahneman and Tversky:

Two cab companies operate in the same city, the Blue and the Green (according to the color of the cab they run). 85 percent of the cabs in the city are Green, and 15 percent are Blue. A cab was involved in a hit-and-run accident at night in which a pedestrian was run over. An eyewitness identified the cab as a Blue cab. The court contested the witness’s ability to distinguish between Blue and Green cabs under nighttime visibility conditions. It found that the witness was correct 80 percent of the time but confused it with the other color 20 percent of the time. What is the probability that the hit-and-run cab was Blue?

The correct answer to this problem can be found by applying Bayes’ Rule, which serves to update a belief in $X$ after observing $M$.6 However, computing probabilities according to Bayes’ rule is complicated. In fact, the median and modal response in this individual decision making task is 0.8 (Camerer, 1995, p. 597). It seems that subjects believe the witnesses’ judgment to be representative of the actual color of the cab, and that they tend to neglect the low base rate that only 15 percent of the cabs are blue – hence the name, base rate fallacy. The calculation according to Bayes’ rule points out that if 100 cabs from this city went past this eyewitness, 85 of them green and 15 of them blue, the 80 percent accurate witness would actually identify 12 of the blue cabs correctly but would

5 Oddly enough, the base rate fallacy was discussed in the first issue of this Journal in Salop (1987), while the Monty Hall problem appeared in the second issue of this journal in Nalebuff (1987).

6 Bayes’ Rule states that $P(X | M) = P(M | X)P(X)/P(M)$. If this formula is applied to the cab-problem above, the correct answer is $P(Blue | identified as Blue) = (0.8)(0.15)/[(0.8)(0.15) + (0.2)(0.85)] = 0.41$. 

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also identify 17 of the green cabs as blue cabs. Therefore, the chance that the witness who claimed to see a blue cab actually saw one is 12/29, or approximately 41 percent.

In a pioneering study, Camerer (1987), and later Ganguly, Kagel and Moser (2000), neatly translated the Bayesian updating problem into a financial market context. Their experiments are particularly interesting for our purposes because the subjects in their experiments first solved the updating problem individually, giving us information about their individual rationality, and then traded a one-period asset in a financial market in which the correct updating was crucial. The market participants could buy and sell the asset in a competitive double auction market. In the experiments of Ganguly et al. (2000), which we present in the following, the asset was framed as a security of a company and paid a dividend of either 500 or 50 francs depending on the actual state of the world, but subjects could not observe directly whether a ‘high’ or a ‘low’ state occurred. Instead, they had to infer this from prior probability information and a signal that came in the form of an analyst’s prediction about whether a ‘high’ or a ‘low’ state prevailed. The asset became worthless at the end of the period, after the dividend payments – that is, the dividend payments in the period in which the asset was traded were the sole determinant of the value of the asset. The subjects knew that the prior probability of a good state was 85 percent in the ‘high’ treatment and 15 percent in the ‘low’ treatment. In addition, subjects were given a signal in the form of an analyst’s prediction of either “success” or “failure” for the company, and they knew that the analyst had an accuracy rate of 80 percent in identifying successful and failing firms. The subjects had to estimate the value of the asset on the basis of this information by updating the prior probability of “success.”

Consider first the ‘low’ treatment, where the prior probability of success is 15 percent, and assume that the analyst predicts that the firm succeeds. This situation exactly corresponds to the previous Cab problem. Thus, traders who follow the base rate fallacy will assume that the analysts prediction is correct 80 percent of the time, while Bayesian traders will recognize that this statement is correct only 41 percent of the time. Therefore, base rate fallacy traders will have a higher valuation of the asset in the ‘low’ treatment. Consider next the ‘high’ treatment with a prior probability of success of 85 percent and assume that the analyst predicts again success. Traders who

7 After a success signal the base rate fallacy traders expect a dividend payment of $500(0.8) + 50(0.2) = 410$ whereas the Bayesian traders’ expected dividend is $500(0.41) + 50(0.59) = 234.5$. After a failure signal the base rate fallacy traders expect a dividend of $500(0.2) + 50(0.8) = 140$ while the Bayesian traders’ expected dividend is $500(0.04) + 50(0.96) = 68$. 

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follow the base rate fallacy will again assume this estimate is correct 80 percent of the time, while Bayesian traders know that if the analyst predicts 100 times, 85 times in a high state and 15 times in a low state, he will correctly predict ‘success’ 68 times (85×0.8) but he will also incorrectly predict success 3 times (15×0.2). Thus, 96 percent (68/71) of the ‘success’ signals will be judged correct by the Bayesian traders. As a result, they have a higher estimate of the value of the security in the ‘high’ treatment.

Each subject is endowed with two units of the asset and with experimental money in the experiment. The amount of money provided to each individual subject is large enough to buy all units available on the market. The experiment lasts for 16 periods; in every period, the subjects first are given the prior probability and the analyst’s signal after which they write down their posterior probability estimate for “success”. Then subjects can trade the asset according to the rules of the double auction. The experimenters provide traders with information about market prices and their own profits but not about the competitors’ profits. Hence, subjects cannot imitate the actions of those traders with higher profits.

The elicited probability of “success” confirms that most subjects did not update in a Bayesian way. Subjects’ individual probability estimates are typically closer to the base rate fallacy prediction; in fact, the median probability estimate is exactly or very close to the base rate fallacy prediction in most periods, indicating that there is a lot of individual judgment error among the subjects. Market experience and interaction thus did not eliminate the base rate fallacy, although some individuals made estimates that were close to the Bayesian prediction. This environment, therefore, enables us to examine whether a small number of rational individuals can drive prices towards the rational level. As the experiment proceeds through 16 periods, do market prices ultimately approximate the base rate fallacy prediction or the Bayesian model?

Before discussing the results, it is useful to view the market experiment through the lens of the substitutability/complementarity framework (assuming that both types of traders are risk neutral). Strategic substitutability applies in the ‘high’ treatment, in the sense that the base rate fallacy traders, who value the asset less than the Bayesian traders do, want to sell the asset while the rational traders have an incentive to buy it. As each trader has enough cash to buy all of the assets, we therefore predict that the rational traders will bid up prices to the rational price. Strategic substitutability also applies in the ‘low’ treatment, but now the rational traders, who value the asset less than the base rate fallacy traders do, want to sell the asset while base rate fallacy traders have
an incentive to buy the asset. If short-selling the asset is possible, the rational traders’ short selling activities will push the price of the asset down to the rational level. However, if short selling the asset is ruled out, the rational traders can only sell the assets they possess, which means that there is a constraint on strategic substitutability: the rational traders would like to sell further assets but cannot. In this case, the base rate fallacy traders are likely to dominate the market price because the rational traders have no means for ensuring that the actual price falls to the rational price.

Ganguly, Kagel and Moser (2000) did not implement a treatment in which they allowed for short selling, but their results are nevertheless very informative. In the ‘low’ treatment where the Bayesian traders value the asset less than the base rate fallacy traders, the latter indeed dominate the market outcome because trading prices are much closer to the base rate fallacy prediction. However, despite the continued prevalence of individual judgment errors, the rational prediction does much better in the “high” treatment when the analyst signals failure – the mean market price is much closer to the rational prediction in this situation, suggesting that the rational traders compensated for the irrationality of the base rate fallacy traders.

The results are more ambiguous in the ‘high’ treatment if the analyst signals “success.” Prices then settle between the Bayesian and the base rate fallacy prediction in such a way that no model is better than the other. However, this result could be due to the fact that when the base rate fallacy traders see a signal of “success,” they assume it is correct 80 percent of the time, while if the rational Bayesian traders see a signal of success, they infer that it is correct 96 percent of the time. These probabilities are relatively close, making it difficult to discriminate between the two models in this case.

Ganguly, Kagel and Moser (2000) do not explicitly examine how many Bayesians it takes to render the market outcome rational. Kluger and Wyatt (2003) provide evidence on this question in a recent paper in the context of the so-called three door anomaly which is inspired by a once-popular TV game-show hosted by Monty Hall. In the stylized version of the game, a subject first chooses between three options called doors. Only one of these doors hides a prize while the other two doors return no payoff. However, the chosen door is not opened immediately. Rather, the experimenter

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8 Short selling could be implemented in the experiment as follows: the experimenter plays the role of a broker who lends the asset to the short seller who sells this asset to another trader in the market. The short seller must both pay the asset’s dividend to the experimenter at the end of the period and also provide the asset to the experimenter at the end of the period. However, since the asset is worthless at the end of the period this latter requirement is trivial. This requirement would not be trivial for multi-period assets.
opens one of the remaining two doors which does not contain the prize. Next, the subject is asked whether he or she would like to remain with the initial choice or switch to the other unopened door.

The rational choice is to switch to the door not initially chosen, since switching doubles the odds of winning the prize from $\frac{1}{3}$ to $\frac{2}{3}$. This follows from an application of Bayes’ Law. At an intuitive level, the argument goes like this: if the subject initially chooses randomly among the three doors, he or she finds the prize with a probability of $\frac{1}{3}$, and does not find the prize with $\frac{2}{3}$. Since the experimenter must open a non-chosen door that does not contain the prize, the strategy of switching will yield the prize whenever the door containing the prize was not chosen in the initial choice (that is, in $\frac{2}{3}$ of the cases). As a consequence, the strategy of sticking with the original choice yields the prize only when the door containing the prize was randomly chosen in the first trial (that is, in $\frac{1}{3}$ of the cases). However, many studies have shown that 70 – 80 percent of subjects stick with their initial choice (Friedman, 1998, Slembeck and Tyran, 2004). The pervasiveness of this anomaly may explain why some Wall Street firms use this anomaly as a screening device for evaluating job candidates (Crack, 2000, pp. 201-3).

Kluger and Wyatt (2003) embedded the Monty Hall problem into an asset market experiment in which assets with and without the right to “switch doors” could be traded. Since the odds of winning the price is twice as high for an asset that confers the right to switch, this asset’s price should be twice that of the asset which does not confer the right to switch. Kluger and Wyatt show that if at least two out of six traders are rational Bayesians, the market typically prices the two assets close to their fundamental value. However, this seems to be a rather rare event which is only observed in 25 percent of the groups. Thus, there is clearly no “market magic” at work. If almost all traders are biased, market outcomes reflect the bias.

**Endogenous complementarity in competitive markets**

The previous examples involve very simple asset markets. However, most assets pay dividends for many periods; indeed, for practical purposes, many assets may be viewed as potentially infinitely lived assets with a random life span. In this case, strategic complementarity between the rational and the irrational traders’ actions may arise because the resale value of the asset becomes a crucial determinant of its return. Thus, whenever there is some momentum in an asset’s price, rational traders have some incentive to mimic the irrational traders’ behavior for some limited time, and
strategic complementarity prevails. For example, if the irrational traders’ buying drives the price of an asset above its fundamental value and the price is expected to rise even further, rational traders who perceive the fundamental value can make positive expected profits by also buying the overvalued asset and selling it just before the upward price trend reverses.

Momentum can build up if a fraction of the traders have biased expectations and naively extrapolate past price changes (“positive feedback traders”). There seems to be compelling evidence that some traders form such biased expectations. For example, Barsky and De Long (1993) suggest that when earnings grow rapidly, investors extrapolate these recent growth rates into the long-term future. Case and Shiller (1996) show that homeowners in cities where house prices have risen sharply in the past expect much greater future price appreciation than homeowners in cities where prices have been stagnant or fallen. Frankel and Froot (1993) document the tendency to extrapolate trends in exchange rate markets. For our purposes, the key point is that the activities of rational traders who anticipate the behavior of positive feedback traders may drive prices away from fundamental value because of strategic complementarity (DeLong et al., 1990b).

A variety of empirical evidence suggests that investors who know that assets are overpriced retain or even buy the overpriced assets and thus ride the bubble. For example, Vissing-Jorgensen (2003) finds in a large survey of U.S. investors that 50 percent thought the stock market was overvalued in the last two years of the boom (1999-2000) but only 25 percent of these investors thought that it would decline. A recent paper by Temin and Voth (2004) provides a case study of a well informed investor – Hoare’s bank – in the famous South Sea bubble. This investor knew that stocks were overvalued and did not face a binding short selling constraint. The bank earned more in two years by riding the bubble than it did in the previous 20 years. Experimental evidence on multi-period asset markets also shows that bubbles can easily arise. Smith, Suchanek and Williams (1988) have shown that bubbles can exist even if every market participant knows the expected future dividend payments of the asset. Traders can buy or sell an asset in a competitive double auction market in their experiments. The asset pays an uncertain dividend at the end of each of 15 periods. The asset can be traded in each period prior to the realization of that period's dividend payment. The expected dividend payments from the remaining periods determine the expected value of the asset. The traders are informed about the asset’s expected value at the end of each period. Since the stake levels in these experiments make it unlikely that risk aversion plays a big role, the subjects' main motivation for trading is their expectations about future price movements. Since the expected value
of the asset is common knowledge, standard theory implies little trading activity, and if trading occurs, prices should be close to the asset’s expected value.

In fact, however, a large volume of trading and vast overpricing is observed in these experiments. Prices typically rise from a level below or close to the expected value to very high levels far above the expected value in intermediate periods before collapsing towards the end of the experiment. Prices in some periods even exceed the maximum possible dividend payments for the remaining periods. Thus, a trader who buys this asset and keeps it until the end is sure to incur losses even if the highest possible dividend payment is realized in every remaining period! Such a trader must either be irrational or believe that others are irrational. Lei, Noussair and Plott (2001) show these markets often have irrational traders, who provide the basis for the rational traders to ride the bubble by buying overpriced assets and selling them before the bubble bursts.

Overpricing in these “bubble experiments” is a surprisingly robust phenomenon (Porter and Smith, 2003). For example, professional stock market traders are as likely to generate the bubble as are student subjects. One reliable method for removing the bubble seems to be conducting the identical experiment with the same group of subjects three times (Smith, Suchanek and Williams, 1988) or creating mixed groups of inexperienced and experienced traders where the latter have just experienced the bubble breakdown twice (Dufwenberg, Lindqvist and Moore, in press). When at least one third of the subjects have experienced the bubble and its breakdown twice, prices revert close to the expected value of the asset in subsequent experiments. However, it would be premature for believers in the efficient market hypothesis to claim victory on this basis. After all, how often does the same group or parts of the same group have the chance of facing identical conditions three times? How often does everyone definitely know the expected value of an asset?

What should we expect if the short selling constraint is removed in Smith-Suchanek-Williams type of experiment? In Ackert, Charupat, Church and Deaves (2002), each of nine traders is endowed with two units of the asset and can short sell up to five units. Short sellers can borrow the asset from the experimenter at zero transaction cost and pay only the realized dividend to the experimenter at the end of each period. They have to give back the then valueless asset to the experimenter at the end of period 15. In these experiments, the possibility of selling short strongly decreases the size and the likelihood of bubbles. In fact, the average trading price over all their sessions comes very close to the expected value if short selling is possible.
There has been a long-standing argument in financial economics between proponents of the “efficient market hypothesis,” which predicts that there should not be any predictable price trends in asset markets because the actions of rational arbitrageurs will keep prices always close to their fundamental values, and recent theories in behavioral finance, which stress the limits to arbitrage (DeLong et al. 1990a, 1990b; Shleifer and Vishny, 1997). This debate is essentially a debate about whether the actions of “smart” traders and “noise” traders are strategic substitutes or complements.

The closer arbitrage comes to the textbook notion of a sequence of exchanges which create a riskless profit, the more plausible it is that strategic substitutability holds and that rational traders will offset the actions of the irrational ones. But in the real world, arbitrageurs face at least three kinds of risk: fundamental risk, noise trader risk, and synchronization risk.

Fundamental risk arises because the fundamental value of an asset may be subject to random shocks. Thus, anyone considering buying an underpriced asset or selling an overpriced asset short must be concerned that the fundamental value may shift while they own the asset. Noise trader risk refers to the idea that an overpriced asset may become even more overpriced or an underpriced asset may become even more underpriced at least in the short and medium runs (DeLong et al., 1990a). Unless arbitrageurs have long time horizons, they face a risk of liquidating their positions at a loss. For example, professional fund managers may have short time horizons because their customers evaluate them on the basis of their short-term performance and relatively poor performance leads to outflow of funds (Shleifer and Vishny, 1997). Synchronization risk arises because individual arbitrageurs become aware of the mispricing at different points in time and each one faces uncertainty about when the other arbitrageurs will react (Abreu and Brunnermeier, 2003). Thus, arbitrageurs do not know when their position will become profitable. When arbitrage is risky and potentially costly for these reasons, smart traders may have strong incentives to mimic noise traders’ behavior, instead of counteracting their activities; strategic complementarity will thus hold.
Concluding remarks

We have discussed empirical evidence indicating that a small amount of individual irrationality can have large aggregate effects under strategic complementarity whereas a small share of rational individuals may generate an aggregate outcome close to the rational prediction under strategic substitutability. Thus, the presumption that individual irrationality does not matter for the aggregate outcome of social interactions can be either true or false, which suggests three general lessons for economists. First, economists should routinely ask themselves which forms of irrational behavior might play a role in the situations they analyze, instead of assuming full rationality of all agents. To do this, they can consult the menu of individual anomalies that have been documented in psychology and experimental economics. Second, we suspect that in many cases economic models should assume that players differ in their degree of rationality (as, e.g., in Camerer, Ho and Chong 2004). It seems rather unlikely that economic forces wipe out all individual irrationality. Third, whether the rational or the irrational players dominate the aggregate outcome depends on the strategic environment. In particular, it is likely to depend on whether strategic substitutability or complementarity prevails.

Previous research has identified a number of economic reasons for strategic complementarity (Cooper 1999). Imperfect product market competition, thick market externalities in the presence of costly search for trading partners, technological externalities, preference externalities and the interdependence between wage and price setting at the macroeconomic level tend to generate strategic complementarities. The existence of non-selfish preferences may also generate important complementarities because they provide incentives for purely selfish agents to behave nonselfishly in order to acquire a reputation as a trustworthy agent (Kreps et al., 1982). Thus, there is little reason to assume that economic forces generically render individual irrationality unimportant. However, in view of the powerful effects of substitutability on aggregate outcomes it would also be important for future research to characterize the economic factors behind strategic substitutability in more detail.
References


Table 1 – Parts of typical payoff matrices in the price setting game
(Best replies are shaded, equilibrium state is circled)

Payoffs are in nominal terms and based on high (pre-shock) money supply

<table>
<thead>
<tr>
<th>Selling price $P_i$</th>
<th>Average price of others ($\bar{P}$)</th>
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<tbody>
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Payoffs are in nominal terms and based on low (post-shock) money supply

<table>
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<th>Selling price $P_i$</th>
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<td>7 18 36 63 89 121 189 284 357 348</td>
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<td></td>
<td>5 13 26 45 62 84 131 210 316 395</td>
</tr>
</tbody>
</table>
Figure 1a – Equilibrium adjustment in treatments with human players

Figure 1b – Equilibrium adjustment in treatments with computer players
Figure 2

Money illusion and coordination failure
Figure 3

Predicted post-shock price adjustment with a minority of rational players in the price setting game of Fehr and Tyran (2002)